A CORRECTION TO MY PAPER "ON THE NON-COMMUTATIVITY OF PONTRJAGIN RINGS MOD 3 OF SOME COMPACT EXCEPTIONAL GROUPS"

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This note is a correction of an error of the author's paper mentioned in the title. (The reference [1]). The proof of the Prop. 6 of [1], Chap. II, p. 247, is an error. And the propositions and formulas in pp. 247-249 of [1] depending on this Prop. 6 must be corrected. All notations are referred to [1].

1. We continue the discussion of [1, p. 246]. The singular planes of $Q$ are partially ordered by the ordering of associated planes in $P$. Give a linear order in $Q$ compatibly with this partial order. Then any subsequence $Q_k$ of length $k$ gives a $2k$-dimensional sub $E_k$-cycle $\Gamma(Q_k)$ of $\Gamma(fP)$. The totality of these $2k$-dimensional $E_k$-cycles forms an additive basis of $H_{2k}(\Gamma(fP) : Z)$. The dual cohomology class of $\Gamma(Q_k)$ is $y_{i_1}^{(f_1)} \cdots y_{i_k}^{(f_k)}$ for $Q_k = \{q_1^{(f_1)}, \ldots, q_k^{(f_k)}\}$ where $\epsilon_s = 0$ if $\mu_s$ is a long root of $F_4$ and $\epsilon_s = 1$ or $2$ if $\mu_s$ is a short root.

Now the Prop. 6, Chap. II of [1], must be corrected as follows:

**Proposition 6.** The $2k$-cycles $f_r \Gamma(P)$ and

$$\sum x_{i_1} \cdots x_{i_k} (\Gamma(P)) \cdot \Gamma(q_i^{(f_1)}, \ldots, q_i^{(f_k)})$$

represent the same class in $H_{2k}(\Gamma(fP) : Z)$, where the summation runs over all subsequences $\{q_i^{(f_1)}, \ldots, q_i^{(f_k)}\}$ of length $k$ of $Q$.

Since $f_r^k(y_{i_1}^{(f_1)} \cdots y_{i_k}^{(f_k)}) = x_{i_1} \cdots x_{i_k}$ by (11) of [1, p. 246], a standard argument proves this proposition immediately. The crucial of the erroneous statement of the Prop. 6 lies in what the author had overseen that the $2k$-cohomology class such as $(x_2)^2x_3 \cdots x_k$ is not necessarily zero in general.

The Prop. 7 of [1, p. 547] should be corrected as follows:

**Proposition 7.** $\Omega f_*(P_e) = \sum x_{i_1} \cdots x_{i_k}(\Gamma(P)) \cdot \{q_i^{(f_1)}, \ldots, q_i^{(f_k)}\}$, where $\Omega f_*$ denotes the homology map induced by $\Omega f$ and the summation is the same as in

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the Prop. 6.

The proof is entirely the same as in the proof of the Prop. 7 of [1, p. 247].

2. The formula (12) of [1, p. 247] is correct as is easily seen from the corrected Prop. 7.

The formulas (12') and (12'') of [1, p. 248] are incorrect. If we compute by making use of the Prop. 4.2 of [2, Chap. III], then we see that the cohomology rings $H^*(\bigcap P_i^*(F_i) : \mathbb{Z})$ and $H^*(\bigcap P_i^*(F_i) : \mathbb{Z})$ have the relations

(*) $x_1^2 = 0, \quad x_2(x_1 + x_2) = 0$

among others, and the cohomology fundamental classes are $x_1 x_2 x_3 x_4 x_5$ for both rings. Then the corrected Prop. 7 and the relations (*) prove the following corrections of the formulas (12') and (12''):

(12') $\varphi f_*(P_{sv}^1(F_i)) = P_{sv}^1(E_8) + P_{sv}^2(E_8) - P_{sv}^3(E_8) + \{(\mu_4, 1), P_4^i\} + \{(\mu_4, 1), P_4^i\}$

(13') $\varphi f_*(P_{sv}^4(F_i)) = \{(\mu_3 - \varphi_3, 1), P_3^i\} + \{(\mu_3 - \varphi_3, 1), P_3^i\} - P_{sv}^3(E_8)$.

The same argument as in pp. 248-249 above the Prop. 8 of [1], with the corrected (12') and (12''), prove the following corrected formulas of (13) and (14):

(13) $\varphi f_*(P_{sv}^1(F_i)) = P_{sv}^1(E_8) + P_{sv}^2(E_8) + P_{sv}^3(E_8),$

(14) $\varphi f_*(P_{sv}^4(F_i)) = P_{sv}^4(E_8)$.

The Prop. 8 of [1, p. 249] is exact and the Prop. 8' is false as is easily seen from the formula (12) and the corrected formulas (13) and (14). We can state the Prop. 8 in a more stronger form as follows:

**Proposition 8''.** The homology map $\varphi f_*$ is injective in $\deg \leq 10$ for any coefficients.

In the discussion in Chap. III of [1] only the Prop. 8 is refered from pp. 247-249 so that no more related corrections are needed.

3. We can prove the above proposition in its most general form.

The diagram of the symmetric space $E_6/F_4$ is of type $A_2$ and all roots have multiplicity 8 ([3], p. 422). The $K$-cycles of [2] describing the additive basis of $H_*(\varphi(E_6/F_4) ; \mathbb{Z})$ are all iterated 8-sphere bundles over 8-spheres,
whence in particular orientable. Then $H_*(\Omega(E_6/F_4); \mathbb{Z})$ has no torsion and $H_i(\Omega(E_6/F_4); \mathbb{Z}) = 0$ if $i \neq 0 \pmod{8}$ by [2].

The spectral sequence associated with the fibration $\Omega(E_6) \rightarrow \Omega(E_6/F_4)$ (fibre $\Omega(F_4)$) is collapsed for any coefficients since odd degree homologies vanish for all three involved homology groups. Hence $\Omega(F_4)$ is totally non homologous zero in $\Omega(E_6)$ for any coefficients, i.e., we obtained the

**Proposition.** The homology map $\partial f_\bullet: H_*(\Omega F_4; G) \rightarrow H_*(\Omega E_6; G)$ is injective in all degrees and for any coefficient group $G$.

A related question will be discussed elsewhere.

**References**


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