century’ will need to be replaced by ‘the twentieth century’. Original quotes, given in English in the text, appear as footnotes in their native language.

This is a detailed, thorough, closely argued text and an enjoyable one. Clearly, those who contributed to modern set theory count among the greatest mathematicians of their time. Those who care for the history of the subject will have a feast here.

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This advanced textbook on infinite cardinals is structured around a particular view of the history of the subject, as the authors explain. As they see it the basic theory is due to Cantor, Bernstein and Tarski, and so runs from 1872 to about 1930. This was how the reviewer first learnt the theory in the sixties because, in the authors’ view, the next spurt came in the seventies with the results about singular cardinals which can be deduced from the Galvin-Hajnal theorem. (Being a singular cardinal says something about the co-finality of the cardinal, but for now let us just remark that the smallest singular cardinal is that aleph in the sequence of alephs with the suffix omega.) The basic theory is set out in a rather long Chapter 1 and the second part is a Chapter 2 of about twenty pages. The remaining seven chapters, rather more than half the book contains an exposition of the investigations of Shelah in the eighties and nineties, when it had become clear that model theory (and logic generally) had an important part to play. It follows from all this that the framework of the exposition is Zermelo-Fraenkel set theory with the axiom of choice added (ZFC).

So what is really at issue here? That finite sets have a cardinality is commonplace. Cantor showed how it was possible to generalise this to infinite ones. But in ZF numbers are sets so the cardinality is a set. And one can calculate with these straightforwardly as far as addition and multiplication are concerned. Next, the power set of a finite set of \( n \) members (the set of all subsets) has cardinality \( 2^n \). Generalising to the infinite case, Cantor showed that the power set of the integers was larger than the original set of integers (as he would put it, \( \aleph_1 = \aleph_0 > \aleph_0 \)). Cantor believed (the continuum hypothesis, CH) that there was no cardinal between \( \aleph_0 \) and \( \aleph_1 \). This was later turned into the generalised continuum hypothesis (GCH): no cardinal between \( c \) and \( 2^c \) for any cardinal \( c \). Gödel showed in the 1940s that if ZFC were a consistent system, then so was ZFC and GCH; there the matter rested for about thirty years. Then Cohen came in from another field and showed that if ZFC were consistent, then so was ZFC and NOT(GCH). In other words, the reader who believes in ZFC can choose to have GCH or not. Now here is the point: if she chooses to accept GCH, then exponentiation is also easy with infinite cardinals. This book becomes necessary only if GCH is not added to ZFC.

The authors have tried hard to make this rather abstract material clear with differing degrees of success in the three parts. The first chapter has plenty of sets of exercises for the reader which are very valuable. These continue into the second chapter where, despite the hints given, they get rather hard. So it goes on, until halfway through chapter 4 the exercises peter out altogether. I would not mind this so much if it were not for the fact that very little wordy explanatory material is given. Apart from a short paragraph at the beginning of each chapter, we have nothing but a succession of definition, lemma, theorem... On the positive side, what the book does indeed provide is a pretty complete survey of all the results on infinite...
cardinals proved up to now in ZFC alone; this is a very valuable thing for postgraduate students in the field. The publisher's hope that it could be aimed at undergraduates would have been over-optimistic half a century ago and must now be quite wrong.

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The stated objective of the authors is to provide an introductory account of the subject of elliptic curves suitable for students of mathematics and physics.

Following an introductory chapter embracing complex manifolds, Riemann surfaces and projective curves the historical development style is used to attain this objective, following closely the work of Abel, Gauss, Jacobi and Legendre. This approach is augmented by substantive recognition of more recent developments, numerous exercises (with hints but no solutions! ... although readers are implored to 'take them as an important part of the discussion and do them faithfully. It will pay off.' ) and an extensive list of references. The assertions that this text requires 'only a first acquaintance with complex function theory' and that 'a first acquaintance with the topology of curves and surfaces and with algebraic number fields, and Galois theory would be helpful, but is not really necessary' are, to say the least, economical with the truth.

The presentation of elliptic integrals and functions is characterised by a historical approach leading from the elliptic integrals to a study of the field \( \mathbb{K} \) of functions of rational character on the complex torus \( X = \mathbb{C}/\mathbb{Z} \) i.e. elliptic functions.

There follows 'an interlude' on theta functions; where the approach of basing the whole story of elliptic curves upon these functions is omitted in favour of developing their connections with the Weierstrassian and Jacobian functions. Extensive references are given to the alternative developments of this material.

The treatment of modular groups and modular functions in chapter four is essentially preparatory material for the final quarter of the text. This latter consists of ikosäder (Klein) and the quintic, telling the complete story of the extraction of roots of the general quintic starting with Ruffini (1799). The text culminates in a study of the arithmetic of elliptic curves per se and a proof of the Mordell-Weil theorem. This paperback (1999) includes corrections to the first edition of 1997 and it certainly achieves (and in the opinion of the reviewer, exceeds) its stated objective. The presentation is at the highest professional level making the material as 'reader-sympathetic' as is possible, given the analytical complexity of many of the results.

This text can be heartily recommended for those readers embarking upon a serious and well motivated quest for an understanding of the origins, evolution and applications of elliptic curves.

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This is a real blockbuster of a reference book. With over seventy contributors and more than 1200 pages, it covers the whole gamut of discrete and combinatorial mathematics: an immensely ambitious undertaking and a significant achievement.