The Factorisation of $1 - 2x^n\cos\alpha + x^{2n}$.

By Professor JACK.

Let $S = \sin\theta + x\sin 2\theta + x^2\sin 3\theta + x^3\sin 4\theta +$ *ad infinitum*

multiply by $2x^n\cos n\theta$

$$\therefore\ S.2x^n\cos n\theta = x^n(\sin\overline{n+1}\theta - \sin\overline{n-1}\theta) + x^{n+1}(\sin\overline{n+2}\theta - \sin\overline{n-2}\theta)$$
$$+ x^{n+2}(\sin\overline{n+3}\theta - \sin\overline{n-3}\theta)$$

$$\therefore\ S.2x^n\cos n\theta = S - (\sin\theta + x\sin 2\theta + \ .\ .\ .\ + x^{n-1}\sin n\theta)$$
$$+ S.x^{2n} - (x^n\sin\overline{n-1}\theta + x^{n+1}\sin\overline{n-2}\theta + \ .\ + x^{2n-2}\sin\theta)$$

Transpose, etc.

$$\therefore\ (1 - 2x^n\cos n\theta + x^{2n})S =$$
$$\left\{\begin{matrix}\sin\theta + x\sin 2\theta + \ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ + x^{n-1}\sin n\theta \\ + x^n\sin\overline{n-1}\theta + x^{n+1}\sin\overline{n-2}\theta + \ .\ .\ .\ + x^{2n-2}\sin\ \theta\end{matrix}\right\}$$

Let $n = 1$

$\therefore\ (1 - 2x\ \cos\ \theta + x^2\ )S = \sin\theta$ (Bracket reduces to one term here.)

DIVIDE.

$$\therefore\ \frac{1 - 2x^n\cos n\theta + x^{2n}}{1 - 2x\ \cos\ \theta + x^2} = \frac{\{\sin\theta + x\sin 2\theta + \ .\ .\ + x^{2n-1}\sin 2\theta + x^{2n-2}\sin\theta\}}{\sin\theta.}$$

and when $\cos n\theta$ is given there are n values only of $\cos\theta$
$\therefore$ there are n quadratic factors similar to the above.

The C-Discriminant as an Envelope.

By Mr JAS. A. MACDONALD.

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