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ABSTRACTS OF AUSTRALASIAN PHD THESES

ON REGULAR SEMIGROUPS, SEMIRINGS AND RINGS

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This thesis is concerned with semirings in which both the additive and multiplicative semigroups are regular.

We first consider the multiplicative semigroup of a ring. We show that a ring, in which the multiplicative semigroup is completely semisimple and satisfies the ascending chain condition on principal two-sided semigroup ideals, is a finite direct sum of matrix rings over division rings.

For any ring $(R, +, \cdot)$, it is shown that if the multiplicative semigroup is orthodox, then the multiplicative semigroup is inverse and hence by Chaptal [*1*], a semilattice of groups.

In an additively inverse semiring, we derive several equivalent conditions, each of which is implied by the multiplicative semigroup being orthodox. In a ring, each of these conditions is shown to be equivalent to the multiplicative semigroup being inverse.

In Chapter Three, we consider additively inverse semirings. Our major result is that an additively inverse division semiring is either a division ring, a lattice ordered group or a lattice ordered group with zero adjoined.

Six conditions which are equivalent for partially ordered groups, are considered for partially ordered inverse semigroups.

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In Chapter Four, semirings in which the additive and/or multiplicative semigroup is simple or 0-simple, are studied.

We first show that if the multiplicative semigroup of a semiring with double zero is primitive inverse, then it is a group with zero adjoined.

A semiring in which the additive semigroup is regular and the multiplicative semigroup is completely [0-] simple, is shown to be either a division ring or a partially ordered completely [0-] simple semigroup.

We then prove that if the multiplicative semigroup of a semiring is a rectangular band, then the additive semigroup is a band.

A semiring in which the additive semigroup is completely simple and the multiplicative semigroup is either completely simple or completely 0-simple, is defined to be *completely simple*. Such a semiring is either a division ring, or has a rectangular band as its additive semigroup; a complete structure theorem is given for the latter case.

We define a semiring to be a *mono-semiring* if its additive and multiplicative semigroups coincide. The semigroup of a mono-semiring is both left and right quasinormal, and any left and right quasinormal semigroup is the semigroup of a mono-semiring.

Since in any semiring, the additive Green's relations are multiplicative congruences, we consider the structure of the additive semigroup of a semiring in which the multiplicative semigroup is either congruence-free or has a unique non-trivial congruence.

In Chapter Five we summarize our results in the form of tables.

Reference

[1] Nicole Chaptal, "Anneaux dont le demi-groupe multiplicatif est inverse", *C.R. Acad. Sci. Paris Ser. A* **262** (1966), 274-277.