SHEAFIFICATION AS A DESIGN TECHNIQUE FOR CREATIVE PRESERVATION – PRINCIPLES, ILLUSTRATIONS, AND FIRST APPLICATIONS

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ABSTRACT

In times of 'grand challenges', design theorists dealing with complex systems are facing a dilemma: grand challenges require rule breaking, but they also require the preservation, as much as possible, of existing resources, systems, know-how and societal values. Design for transition calls not for 'creative destruction', but for 'creative preservation'. How do we model a design process that involves 'creative preservation'?

Today, it is recognized that category/topos theory provides a solid foundation for modelling complex systems and their evolution in design processes. Category theory can account for a design process inside a given 'theory of the object', while topos theory and design theory can account for the phenomena whereby a design process is innovative to preserve the knowledge structure. At the heart of this creative preservation is sheafification.

In this study, we analyse the sheafification process using design theory. First, we characterize sheafification from a design perspective. Next, we propose a very simple illustration involving the sheafification of an ordinal 2 category presheaf. Finally, we show how sheafification can be used to enable 'creative preservation' in specific complex systems.

Keywords: category theory, Innovation, Design theory, creative preservation, Knowledge management

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1 INTRODUCTION: MODELLING CREATIVE PRESERVATION

At the heart of many contemporary design issues is '*creative preservation*'. On the one hand, design issues such as 'transitions' (environmental transition, digital transition…) and 'grand challenges' (mobility for all, renewable energies, decarbonated chemistry…) require disruption and rule breaking, while on the other hand, 'transitions' and 'grand challenges' require the preservation, as much as possible, of existing resources, systems, know-how, societal values and relationships. Design for transition calls for 'creative preservation' rather than Schumpeterian 'creative destruction'. Creative preservation is expected and, in some cases, can be empirically confirmed, but we lack the necessary model to understand how we can logically overcome the dilemma of 'disruption and preservation' and its apparent contradictions: could it be possible to combine fixations and the exploration of the unknown, to retain traditions while changing the rules? How do we model a design process that involves 'creative preservation'?

Previous engineering design studies have explored either routine-based design (within a given knowledge-based system) or innovative design (considered as a creativity-inspired, rule-breaking process). At the intersection of these research streams, a model of a design process for 'creative preservation' has appeared at best as a compromise (keep one part of the heritage, destroy the other) or at worst as a logical contradiction (how can one simultaneously retain what exists and innovate?). However, contemporary approaches based on category theory and topos have opened up new research directions enabling designers to overcome the dilemma and move beyond a compromise.

Today, it is recognized that category theory with a topos perspective provides a solid foundation for modelling complex systems (Breiner et al. 2017; Giesa et al. 2015; Spivak 2013; Short et al. 2022), and their evolution in relation to design processes. For instance, Breiner et al. showed how category theory could account for a (C-K) design process inside a given 'theory of the object'. More recently, Harlé et al relied on topos theory and design theory to propose a design model that accounts for the (counterintuitive) phenomenon whereby the design process is innovative while simultaneously preserving the knowledge structure, indeed is innovative *to* preserve the knowledge structure. They showed that the process at the heart of this model is sheafification (Mac Lane and Moerdijk 1992), a critical process in topos theory that can be likened to 'creative preservation' (Hatchuel et al. 2019). They identified sheafification as the key operation in topos theory that enables 'creative preservation'. In this study, we aim to open up the black box of sheafification and show how it can be used to explain a series of enigmas (detailed later) related to creative preservation.

In this study, we propose an analysis of sheafification. First, we characterize the principles of sheafification from a design perspective. Next, we analyse key design properties, based on a simple case involving the sheafification of an ordinal 2 category. We conclude with some critical properties of sheafification and their implications for 'creative preservation' in complex systems.

2 THE DILEMMA OF CREATIVE PRESERVATION

Grand challenges can lead to grand designs such as positive energy building, low-CO_2 mobility and decarbonated materials, which are both highly challenging and reliant on existing complex engineering systems. They are often perceived as presenting a dilemma. Should we rely on optimization within a knowledge-based system, which might be too slow or inadequate, or on a creativity-inspired rule-breaking process, which might be socially, technically or politically unsustainable.

An alternate path involves being able to create while preserving, that is, innovating within tradition. At first, this seems like an impossible contradiction: how can one both retain tradition and innovate? A compromise between tradition and preservation seems unavoidable, resulting in too limited solutions.

How can one move beyond the dilemma of creative preservation to achieve both intensive creation and complete preservation? It is possible to be relatively optimistic because there are well-known examples:

a) Science is full of creative preservation. Beyond a Kuhnian paradigm shift, there is always a strong effort to make the new paradigm account for the results of the previous paradigm. See the Lorenz equations used by Einstein in his relativity theory, which provide a means of retaining Newtonian physics in the field of relativity (Einstein 2011).

b) Art is also a place of creative preservation. See, for instance, culinary art that has been both preserved and enriched over several centuries (Hatchuel et al. 2019).

We have empirical cases, but we lack a design model to account for the process of creative preservation. In particular, we would like to be able to account for four enigmas associated with creative preservation:

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- The preservation of knowledge contains fixations (Crilly 2015), but creativity involves exploration. Can creative preservation combine fixation with the exploration of the unknown?
- Preservation implies a fixed ontology (Ullah 2020), and yet creation should result in a revision of the identity of objects. How can creative preservation combine a fixed ontology with a revision of the identity of objects?
- A design process should be sufficiently systematic to both delve into the unknown and return to the known (McMahon et al. 2021). How can creative preservation lead to conjunctive expansion?
- Is the process of creative preservation repeatable? Is there a limit? Is it possible that after a series of systematic conjunctive expansions, new expansions become impossible? Intuitively, it seems reasonable that a design 'recipe' might be used several times to obtain a range of outcomes. However, it is also known that some design solutions involved breakthroughs that rendered large sets of design alternatives obsolete (Kokshagina et al. 2013). How does creative preservation (and its underlying creative heritage) support repeated design expansions?

The objective of this study is to contribute to our understanding of how the process of creative preservation addresses these four research questions.

3 METHOD: LEARNING FROM A SHEAFIFICATION MODEL USING AN ENGINEERING DESIGN PERSPECTIVE

We need to model how a knowledge structure is transformed and preserved through a design process. Hence, our method is to apply design theory to a particular knowledge structure in the hope that we can account for both its transformation and its preservation through the design process. We follow a method previously used to study specific design regimes (Hatchuel et al. 2013; Le Masson et al. 2016; Le Masson et al. 2019). We rely on C-K design theory because this theory is largely independent of knowledge structures, and thus is compatible with many models of K. The critical issue is the knowledge structure: what is a relevant 'knowledge model' that enables us to answer our research questions?

There is a long history of knowledge representation. The first tradition developed after Herbert Simon described knowledge as sets of rules. Later, knowledge was described using ontologies or type models. In recent decades, mathematics has assessed the generality, richness and power of topos theory, which introduces very flexible representations of knowledge (Rosiak 2022) and has two main properties from the perspective of engineering design:

1. Topos theory has generative power: it has been demonstrated that it corresponds to models of set theory (cf. (Lawvere 1964; Tierney 1972)). In particular, it was shown that the technique of 'forcing' (Cohen 1963), which is a form of design used in models of set theory, can be generalized in topos theory (Tierney 1972).
2. Topos theory captures universes that present layers of information and are too complex to be described by sets and standard logic. As explained by Prouté, "Set theory is too lax and requires a capacity to name and handle 'types'" (Prouté 2007) Prouté also showed that topos theory was particularly well-suited to this end (to an even greater degree than type theory). This issue with mathematical 'types' is self-evidently true for the objects of art, engineering or architecture. Hence, topos theory is useful for analysing design regimes.

In this study, we examine a particular model of C-K theory, namely, C-K/topos, which is a C-K model in which the K base is a topos structure. This will help us to understand creative preservation. We also analyse this C-K/topos model using a simple case in which the topos is an ordinal 2 category, denoted as Set^{2op}, building on the work of Prouté (Prouté 2016). The topos models the *attribution*, that is, how an attribute is added to a base to form a new entity. For instance, ordinal 2 can represent how, in culinary art, a sauce (base object) is accompanied by a meat (an attribute or a functional group) to form a dish[1]. It can also describe how a car body (base) is attributed with an engine (functional group) to form a car. In each case there is only one constraint, the 'attribution' constraint, whereby the functional group must be associated with a single base. This simple case will help to describe sheafification and the process of creative preservation.

[1] In this illustrative case, we chose to consider the meat as the functional group. However, an alternative representation of a dish could logically be the reverse, with the sauce becoming the functional group of the meat.

In the rest of the paper, we first present sheafification from a design perspective, and then illustrate this creative preservation with sheafification processes using Set^{2op}. Based on this sheafification analysis, it is then possible to elicit some novel properties of creative preservation processes.

4 THE PRINCIPLES OF SHEAFIFICATION FROM A DESIGN PERSPECTIVE

4.1 Topos as a descriptive language for potential artefacts

By definition, a (Grothendieck) topos is a category of sheaves on a site (C, J), where C is a category and J is a Grothendieck topology. We describe step by step the notions of category, presheaves and sheaves step by step and we how it can help us model creative preservation.
1.1 A topos initially comprises a category Cat[2], which represents the basic knowledge that is used to describe any object. It can be seen as a 'geometry' or 'space' in which one 'describes' specific artefacts, that is, the dimensions of the space are the description parameters.
Take, for example, ordinal 2: in culinary art, the ordinal 2 Cat contains the rule that 'any dish is made of meat and sauce and the meat never goes without a sauce', or in automotive engineering, 'any car is made of an engine and a body and the engine never goes without a body'.
1.2 As shown by the above examples, Cat is a language that is used to *describe*. The next step is to be able to speak of particular sets of artefacts that are described using the Cat language. Based on the building category Cat, a topos consists of all applications ζ(Cat), called presheaves on Cat (PSh(Cat)), from Cat to any set of values, for example, the category Set. Presheaves describe properties of sets of objects. Numerous presheaves correspond to any unique category Cat, that is, numerous layers of knowledge that share a reference to Cat, or 'speak' about Cat. Figure 1 shows the structure of a topos on Cat (Kostecki 2011).

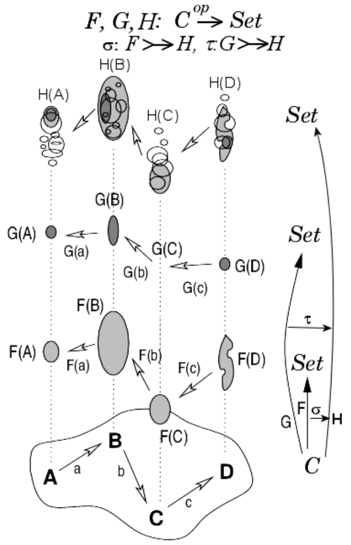

Figure 1.: A category of presheaves. The category Cat has the objects A, B, C and D and the arrows a, b and c, while F, G and H are presheaves of Cat on Set. (Kostecki 2011)

For example, on ordinal 2, the topos is denoted as Set^{2op} (Set to the power of ordinal 2 with a contravariant functor), and using Set^{2op}, we can formulate the following examples: in a culinary Set^{2op}, a presheaf can relate to 'dishes with a red sauce and beef meat' or to 'luxury dishes' or 'diet dishes'. Similarly, in the car example, we can have cars 'with a painted body and a diesel engine' or 'sustainable cars'. It can be seen that in a topos, presheaves describe sets of artefacts with either more or less precision: 'red sauce with beef meat' provides a precise description of the dish, whereas 'luxury dish' leaves the related meat and sauce unknown. Thus, with Set^{Catop}, we have a descriptive language in which all potential 'artefacts' are described. Next, we describe the design process using Set^{Catop}.

[2] Reminder: by definition, a category consists of objects and arrows (or morphisms, from one object to another), and thus there is an identity arrow for each object and arrows are stable by composition. A category is a very general notion, used in many disciplines, in particular engineering science (Breiner et al., 2017, Spivak, 2013).

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4.2 A topos as a design space

To consider a topos as a design space, we need to account for partially *unknown* artefacts (concepts in C-K theory), as opposed to completely known ones (knowledge in C-K theory). What is 'unknown' in a topos? We will now introduce the notions of sheaves, topology and site to have a full view of 'topos'.

Suppose that in Cat, the 'object' U is described by facets (or 'points of view') U_i, and these facets are such that U is the result of the 'points of view' $U_{i,\text{ of }J}$, denoted $U_{i,\text{ of }J} \rightarrow U$ (U is the summit of a 'cone' made up of $U_{i,\text{ of }J}$, see Figure 2), where J is a selection of the facets of U_i that are considered indispensable for characterizing U. Hence, J defines gluing (J is called a Grothendieck topology and follows specific rules, as described below). A presheaf ζ might be defined for some or all facets U_i of a U in a Cat, *but there is no guarantee that the values ζ(Ui) on each facet Ui can be J-glued together to form ζ(U) without contradictions or holes.* If this J-gluing can be done properly, the presheaf is said to be a J-sheaf. In mathematical formulation, a presheaf ζ is a J-sheaf iff ζ sends any cone $(U_i)_{i\text{ of }J}$ of limit U into a co-cone $\zeta(U_i)_{i\text{ of }J}$ of limit ζ(U) (see Figure 2). In that case, we can consider that *the design process is finished*. In the case where a presheaf ζ is not a J-sheaf, there is *some unknown left* to 'well-define' the J-sheaf associated with ζ. This can be a missing facet $\zeta(U_{i^*})$ or an issue in the J-gluing of the facets $\zeta(U_i)_{i\text{ of }J}$. Thus, design is required. These results illustrate why a design process is possible in a topos:

- in a topos, it is possible to account for partially unknown sets of artefacts, which are presheaves that are not J-sheaves
- in a topos, it is also possible to account for 'well-known' sets of artefacts, which are J-sheaves
- in a topos, the design process consists of transforming a presheaf that is not a J-sheaf into a J-sheaf.
- the design space depends not only on Set^{Catop}, but also on the *choice of the topology J in* Set^{Catop}.

Intuitively, if J is a system of facets consisting only of 'small' independent facets with empty intersections, then gluing occurs easily in the juxtaposition of facets. Hence, with such a J (called discrete topology), each presheaf is a J-sheaf and there is no unknown and no design process. If there are 'larger' facets with non-empty intersections, then gluing is less self-evident and there are presheaves that are not J-sheaves. Hence, a design space is more precisely given by the base category Cat and a Grothendieck topology J defined on Cat. The pair (Cat, J) is called a site.

$$\zeta\left(\lim_{\overrightarrow{J}}(U_i)\right) = \lim_{\overleftarrow{\zeta(J)}}(\zeta(U_i))$$

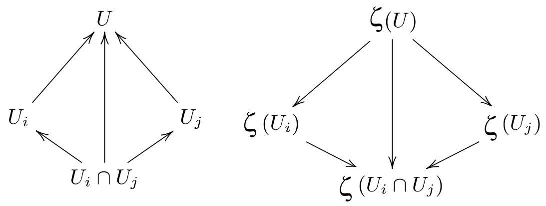

Figure 2. Definition and representation of a presheaf ζ that is a sheaf.

4.3 Designing in a topos: the critical notion of sheafification as creation heritage

How does design occur in a topos? A powerful theorem in topos theory states that in a site (Cat, J), for any presheaf ζ that is not a J-sheaf, it is possible to associate a J-sheaf that contains the presheaf ζ. The mathematical formulation is that for any site (Cat, J), the inclusion functor $Sh_J(Ens^{Catop}) \rightarrow Ens^{Catop}$ has a left adjoint. This is called a *sheafification*.

The key properties of the sheafification process from a design perspective are as follows:

- sheafification transforms the unknown ζ (a presheaf that is not a J-sheaf) into the known (a J-sheaf)
- sheafification preserves the base category Cat, and thus *sheafification is a creative preservation of Cat*
- sheafification depends on the topology J, which *controls the logic of creative preservation*.

Hence, sheafification is a relevant model for creative preservation. Furthermore, it is clear (at least theoretically) that creative preservation depends on a very specific knowledge structure: a site (Cat, J) consisting of a category Cat and its associated topos Set^{Catop}, and a Grothendieck topology J that is itself based on Cat. This site, unless J is discrete (in which case there is no design), can be termed a *creation heritage* because it is preserved in the design process, and yet it enables creation.

In the rest of this paper, we illustrate and analyse the sheafification process in an effort to elicit the properties of creative preservation. How can creative preservation a) combine fixation and the exploration of the unknown, b) combine fixed ontology and the revision of the identity of objects, c) systematically lead to a conjunctive expansion, and d) support repeated design expansions?

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5 STUDYING SHEAFIFICATION ON ORDINAL 2 PRESHEAVES

We examine sheafification using the topos Set^{2op}, which is a topos built on ordinal 2. This relatively simple yet exemplary illustration will help to elicit the basic properties of creative preservation.

5.1 The base category, the topos and the description logic within an ordinal 2 topos

The base category ordinal 2 consists of pairs (B, F) wherein B is a base and F is a functional group, with the morphism *l*. Each F is linked (*l*) to one and only one B (see Figure 3). For example, a culinary dish is given by (sauce, meat), where a meat only comes with one sauce, a car is given by (body, engine), where an engine comes with only one body. The topos consists of presheaves such as 'dishes that are considered luxury' or 'cars that are considered sustainable'.

Following the ordinal 2 base category, describing luxury dishes involves describing the sauces for such dishes, the meats associated with such dishes, and the link that relates each meat to a sauce in such dishes. It can immediately be seen that there is a strange property in relation to the logic of describing the luxury dish, which is known as ternary logic. If we examine the sauce/meat possibilities regarding luxury dishes, the sauce can be non-luxury (sauce = 0), in which case it cannot be used for the dish, and thus the meat that is associated with this sauce cannot be used. Alternatively, the sauce can be luxury, as can the meat associated with this sauce (sauce = 1, meat = 1). However, there is a third possibility, whereby the sauce makes the dish luxury even though the meat associated with this sauce was not luxury (sauce = 1; meat = 0). This ternary logic is created by Cat, the base category. More generally, using the descriptive logic of the presheaf luxury dish, we have described the membership logic in relation to ordinal 2, which can be represented by a specific presheaf inside Set^{2op}, called the sub-object classifier Ω (see Figure 3).

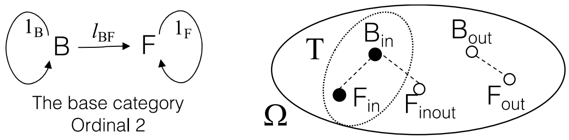

Figure 3. The base category ordinal 2 (left) and its classifier of sub-objects Ω (right). For any topos in Set^{2op}, the base can be in or out and the functional group can be either in when its base is in, out when its base is out, or out when its base is in.

5.2 The Grothendieck topologies and their presheaves that are sheaves

Given that a topology is a way to glue pieces together into a well-formed object, it is closely related to sub-objects, which can be represented by a sub-object classifier to identify a topology by identifying the sub-objects that it can glue together. On ordinal 2, we can show that *there are four (and only four) Grothendieck topologies J*, which can be represented graphically (see Figure 4).

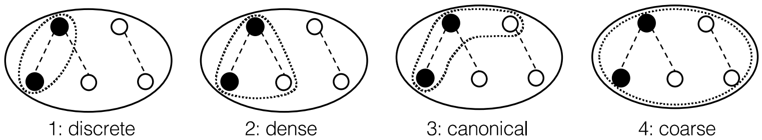

Figure 4. The four Grothendieck topologies on ordinal 2.

We analyse the presheaves that are sheaves and those that are not sheaves for each of these topologies:

- In discrete topology (J=1) one can only glue together B=1, which means B for which ζ (B) is known, and F=1, which means F such that a) F is linked to a B whose ζ(B) is known, and b) ζ(F) is also known. It is self-evident that in this case ζ(F–B) is perfectly known because we only glue pieces that are perfectly known. *Hence, in the discrete topology, every presheaf is a sheaf.*
- The dense topology (J=2) contains cases such as J=1 (B=1, F=1), but can also contain cases where one glues B for which ζ(B) is known and F is linked to B *but for which ζ(F) is unknown*. In the general case, in J=2, presheaves are not necessary sheaves (because ζ(F) can be unknown). However, we can consider presheaves that we call ζ_2, which have one additional rule: in ζ_2, each B has one and only one F. Then, in each $\zeta_{2,}$ B determines F, and thus ζ(F) becomes known.

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Hence, ζ_2 are presheaves that are sheaves in J=2. It can be shown that ζ_2 are the only presheaves that are sheaves in J=2. By contrast, consider two simple presheaves that are not sheaves in J=2: a base B without F, and a base B with two different values of F (see Figure 5).

- The canonical topology (J=3) contains cases such as J=1 (B=1, F=1), but it can also contain cases where we glue B for which ζ(B) is unknown. In the general case, in J=3, presheaves are not necessary sheaves (because ζ(B) is unknown). However, consider presheaves, which we label ζ_3, with one additional rule: there is only one unique B. This B is known, and thus it becomes a case in which ζ(B) is known, as well as ζ(F). Hence, ζ_3 are presheaves that are sheaves in J=3. It can be shown that ζ_3 are the only presheaves that are sheaves in J=3. By contrast, consider one simple presheaf that is not a sheaf in J=3: two different bases (with, or without F).
- The coarse topology (J=4) contains all the previous topologies. It can easily be shown that the only presheaves ζ_4 that are sheaves are those with a unique B for which there is only one F.

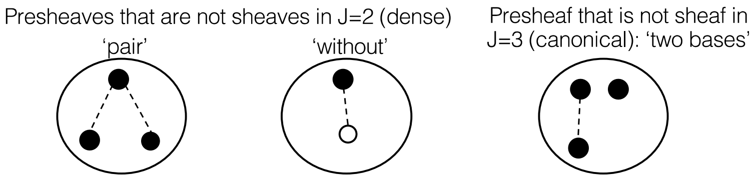

Figure 5: Presheaves that are not sheaves in specific topologies. Left, in dense topology: a dish with one sauce and two meats ('pair'); Centre: a dish with a sauce but without meat ('without'); Right, in canonical topology: a dish with two sauces, one of which has meat.

5.3 Designing a dish without meat: an illustration of the design process for J=dense

Suppose that, using C-K theory, we design the 'dish' category in K-space (an ordinal 2 category where B represents a sauce and F represents a meat, and one meat is paired with one and only one sauce) and the concept is 'a dish without meat' (see Figure 6). We can keep the base category as ordinal 2 and choose a topology to express the concept as a presheaf. Assume that we choose the discrete topology. In this topology, the presheaf expression requires a value for the sauce and a value for the meat. However, the dish is 'without meat', and thus this presheaf is impossible in the topology J=1. It can be considered that under J=1, preservation kills innovation. By contrast, it might be possible to change the base category and consider a simple set composed only of sauces. In that case, innovation kills tradition.

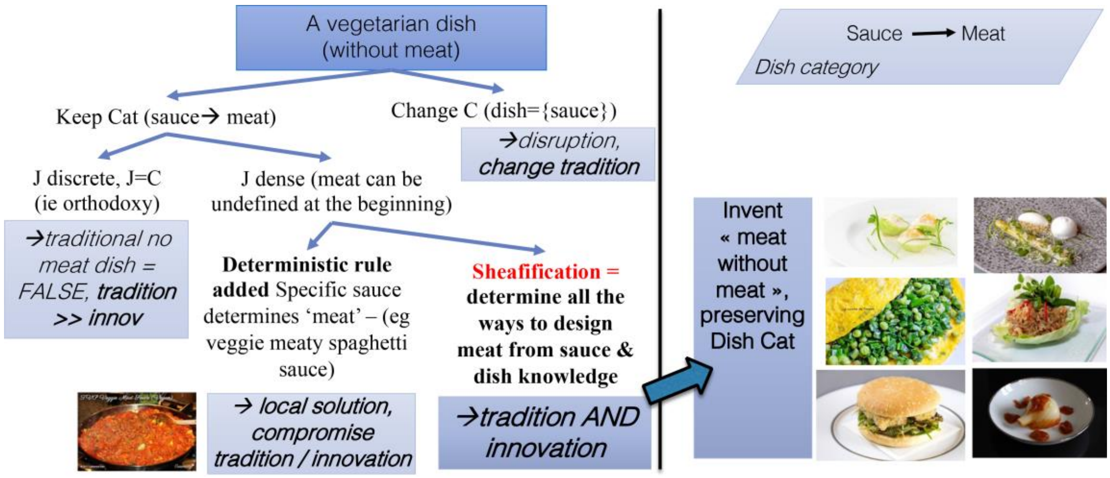

Figure 6: Designing a dish without meat

Suppose we choose the topology J=2. Then, it is possible to express a concept based on the value of the sauce without knowing the value of the meat. Here, the presheaf can be expressed but is not yet a sheaf (because the value of the meat is unknown). Following the above analysis, in an effort to transform the presheaf into a sheaf, it is possible to add the constraint 'each sauce is paired with one and only one meat' and arbitrarily decide that one sauce provides a meaty taste. Then, we can create a 'local' (using a specific sauce) vegetarian dish. This appears as *a local trade-off between tradition and innovation*. Another solution

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is to very generally sheafify the presheaf, which involves determining all the ways in which we can design a 'meaty' dish from the available set of sauces. In that case, we have creative preservation in the sense that the preservation of the base category leads to the (systematic) creation of all possible vegetarian dishes *using the sauce to create a meaty taste without meat*, accepting that the notion of a sauce is extended to include any ingredient that accompanies a meat but is not meat (see Figure 5).

5.4 Sheafification of presheaves in the attributive category

Here, we describe the sheafification of the presheaf 'a dish without meat' in ordinal 2 with J=dense in more depth. Generally speaking, the sheafification of a presheaf corresponds to the construction of the power set of the presheaf and its subsequent completion using the site topology (Prouté 2016). To construct the power set of a presheaf, we need to consider the applications from the presheaf to the sub-object classifier Ω. For the sub-object sauce, we have the applications from sauce to sauce, and for the sub-object meat, we have the applications that are based on the sauce related to the meat that act on the meat in the presheaf. In the presheaf 'without', there is a sauce for 'without meat', and thus there are applications from sauce to sauce (even without meat) for the sub-object 'meat'. *Hence, in the power set we create 'meat' that is based on applications from sauce to sauce, and thus we create 'meat' without meat!* Figure 7 shows how sheafification transforms a J-presheaf (J=dense) that is not a sheaf into a sheaf (because in the sheafified presheaf, there is only one 'meat' for each 'sauce').

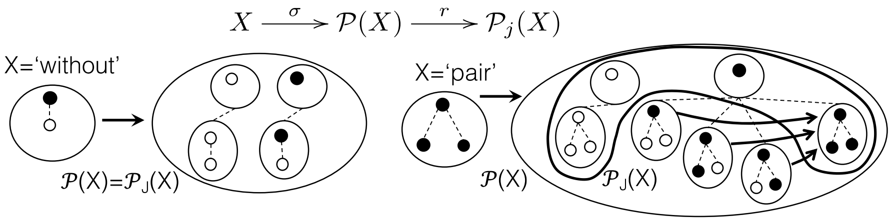

Figure 7: Sheafification of the presheaves 'without' and 'pair' in J=dense. Left: The dish 'without meat' receives a 'meat without meat' which is based on applications among sauces. Right: the dish 'with two meats' receives 'meat' in the form of combinations of two meats.

Similarly, it is possible to sheafify the presheaf 'pair' shown in Figure 3 based on J=2. The power set includes applications based on two meats, that is, hybrids of the two meats. Following sheafification, the sauce is paired with a single new 'meat', which consists of all the combinations of the two initial meats. Again, this is a J=2 sheaf because the sauce has only one (quite complex) meat.

The above example shows how sheafification works for J=2, and can be summarized as follows:

a) it illustrates *how sheafification enables creative preservation: Cat is preserved, and yet new entities are created (e.g., 'meat without meat', 'meat as a combination of meats')*

b) in this process, the topology simultaneously plays two roles:

- *the topology constrains the types of 'unknown' that are acceptable:* in J=2, the sauce is always known, with the unknown being confined to the meat
- *the topology enables the creation of new entities* because it guides us towards the generation of an entity that respects the rule regarding being a sheaf in the topos (e.g., each sauce must have one and only one meat).

c) generally speaking, sheafifying on the 'attributive category' with J=dense enables us to explore the many ways in which we can create a new 'functional group' F when the base B is known, either in the 'without' situation, where the base is used to replace the missing F, that is, a 'functionalization' of the base, or the situation involving the exploration of the possible combinations of two functions with the base, that is, a 'functional hybridization' of the base.

In J=3, it is possible to sheafify the 'two bases' presheaf (see Figure 8).

We confirm the above properties as follows:

a) there is creative preservation: Cat is preserved, and yet new entities are created

b) the topology constrains the unknown: in J=canonical, the bases can be unknown but the sauces of known bases are known. The topology also leads the design process: in the end, there is only one sauce.

c) we explore the ways in which we can design new functional groups when the base is unknown. In that case, sheafification leads to the creation of a known base and the preparation of functional groups for all paired combinations, that is, new functional groups are adapted to the unknown base.

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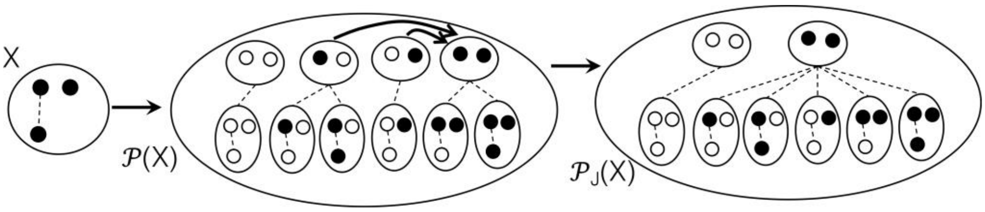

Figure 8: Sheafification of the 'two bases' presheaf of J=canonical. A dish with one 'sauce' (a pair of sauces) emerges whose 'meats' are meat–sauce combinations.

5.5 Synthesis: the design paths in ordinal 2

It is now possible to characterize a design process in ordinal 2. Here, we rely on C-K design theory using the category ordinal 2 in K (i.e., base B and functional group F where each F has one and only one B). In K, this category has four related Grothendieck topologies. If we want to design a new sheaf defined by ordinal 2, by the definition of a sheaf, it requires a specific category, hence the four different partitions (see Figure 9). Following the above analysis, it is clear that for J=1, the expression of a presheaf requires ζ(B) and ζ(F), and thus the presheaf is immediately a sheaf. For the topology J=2, the expression of a presheaf requires us to specify ζ(B) but allows us to leave ζ(F) unknown. There are two possibilities: either we add the deterministic rule 'there is one and only one B for each F' to the presheaf so that the presheaf is a sheaf, or we do not add the deterministic rule. In the latter case, the presheaf is not a sheaf, and thus sheafification is required. Following the same reasoning, it is possible to represent all the different design paths in ordinal 2.

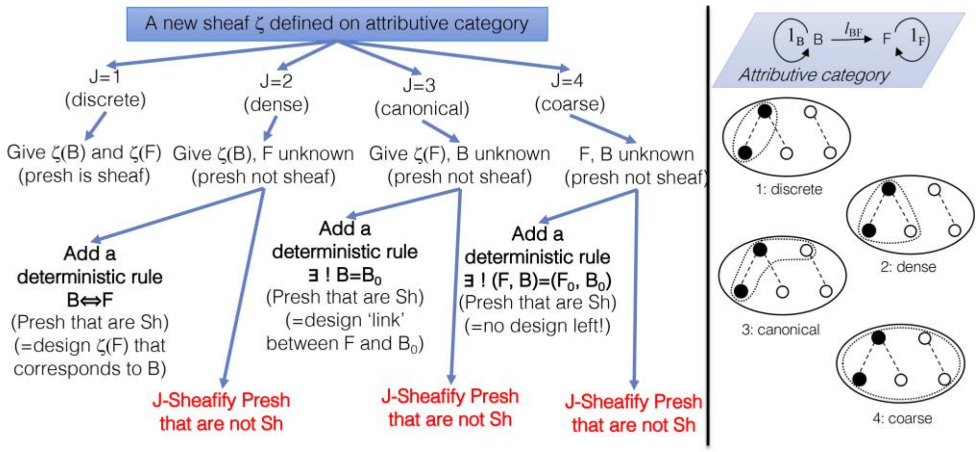

Figure 9: Design paths to create new sheaves in ordinal 2. The design process depends on the choice of the Grothendieck topology.

6 ELICITING THE PROPERTIES OF CREATIVE PRESERVATION

The description of sheafification in ordinal 2 elicits the properties of creative preservation as follows:

a) Fixation and exploration of the unknown: all possible fixations can be represented by Cat and its sub-object classifier Ω. Each J selects some constraints from among Ω, retained as fixation. It also accepts some unknowns in relation to some facets of Cat, and thus it enables exploration. Hence, sheafification uses the law of fixation Ω to guide and structure a defixated exploration of the unknown.

b) This process enables us to retain the Cat (fixed ontology) while also creating new identities for some parts of the category (e.g., new functional groups or new bases). The revision of identity occurs *inside* the fixed ontology, and the fixed ontology (via the topology) helps us to design the new identity. Hence, the Schumpeterian conjecture, which claims that innovation is creative destruction (Schumpeter 1939) is dismissed: there can be innovation enabling creation while preserving a fixed ontology.

c) There is a clear systematic process of sheafification: as soon as there is a topology that enables presheaves that are not sheaves, sheafification (operating by completion of the power set of the presheaf) will systematically create a new sheaf that not only contains but also extends the presheaf.

d) Finally, we have seen that for one Cat, there are multiple, nested design processes, many of which involve creative preservation. In particular, a category can contain several different topologies, each of which (except the discrete topology) enables a different type of creative preservation.

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These theoretical results enable us to dispel a false received wisdom: in C-K/topos theory, it is possible to achieve creative preservation, which overcomes the either-or dilemma: there is both fixation and exploration, and there is both a fixed ontology and a revision of identity because the base category is both creative and systematic. Finally, in any given category, there are multiple nested processes enabling creative preservation.
Hence, in situations where there is an apparently contradictory need for both innovation and preservation, C-K/topos theory enables us to confirm that this is not impossible, and also provides a conceptual model enabling us to develop actionable methods and processes to support creative preservation in the face of grand challenges.

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REFERENCES

Breiner S, Subrahmanian E, Jones A (2017) Categorical foundations for system engineering. In: Madni AM, Boehm B, Erwin DA, Ghanem R, Wheaton MJ (eds) *5th Annual Conference on Systems Engineering Research (CSER)*, 2017.

Cohen P (1963) The independence of the Continuum Hypothesis. *Proceedings of the National Academy of Science* 50:1143-1148.

Crilly N (2015) Fixation and creativity in concept development: The attitudes and practices of expert designers. *Design Studies* 38:54-91.

Einstein A (2011) *Letters to Solovine (1906-1955)*.

Giesa T, Jagadeesan R, Spivak DI, Buehler MJ (2015) Matriarch: A Python Library for Materials Architecture. *ACS Biomaterials Science & Engineering* 1 (10):1009-1015.

Hatchuel A, Le Masson P, Weil B, Carvajal-Perez D (2019) Innovative Design Within Tradition - Injecting Topos Structures in C-K Theory to Model Culinary Creation Heritage (reviewers'favourite award). *Proceedings of the Design Society: International Conference on Engineering Design* 1 (1):1543-1552.

Hatchuel A, Weil B, Le Masson P (2013) Towards an ontology of design: lessons from C-K Design theory and Forcing. *Research in Engineering Design* 24 (2):147-163.

Kokshagina O, Le Masson P, Weil B (2013) How design theories enable the design of generic technologies: notion of generic concepts and Genericity building operators Paper presented at the *International Conference on Engineering Design, ICED'13*, Séoul, Korea,

Kostecki RP (2011) *An Introduction to Topos Theory*. Institute of Theoretical Physics, Warsaw

Lawvere FW (1964) An elementary Theory of the Category of sets. *Proceedings of the National Academy of Sciences* 52 (6):1506-1511.

Le Masson P, El Qaoumi K, Hatchuel A, Weil B (2019) A Law of Functional Expansion - Eliciting the Dynamics of Consumer Goods Innovation with Design Theory (reviewers'favourite award). *Proceedings of the Design Society: International Conference on Engineering Design* 1 (1):1015-1024.

Le Masson P, Hatchuel A, Kokshagina O, Weil B (2016) Designing Techniques for Systemic Impact - Lessons from C-K theory and Matroid Structures. *Research in Engineering Design* 28 (3):275-298.

Mac Lane S, Moerdijk I (1992) *Sheaves in Geometry and Logic. A First Introduction to Topos Theory*. Universitext. Springer, New York

McMahon CA, Eckert CM, Fadel G (2021) Towards a Debate on the Positioning of Engineering Design. *Proceedings of the Design Society* 1 (160011.png):3169-3178.

Prouté A (2007) Sur quelques liens entre théorie des topos et théorie de la démonstration. Paper presented at the *Conférence faite à l'Université de la Méditerrannée le 29 mai* 2007,

Prouté A (2016) *Introduction à la logique catégorique*. CNRS / Univesrité Paris Didrerot, Paris

Rosiak D (2022) *Sheaf Theory Through Examples*. MIT Press, Boston, MA

Schumpeter JA (1939) *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*. first abridged edition 1964, first edition 1939 edn. McGraw Hill Book Company, New York and London

Short R, Hylton A, Cleveland J, Moy M, Cardona R, Green R, Curry J, Mallery B, Bainbridge G, Memon Z (2022) Sheaf Theoretic Models for Routing in Delay Tolerant Networks. In: 2022 *IEEE Aerospace Conference (AERO)*, 5-12 March 2022 2022. pp 1-19. https://dx.doi.org/10.1109/AERO53065.2022.9843504

Spivak DI (2013) *Category Theory for Scientists*.

Tierney M (1972) Sheaf theory and the continuum hypothesis. In: Lawvere FW (ed) *Toposes, algebraic geometry and logic*. Springer Verlag, Heidelberg, pp 13-42

Ullah AS (2020) What is knowledge in Industry 4.0? *Engineering Reports* n/a (n/a):e12217.