THE THEORY OF EXTENDED AND EXPANDING ATMOSPHERES

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Abstract. (A) The possibilities and the difficulties of a theoretical study of extended atmospheres in binaries are briefly discussed.

(B) We try to summarize and discuss critically the present status of the theory of three types of *extended* atmospheres (i.e. atmospheres in which the average photon mean-free-path is the same order of magnitude or larger than the stellar radius):

(1) Extended atmospheres in hydrostatic and in grey or non-grey radiative equilibrium.

(2) Dynamic (expanding) atmospheres which occur if the radiative acceleration is slightly smaller than the acceleration of gravity.

(3) Stellar coronae which are formed in the presence of a mechanical energy flux.

In (1) we study the importance of the 'forward peaking' of the radiation field in the outer layers of the atmosphere. The possibilities for the solution of the non-grey transfer problem in an extended atmosphere are discussed.

In (2) we pay special attention to Marlborough's and Roy's (1970) result that the atmospheric gas cannot be accelerated directly to supersonic velocities by the action of the radiation force.

In (3) the large differences in the coronal properties of stars of different chemical composition are emphasized. We draw attention to the partially unexplored but probably very interesting properties of coronae of helium-rich stars.

1. Introduction

There seems to be no general agreement among theoreticians what they call an 'extended atmosphere'. The definition of this topic becomes even more arbitrary if we exclude problems of 'Outflow of Matter' and 'Expanding Envelopes' which have been treated already in other papers of this symposium. So, I have to beg your pardon for starting out by giving a somewhat subjective definition of my topic.

As I understand it, we want to have a survey of the theory of stellar atmospheres for cases in which the atmosphere can no longer be considered as thin in comparison to the stellar radius. In such a case obviously we have to take into account the change of gravity with radius (which is rather trivial) and we have to solve the problem of radiative transfer for (at least) the case of spherical symmetry and possibly (in the binary case) for more complicated geometries. This increases the difficulty of the problem very considerably. Moreover, during the last few years (Bisnovatyi-Kogan and Zel'dovich, 1968; Kutter *et al.*, 1969; Schmid-Burgk, 1969; Finzi and Wolf, 1971; Cassinelli and Castor, 1972) it has become increasingly clear that once an atmosphere is extended in the above sense we can rather easily have a situation in which hydrostatic equilibrium no longer holds and we get a stationary expansion of the atmosphere. This statement seems to agree rather well with many observational facts. Consequently, we are forced to study not only static spherically symmetric atmospheres but also dynamic ones.

Let us look (from a naive theoretical point of view) at the possible causes for the formation of extended atmosphere. Let us first consider the case in which radiative

Batten (ed.), Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems, 148–170.

acceleration is *not* important. Then the local pressure scale height H must be of the same order of magnitude or larger than the radius:

$$\frac{RT}{\mu g} \gtrsim r_0 ; \; 8.3 \times 10^7 \frac{T}{g} \gtrsim r_0 , \tag{1a}$$

or

$$\frac{1}{g} \left\{ \frac{RT}{\mu} + \frac{\xi^2}{2} \right\} \gtrsim r_0 , \tag{1b}$$

at least somewhere in the atmosphere (R = gas constant, μ = mean molecular weight, g = local acceleration of gravity, ξ = turbulent velocity). As is well known there are very few normal atmospheres of nonrotating single stars in which condition (1) is fulfilled. Only if the effective value of g is strongly reduced by the presence of a strong centrifugal force in a fast rotating star or by the presence of a neighboring star in a close binary can the condition (1) be fulfilled easily for an atmosphere with radiative energy transport. However, (1) can be relatively easily fulfilled for chromospheres and coronae which have a high temperature due to the heating by a mechanical energy flux. (Cf. Kuperus, 1965, 1966; Ulmschneider, 1967; Nariai, 1969; de Loore, 1970; Böhm and Cassinelli, 1971; for a summary of the semiempirical aspects see Praderie, 1970.)

As pointed out by Underhill (1949, 1966) and Mihalas (1969) for normal early type stars and discussed by Böhm (1969) and Cassinelli (1970) for central stars of planetary nebulae, the theory predicts extended atmospheres for hot stars in which the radiative acceleration

$$|g_{\text{rad}}| = \frac{\pi}{c} \int_0^\infty (\kappa_\nu + \sigma_{\text{el}}) F_\nu d\nu \tag{2}$$

becomes comparable to the local gravity. (κ_ν = monochromatic absorption coefficient, σ_{el} = Thomson scattering coefficient, F_ν = monochromatic radiative flux).

It turns out that typically the atmospheres become extended if

$$|g_{\text{rad}}| \approx \frac{\sigma T_{\text{eff}}^4}{c} \bar{\kappa} \gtrsim 0.8 \, g . \tag{3}$$

in the case of central stars of planetaries.

Moreover, if g_{rad} approaches g too closely hydrostatic equilibrium is not even approximately possible (Cassinelli and Castor, 1972). It is believed that a considerable number of stars exist which fall into the range defined by (3).

According to this discussion it seems reasonable to review the theory of the following atmospheres:

(1) Hydrostatic atmospheres in radiative equilibrium which are extended because of the validity of condition (1) or (3);

(2) Dynamic atmospheres (i.e. atmospheres with continuous mass loss in which g_{rad} is important);

(3) Hydrostatic and hydrodynamic chromospheres and coronae which are due to the dissipation of a mechanical energy flux.

As far as I can see this covers the types of extended atmospheres which are at least partially understood from a theoretical point of view. Unfortunately it probably does not include the interpretation of those extended atmospheres in binaries for which we have the most detailed observational material like ζ Aur, 31 Cyg and 32 Cyg (Cf. Groth, 1957, 1970). This is, of course, sad. On the other hand, I think we will not be able to develop convincing theories of these rather complicated cases unless we understand first the theory of certain basic effects which occur in extended atmospheres.

Before we discuss points (1) to (3) in detail we have to ask the following question: To which extent do we have to change our discussion if we talk about extended atmospheres in reasonable close binaries.

In this case obviously the following effects are important (cf. the interesting and detailed discussion by Kopal, 1959):

(1) Gravity darkening as a consequence of stellar rotation as well as the deformation of the star by the gravitational field of the other component.

(2) The generalized 'reflection effect', i.e. the effect of the radiation of the other component on the atmosphere.

It is well known that these two effects do not introduce new basic difficulties if the atmosphere can be considered as 'thin', i.e. plane-parallel. (However, the required amount of computational work is increased very considerably.)

The influence of gravity darkening on the predicted spectra of *single* rotating stars has been studied successfully in recent years by many authors, e.g. Collins (1963), Roxburgh and Strittmatter (1965), Collins and Harrington (1966), Hardop and Strittmatter (1968a, b), Collins (1968a, b), Collins (1970), and others. These calculations are based on the (well-justified) assumption that the atmospheric structure can be calculated everywhere using the local value of g and the local value of T_{eff} as it follows from the gravity darkening law. Since data on gravity darkening in close binaries (at least for some cases of the Roche model) and on the reflection effect are available (Cf. Kopal, 1959; Minin, 1965; Peraiah, 1969; Rucinski, 1969, 1971) analogous calculations have been carried out recently for binaries by Buerger (1969). He has used grey, plane-parallel atmospheres defined by local values of g and T_{eff}. Gravity darkening as well as the reflection effect have been taken into account.

However, in this symposium we are concerned with extended atmospheres. A quantitative treatment of gravity darkening and of the reflection effect in these atmospheres would be much more difficult for these atmospheres and has – to the best of my knowledge – not yet been tried. Obviously the lateral radiative exchange must become very important in this case and a description using local values of g and T_{eff} becomes impossible. Since even the simple problems of extended atmospheres in single stars or in wide binaries (as formulated above) are far from being solved completely, we may doubt whether it will be possible to solve the problem of an extended atmosphere in close binaries in the near future.

2. Extended Atmospheres in Hydrostatic and Radiative Equilibrium

In this chapter we shall assume that (at least some) extended stellar atmospheres can exist in hydrostatic and radiative equilibrium. The interesting question concerning the validity of this assumption will be discussed briefly in the next chapter.

The hydrostatic equation can be written (neglecting turbulence)

$$\frac{dp}{d\tau_0} = \frac{GM}{r^2\kappa_0} - \frac{\pi}{c}\frac{\int_0^\infty (\kappa_\nu + \sigma_{el}) F_\nu d\nu}{\kappa_0}, \tag{1a}$$

or

$$\frac{dp}{dr} = -\frac{GM}{r^2}\varrho + \frac{\kappa_F \varrho L}{4\pi r^2 c} = -\frac{GM\varrho}{r^2}\left\{1 - \kappa_F \frac{L}{4\pi cGM}\right\}, \tag{1b}$$

with p=gas pressure, ϱ=density, M=total mass of the star, κ_ν=monochromatic absorption coefficient, σ_{el}=electron scattering coefficient, F_ν=monochromatic radiative flux, L=total luminosity, r=radial coordinate (distance from the center of the star, κ_0 and τ_0 are the absorption coefficient and the optical depth for some standard frequency. The flux mean κ_F of the absorption coefficient is defined as

$$\kappa_F = \int_0^\infty (\kappa_\nu + \sigma_{el}) F_\nu d\nu \Big/ \int_0^\infty F_\nu d\nu. \tag{2}$$

Obviously the first term in (1b) is the usual gravitational force (per cm^3 of matter), whereas the second term is the radiative force on the same amount of material. The ratio of these two forces is independent of r if κ_F=const. Equation (1b) also shows that a hydrostatic solution is impossible if

$$L > \frac{4\pi cGM}{\kappa_F}. \tag{3}$$

Radiative equibilbrium is described by equations of transfer for every frequency of the form

$$\left\{\mu \frac{\partial}{\partial r} + \frac{(1-\mu^2)}{r}\frac{\partial}{\partial \mu}\right\} I_\zeta(r,\mu) = -\kappa_\nu \varrho \{I_\nu(r,\mu) - S_\nu(r)\}, \tag{4}$$

and the radiative equilibrium condition which may be written

$$L_r = \left(\int_0^\infty L_\nu d_\nu\right)_r = 4\pi r^2 \int_0^\infty F_\nu d\nu = \text{const}. \tag{5}$$

The atmosphere will be really extended if

$$\frac{1}{\bar{\kappa}\varrho} \gtrsim r, \tag{6}$$

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whereas the problem reduces to the plane-parallel case if

$$\frac{1}{\bar{\kappa}\varrho} \ll r\,, \tag{7}$$

(see Chapman, 1964; Cassinelli, 1970). It is important to note that in contradistinction to the plane case neither the hydrostatic Equation (1b) nor the transfer Equation (6) can be transformed into an equation for the independent variable τ_ν only. In other words, though we can e.g. determine the temperature stratification (apart from a scaling factor) of a grey atmosphere once and for all in the plane-parallel case by computing $T(\bar{\tau})$ (Hopf, 1934; Mark, 1947) this is not true for an extended atmosphere. In this case the temperature stratification also depends on the relation between $\bar{\tau}$ and r:

$$\bar{\tau} = \bar{\tau}(r)\,. \tag{8}$$

(Another formulation of the same statement says that the T-stratification depends on the ratio between the radius of curvature and the photon mean path.)

Additional insight into the problem may be gained by writing down the moment equations of the equation of transfer (4). They are found in the usual way by applying the operators:

$$\tfrac{1}{2}\int_{-1}^{+1} \ldots \, d\mu\,, \tag{9a}$$

and

$$\tfrac{1}{2}\int_{-1}^{+1} \ldots \, \mu d\mu \tag{9b}$$

to Equation (4). We find

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 H_\nu\right) = -\kappa_\nu \varrho \left(J_\nu - S_\nu\right), \tag{10}$$

by applying (9a) and

$$\frac{\mathrm{d}K_\nu}{\mathrm{d}r} + \frac{1}{r}\left(3K_\nu - J_\nu\right) = -\kappa_\nu \varrho H_\nu\,. \tag{11}$$

Obviously (10) and (11) can be specialized to the grey case by leaving out the subscript ν and by setting $J = S$. It follows immediately that

$$r^2 H = 4r^2 F = \text{const}\,, \tag{12}$$

(12) is of course intuitively obvious. Many astronomers tend to consider (12) as *the* important condition which distinguishes an extended from a plane-parallel atmo-

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sphere. This approach has led to the use of the Milne-Eddington approximation

$$K_\nu = \tfrac{1}{3}J_\nu, \tag{13}$$

in many of the earlier papers in this field (Cf. Kosirev, 1934; Chandresekhar, 1934, 1945, 1960; see also Pearce, 1967). When (13) is used, (11) reduces to the same equation as in the plane-parallel case. However, in the more modern literature (cf. Chapman, 1964, 1966; Schmid-Burgk, 1970; Cassinelli, 1970, 1971; Hummer and Rybicki, 1971) it has been shown that (at least in very extended atmospheres) (13) is not an acceptable approximation. As is well known the validity of (13) implies that the radiation field is isotropic. On the other hand the 'real star' (defined e.g. by the surface $\bar{\tau}=1$) covers only a very small fraction of the total solid angle as viewed from the outer parts of a very extended atmosphere. In fact in the limiting case that the solid angle covered by the 'star' becomes very small we have to replace (13) by the relation

$$K_\nu = H_\nu = J_\nu. \tag{14}$$

So, from our present point of view spherical symmetry in an extended atmosphere leads to two important effects:

(1) the decrease of F(or H) like $1/r^2$ and 2) the increasing 'outward peaking' (Hummer and Rybicki, 1971) of the radiation field in the outer layers of the atmosphere.

Most authors, with the exception of Cassinelli (1970, 1971), have restricted their actual calculations to strongly simplified models in which grey absorption (or pure coherent scattering) has been assumed and in which also the radiative transfer problem (4), (5) is decoupled from the hydrostatic Equation (1b). Most of these calculations are based on the assumption

$$\kappa\varrho = r^{-n} \tag{15}$$

(Cf. Chapman, 1964, 1966; Schmid-Burgk, 1970; Hummer and Rybicki, 1971; Cassinelli and Hummer, 1972). The most accurate numerical calculation of the radiative transfer in extended atmospheres is due to Schmid-Burgk (1970) and to Hummer and Rybicki (1971), Schmid-Burgk (1970) solves the problem in the following way: From (4) he derives integral equations for the mean intensity $J(r)$. He solves one of these equations by expanding it into a series of known coefficients. The functions are selected in such a way that the integration over r can be carried out analytically. After numerical integration over μ a system of linear equations for the unknown coefficients can be derived from the integral equation.

Hummer and Rybicki (1971) rewrite the momentum Equation (11) by defining the 'Eddington factor' f

$$f = K/J, \tag{15}$$

so that K in (11) can be replaced by fJ. Starting out with a guess of $f(r)$ they integrate (11) numerically and so get a first approximation of $J(r)$. Since in the grey case $J=S$, this approximation of $J(r)$ can be used to find a first approximation of $I(r, \mu)$

from Equation (4). From $I(r, \mu)$ we can recalculate the moments and find an improved value of the Eddington factor. The procedure is iterated. Schmid-Burgk's (1970) and Hummer's and Rybicki's (1971) results for assumption (15) are useful from two different points of view:

(1) The newly developed methods may be applied to more realistic cases of extended atmospheres (though the generalization may not always be trivial).

(2) The numerical results give us considerable insight into the mechanism of the radiative transfer in an extended atmosphere.

As an illustration of point 2 we reproduce in Figure 1 Schmid-Burgk's (1970) results for the angular dependence $I^+(\tau, \mu)/I^+(\tau, 1)$ of the 'outward' intensity in the case $n=3$. The diagram shows very clearly the very strong 'forward peaking' of the radiation field in high layers of the extended atmosphere. It also shows to which extent

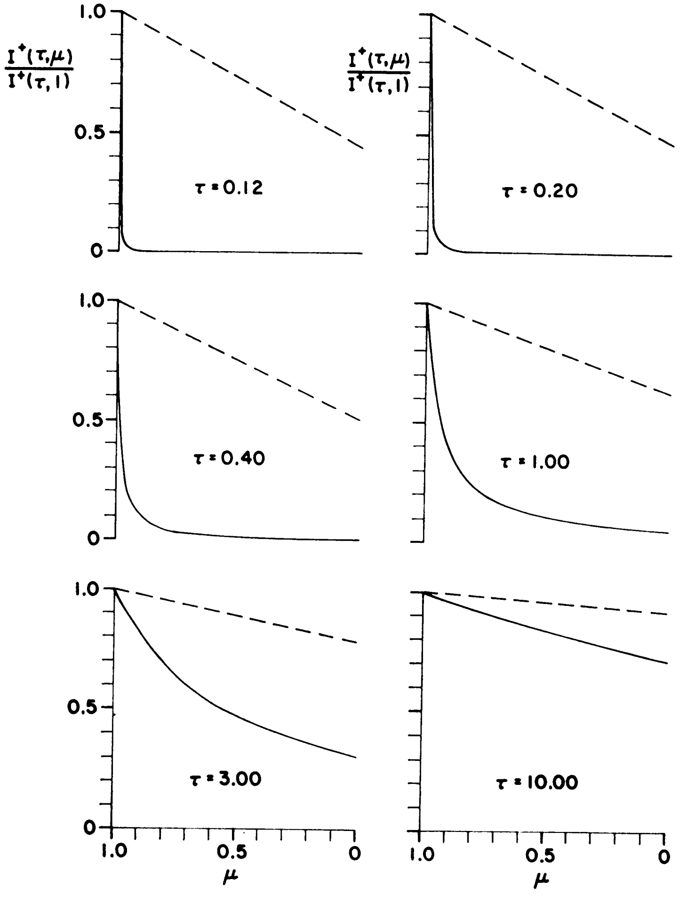

Fig. 1. The outward intensity I^+ as a function of μ for different optical depths τ. The solid curve refers to the spherically symmetric grey atmosphere for the case $n=2$. (See formula 15.) The broken line corresponds to the plane parallel case. The diagram has been taken from Schmid-Burgk's (1969) work (with minor modifications).

Eddington's approximation leads to incorrect results. It is also interesting to note (Schmid-Burgk, 1970) that the spectral energy distribution of the emergent flux from an extended grey atmosphere is much 'flatter' (not so much peaked around one frequency) than the energy distribution from the corresponding plane parallel atmosphere.

Let us now look at the determination of more realistic models of extended atmospheres. Obviously the following points will be important:

(1) Assumption (15) has to be replaced by a realistic opacity law $\kappa(\varrho, T)$. This leads immediately to a coupling between the hydrostatic and the transfer equation. This in turn requires a critical consideration of the boundary conditions at the 'surface' of the atmosphere.

(2) We would like to include the effects of 'nongreyness' in the radiative equilibrium calculation.

(3) As soon as possible deviations from local thermodynamic equilibrium (which are of course usually more important in extended than in plane-parallel atmospheres) should be included (Thomas, 1970).

Cassinelli (1970, 1971) had tried to solve the difficult problem in which points (1) and (2) have been taken into account.

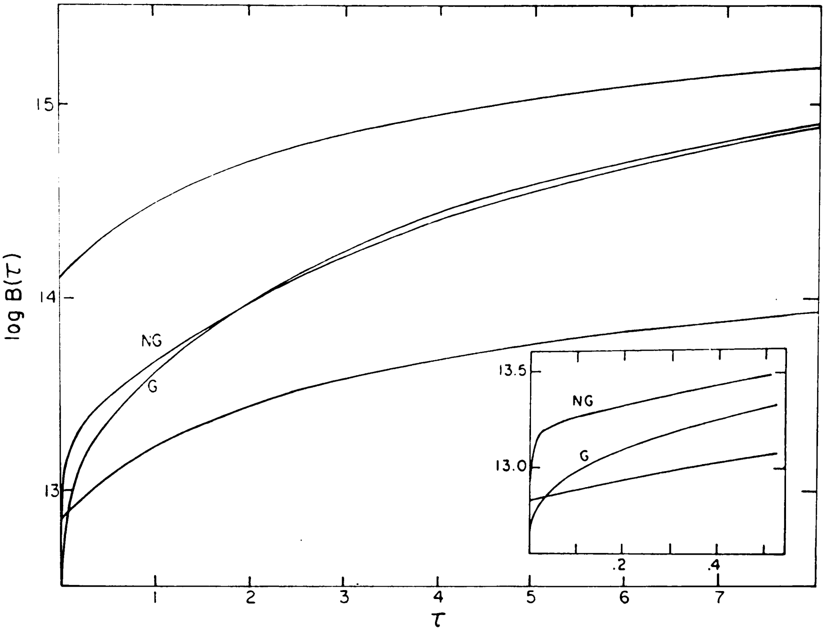

Fig. 2. $B(\tau)$ in an extended atmosphere of a star of $M = 0.6\, M_\odot$ and $L \simeq 2.05 \times 10^4\, L_\odot$, corresponding to $T(\bar{\tau} = \frac{2}{3}) \simeq 37600$ K. The geometrical radius at $\bar{\tau} = 10^{-3}$ is about 4.3 times as large as at $\bar{\tau} = 10$. The two inner curves give the grey and the non-grey models taking into account effects of spherical symmetry. The outer curves are plane-parallel stratifications drawn for comparison purposes. After Cassinelli (1970).

In the formulation of the outer boundary conditions he makes the useful assumption that the hydrostatic atmosphere has to be cut off at the point where the thermal velocity becomes equal to the velocity of escape. Such a precaution is necessary because a spherically symmetric atmosphere with a sufficiently small temperature gradient in its outer layers has a finite density at $r = \infty$. For the solution of the transfer problem Cassinelli uses the so-called S_N-method, a discrete ordinate method developed by Carlson and Lathrop (1968) for the treatment of neutron transport problems. In order to fulfill the condition of radiative equilibrium

$$r^2 \int_0^\infty F_\nu d\nu = \text{const}. \tag{16}$$

Cassinelli developed a temperature correction procedure which is a generalization of the well-known Unsöld-Lucy method (Unsöld, 1951; Lucy, 1964) to the spherically symmetric case. A temperature correction procedure permits us to calculate a correction to a given approximate temperature stratification if we know the derivation of the

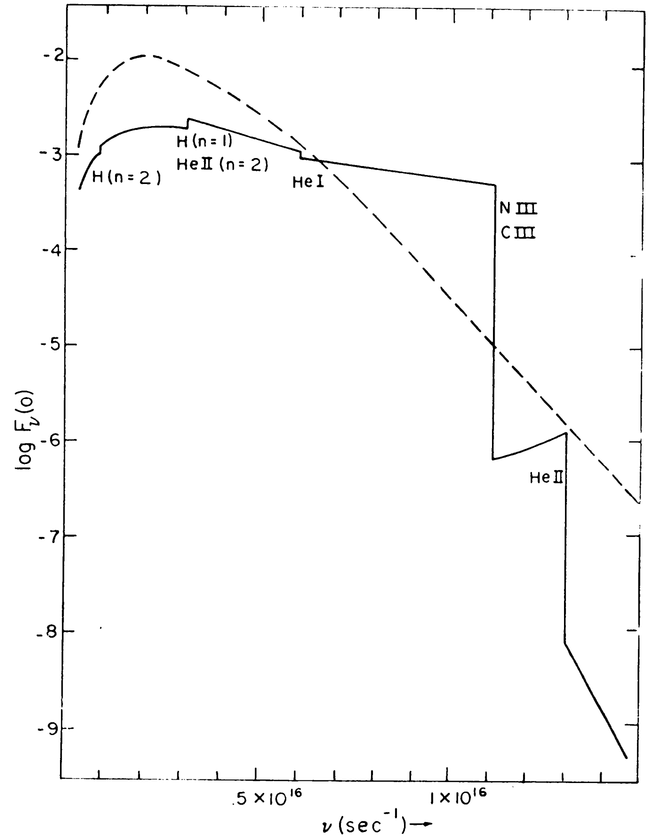

Fig. 3. The energy distribution of the emergent flux $F_\nu(0)$ for the non-grey extended atmosphere shown in Figure 2. After Cassinelli (1970, 1971). (By permission of the *Astrophysical Journal*; copyright 1971; The University of Chicago.)

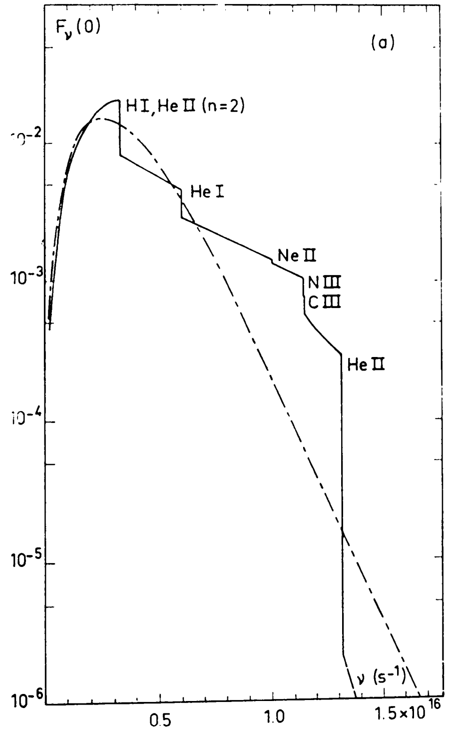

Fig. 4. The emergent flux $F_\nu(0)$ in a plane-parallel non-grey atmosphere of similar temperature ($T_{\rm eff} = 4.3 \times 10^4$ K, $g = 6 \times 10^4$ cm s^{-2}) for comparison with Figure 3. (After Böhm (1969); by permission of the Springer-Verlag.)

total flux for this stratification from the correct value as required by condition (16). Unfortunately Cassinelli (1970) succeeds only in deriving temperature correction formulae in a simple form for the two limiting cases given by the conditions (13) and (14). Suprisingly, the two expressions turn out to be very similar. Moreover, as is well known temperature correction formulae do not have to be very accurate. After every iteration step one checks how well condition (16) is fulfilled and so one can judge the quality of the solution independently. Cassinelli was able to achieve a flux constancy of 1.5% or less in his models.

His calculation shows that an extended atmosphere covers a much larger temperature interval in a given optical depth range than a plane-parellel atmosphere as one would expect. This is illustrated in Figure 2 which shows the grey and the nongrey temperature stratifications for an extended atmosphere with a luminosity $L \approx 2.05 \times \times 10^4\ L_\odot$ and a mass of $M \approx 0.6\ M_\odot$. (Note that the concepts of the effective temperature and surface gravity are of course no longer useful in an extended atmosphere.) The grey and nongrey stratifications have temperatures of 34710 K and 37500 K at $\bar{\tau} = \frac{2}{3}$. Their 'surface temperatures' are 21890 K and 27140 K (with the grey surface temperature lower). The atmosphere is about four times as thick as the 'radius of the star' provided we set the boundary between atmosphere and star at $\bar{\tau} \approx 10$.

The critical luminosity

$$L_{\rm crit} = \frac{4\pi c G M}{\langle \kappa + \sigma_{\rm el} \rangle}, \tag{17}$$

is about $2.3 \times 10^4\ L_\odot$. The energy distribution of the emergent flux is given in Figure 3. In comparison to the situation for a plane-parallel model (cf. Figure 4) for a similar $T(\frac{2}{3})$ the extended atmosphere shows a somewhat 'flatter' energy distribution and leads to an emission edge at $\lambda = 912$ Å in contradistinction to the absorption edge found in the plane-parallel case.

Finally we might ask how these more realistic models compare to the schematic ones described by condition (15). We find that usually the inner parts can be described by condition (15). We find that typically the inner parts can be described by $n \approx 2.5$ whereas the outer parts ($\bar{\tau} < \frac{2}{3}$) require a much larger value of n ($n \approx 14$ according to Cassinelli, 1971).

So far we have discussed the determination of the model atmosphere and of the continuous spectrum only. The calculation of the line spectrum could in principle be based on the same transfer Equation (4) or the corresponding momentum equations. (This was done rather early, cf. McCrea, 1928.) However, today many people feel that such a procedure would not be applicable in many cases. From an observational point of view practically all extended atmospheres do show some motion and even if a considerable part of the atmosphere can be considered to be approximately in hydrostatic equilibrium many lines will probably be influenced by the differential Doppler effect of different layers. After a study of line profiles from moving plane-parallel atmospheres (cf. Abhyankar, 1965), of thin spherical shells (Beals, 1931; Rottenberg, 1952) and certain simplified models (Sobolev, 1960) the complete transfer problem in an expanding and extended atmosphere has recently been considered by several authors (Cf. Rybicki, 1970; Lucy, 1971). Lucy has pointed out that the transfer equation for an expanding extended atmosphere can be brought into a relatively simple form provided

(1) only terms of the first order in (v/c) are retained,

(2) one uses the 'narrow line limit', i.e. one assumes that the thermal velocities are much smaller than the hydrodynamic velocities.

In this case one gets

$$\left\{\mu^2 \left[\left(\frac{\mathrm{d}v}{\mathrm{d}r}\right) - \frac{v}{r}\right] + \frac{v}{r}\right\} \frac{\mathrm{d}I'_{\nu'}}{\mathrm{d}\nu'} = \kappa'_\nu \varrho \frac{c}{\nu_0} (I'_{\nu'} - J'_{\nu'}), \tag{18}$$

where I', J' and v' refer to the comoving frame. Lucy discusses a very effective method for the solution of transfer problems described by Equation (18) or by (4). It is analogous to the classical Schwarzschild method of describing the angular dependence of the intensity by setting the outward intensity $I^+(\mu)$ equal to a constant and inward intensity $I^-(-\mu)$ equal to another constant. However, the switchover from I^+ to I^- does not occur at $\mu = 0$ as in the plane-parallel case but at $\mu = \mu^*$ where μ^* is defined by the angle under which the 'stellar limb' is seen from a point in the extended atmosphere. This method seems to be very promising since it takes into account the 'forward concentration' of the radiation in the high layers in the simplest possible way. The result of such calculations depends of course on the assumed velocity law which can be determined (at least in principal) from hydrodynamical calculations. (See next chapter.)

The numerical calculations lead to P Cygni-type profiles in most cases.

When talking about extended atmospheres we have so far mostly emphasized effects which can be attributed directly or indirectly to the changing geometry from plane-parallel to spherically symmetric. However, there are also effects of simply having large regions with relatively low density instead of small regions with relatively high density. The importance of these effects close to the instability limit is illustrated in Figure 5 which shows the change of the continuous energy distribution of a very hot star as we approach the limit $g_{\text{rad}} = g$.

The interesting problem of the curve of growth in an expanding atmosphere (with constant velocity of expansion) has been investigated by Arakelian (1969).

I should also like to draw attention to the interesting studies of extended atmospheres by the Tartu astronomers (cf. Sapar and Viik, 1968) who have studied the generalization of the Avrett-Krook procedure to the spherically symmetric case.

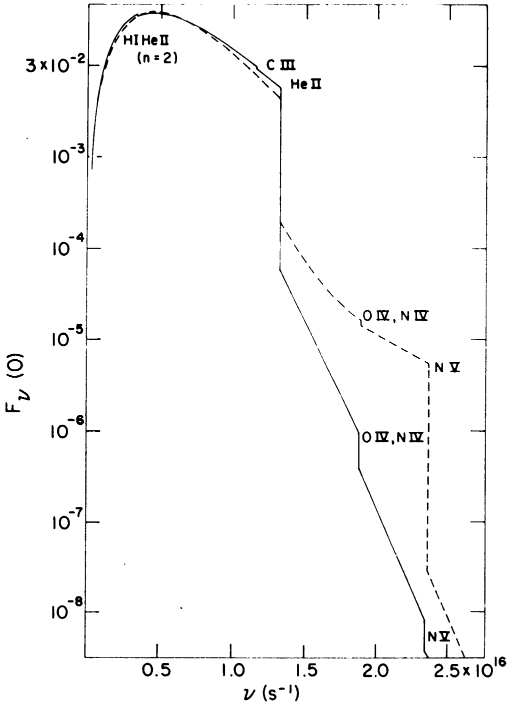

Fig. 5. Comparison of the emergent fluxes $F_\nu(0)$ for two atmospheric models with $T_{\text{eff}} = 63000$ K. The solid curve corresponds to a value g corresponding to $\Gamma = (g_r/g)$ of 0.725. The broken curve refers to a model with $\Gamma = 0.95$.

3. Dynamic Atmospheres

Our understanding of extended atmospheres not in hydrostatic equilibrium is still very limited. Only two cases have been studied to some extent:

(1) stellar coronae which (in analogy to the solar case) may lead to a stellar wind and

(2) expanding atmospheres which occur in stars with sufficiently low g (or sufficiently high temperature) so that the instability limit $g = |g_{\text{rad}}|$ is approached but not reached.

In a certain sense case (2) is simpler than (1) because it occurs in simple radiative transfer atmospheres and does not require a mechanical energy flux to build up a stellar corona. In the present chapter we shall restrict ourselves to case (2).

Practically all theoretical studies in this field have been done with relatively hot stars, like early B-stars, O and Of stars, WR stars and central stars of planetary nebulae in mind. As is well known the interest in this field has increased very considerably because of the interesting observations of very large outward velocities in the uv lines of early-type stars by Morton and his collaborators. (Cf. Morton *et al.*, 1968.) However, as we shall see below, attempts to explain these high velocities in one simple step can be misleading.

Attempts to solve the theoretical problem have (so far) been based on the following assumptions and requirements:

(1) Look for a stationary (steady state) hydrodynamic solution.

(2) Take into account the (r-dependent) acceleration of gravity as well as the radiative acceleration in the equation of motion.

(3) Take into account radiative exchange as fully as possible in the energy equation.

(4) Treat the radiative transfer as a spherically symmetric problem and take into account terms proportional to (v/c) (due to the motion of the gas) in the transfer equation.

Because of the complexity of the problem all authors had to restrict themselves to the grey approximation in the treatment of the radiative transfer problem.

It seems to us that so far the relatively most complete treatment of the problem is due to Schmid-Burgk (1969) and to Cassinelli and Castor (1972).

Using the above assumptions the problem can be formulated as follows:

$$\frac{d}{dr}(\varrho v r^2) = 0\,; \quad \text{(equation of continuity)}, \tag{19}$$

$$v\frac{dv}{dr} + \frac{1}{\varrho}\frac{dp}{dr} = -\frac{GM}{r^2} + \frac{\langle \kappa + \sigma_{\text{el}} \rangle L}{4\pi c r^2}\,; \quad \text{(equation of motion)} \tag{20}$$

$$v\frac{dE}{dr} + pv\frac{d}{dr}\left(\frac{1}{\varrho}\right) = 4\pi \int_0^\infty \kappa_\nu (J_\nu - S_\nu)\, d\nu\,;$$

$$\text{(energy equation, including radiative exchange)} \tag{21}$$

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(cf. Cassinelli and Castor, 1972), where v is the gas velocity, E is the internal energy per gram (of the gas only). It is important to note that the radiative quantities, κ_ν, S_ν, J_ν, refer to the comoving frame of reference. The momentum equations for radiative transfer [corresponding to Equation (10) and (11)] become somewhat complex if (v/c) terms are included. They have been given by Cassinelli and Castor (1972).

The radiative acceleration term in (20) can of course also be written as

$$\frac{\pi}{c}\int_0^\infty \kappa_\nu F_\nu d\nu\,. \tag{22}$$

Before saying something about the solutions of the system (19), (20), (21) we should try to be aware of one possible misunderstanding to which one is easily lead if the problem is approached in a naive way. Since these expanding atmospheres occur as we get close to the situation where the absolute value of (22) becomes comparable to the local gravity one might think that the gas is directly accelerated by the 'radiation force' (which is proportional to (22)). It was recognized by Schmid-Burgk (1969) and clearly analyzed and discussed by Marlborough and Roy (1970) and Cassinelli and Castor (1972) that this is impossible (for some early remarks on this problem see Paczyński, 1968).

The essential point is the following: as is well known from the theory of the solar wind (cf. Parker, 1963; Holzer and Axford, 1970; Brandt, 1970) the *interior* boundary conditions for the flow can be fulfilled only by solutions which are subsonic throughout or by the so-called critical solution which makes a smooth transition from subsonic to supersonic flow at the sonic point. Moreover the *outer* boundary condition ($p \to 0$ at sufficiently large r) can be fulfilled only if the solution is supersonic for large r (see Figure 6). Consequently the critical solution is the only one which fulfills both boundary conditions. In other words the correct solution always has to pass through

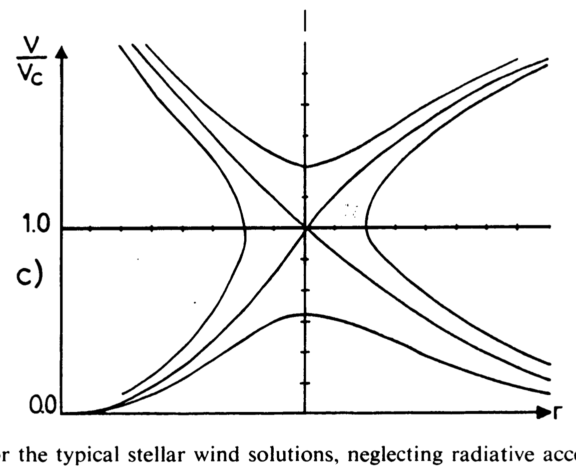

Fig. 6. $v(r)$ for the typical stellar wind solutions, neglecting radiative acceleration. After Schmid-Burgk (1969).

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a point $v = c_s$ (c_s = local velocity of sound) continuously. Let us now look at a case in which the radiative acceleration term is important but in which the radiative exchange term in (21) can be neglected (adiabatic case). Then Equations (19), (20), (21) can be reduced to the following relation, provided the perfect gas law holds. (Marlborough and Roy, 1970; Cassinelli and Castor, 1972.)

$$\frac{r\,\mathrm{d}v}{v\,\mathrm{d}r} = \frac{2c_s^2 - \{GM(1-\Gamma)/r\}}{v^2 - c_s^2}, \tag{23}$$

when the Γ is the ratio of the radiative acceleration to the local gravity:

$$\Gamma = \frac{\langle \kappa + \sigma_{\mathrm{el}} \rangle L}{4\pi cGM}. \tag{24}$$

Obviously $\Gamma = 1$ corresponds to the instability limit $|g_{\mathrm{rad}}| = g$. In order to get a smooth transition from subsonic to supersonic velocities the numerator in Equation (23) must go from negative to positive values exactly at the point $v^2 = c_s^2$. ($\mathrm{d}v/\mathrm{d}r$ is positive everywhere). This condition can be fulfilled only if $\Gamma < 1$ in the subsonic region (See Equation 23). In other words, the radiative acceleration cannot be used to accelerate the gas in the subsonic region. However, as pointed out by Marlborough and Roy (1970) the flow can be occeterated in the supersonic region by radiative acceleration (as described e.g. by Lucy and Solomon, 1970). Cassinelli and Castor (1972) call this the 'afterburner' mechanism.

How can we then accelerate the gas up to the sonic point?

As we saw in the preceding chapter a *hydrostatic* atmosphere becomes more and more extended (it is less and less bound) as we approach the limit $\Gamma = 1$. Consequently less and less energy is needed to drive an outflow of the gas. It is implicit in part of the earlier work (Bisnovatyi-Kogan and Zel'dovich, 1968; Schmid-Burgk, 1969) and it was very clearly discussed and emphasized by Cassinelli and Castor (1972) that the acceleration is possible only because energy is transported by radiation to the outer layers and deposited there. This energy is used to heat up these layers sufficiently so that they can escape eventually.

This shows that:

(1) this type of expanding atmosphere can be understood only if the absorption and emission of radiation in the relevant (usually the outer) layers is treated in sufficient detail,

(2) '*true*' *absorption* of radiation must occur in the relevant layers, because a pure scattering process (specifically Thomson scattering) would not lead to a deposition of energy. (Strictly speaking this applies to coherent scattering only.)

Cassinelli's and Castor's (1972) calculations show that (at least in the optically thin case which can be treated easily) the transition from negative to positive total energy of the gas (i.e. kinetic energy plus gravitational energy plus enthalpy) occurs just somewhat below the critical (sonic) point in all interesting cases.

Cassinelli and Castor (1972) find that strictly speaking all spherically symmetric stellar atmospheres show an outflow of matter. However, if Γ is not sufficiently close to 1 the sonic point will occur at such a large distance from the star that the total outflow is completely negligible. Only as we approach $\Gamma = 1$ does the outflow become considerable and the atmospheric structure becomes very different from the hydrostatic case. Atmospheres which are not in hydrostatic equilibrium show in general a flatter temperature stratification especially in their upper layers, than hydrostatic atmospheres. (Cf. Bisnovatyi-Kogan and Zel'dovich, 1968; Schmidt-Burgk, 1969; Cassinelli and Castor, 1972; see also Böhm, 1968.)

As especially Schmid-Burgk (1969) has pointed out the topology of the stellar wind solutions $v(r)$ can be much more complicated than in the simple cases which are usually discussed provided Γ is not too much smaller than 1 and $\bar{\kappa}(r)$ is not a monotonic function of r. The topology of the flow solutions for the case in which $\bar{\kappa}(r)$ has a maximum somewhat above the critical point is illustrated in Figure 7 (taken from Schmid-Burgk's thesis).

Finally I should like to emphasize again that all the models discussed here refer to situations in which $\Gamma < 1$ everywhere. In other words, we get dynamic atmospheres (with outflow) though the effective gravity

$$g_{\text{eff}} = g - g_{\text{rad}}, \tag{25}$$

is directed inward everywhere.

4. Stellar Coronae

Instead of the above chapter title the observer would rather see one indicating a chapter on *chromospheres and* coronae. However, it seems that we really understand too little about the formation of chromospheres to include it in our discussion unless

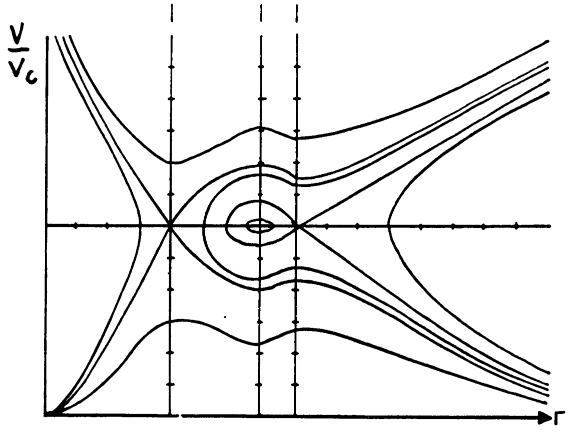

Fig. 7. Typical stellar wind solution $v(r)$ for an atmosphere in which the radiative acceleration has its maximum somewhat above the sonic point. After Schmid-Burgk (1969).

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you would just call the transition region from the photosphere to the corona a chromosphere. On the other hand, it seems rather obvious that the extended chromospheres observed in stars like ζ Aurigae are something very different from such a transition region.

The prediction and computation of stellar coronae is based on ideas developed for the calculation of the transition region and the corona in the solar case which seems to work surprisingly well there (at least according to the opinion of many astronomers). However, recently some astronomers (cf. Ulrich, 1972) have expressed some doubts concerning the validity of the 'standard' theory of coronal heating. It is not yet quite clear how serious these objections are (see below). Moreover, the standard (shock-wave heating) approach is the only one which has been worked out at least in some detail and which consequently can be applied to stars other than the sun. Therefore, we shall restrict ourselves to the discussion of coronae (and transition regions) which are due to the heating by shock waves. The shock waves are thought to come from the continuous steepening of acoustic waves which are generated in the stars outer convection zone. Obviously, a necessary condition for the existence of a corona is the presence of an outer convection zone and the generation of a sufficiently strong acoustic noise flux in this zone. The calculation of the acoustic noise flux has usually been based on the Lighthill-Proudman theory (cf. Lighthill, 1954, 1955; Proudman, 1952). Though Lighthill (1967) himself has raised some objections against the astrophysical application of his theory and has strongly urged us to consider the generation of gravity waves by the convection zone, it is true that only acoustic waves have been seen in the solar atmosphere. As pointed out by Souffrin (1966) this must be due to the short radiative relaxation time of the gravity waves. With these facts in mind we can easily calculate the acoustic ('mechanical') flux

$$F_m \simeq 19 \int_{z_0}^{z_1} \varrho M_*^5 \frac{v^3}{l} \, \mathrm{d}z . \tag{26}$$

F_m is the acoustic energy flux, v is the local convective velocity, M_* the corresponding Mach number, l is the characteristic length of the flow generating the acoustic noise. z is the geometrical depth, z_0 and z_1 are the coordinates of the upper and lower boundaries of the convection zone. Equation (26) can be evaluated easily for any star for which a model of the outer convection zone is available. (See e.g. Biermann and Lüst, 1960). Calculations of acoustic fluxes in stars of different type have been carried out e.g. by Kuperus (1965), (stars of solar composition with 4400 K $\leqslant T_{\mathrm{eff}} \leqslant$ 7000 K), Nariai (1969) (He-rich stars), de Loore (1970) (stars of solar composition, 2500 K $\leqslant T_{\mathrm{eff}}$ $\leqslant$ 16630 K), Böhm and Cassinelli (1971) (He-rich white dwarfs, 5790 K $\leqslant T_{\mathrm{eff}} \leqslant$ $\leqslant$ 30000 K). As is well known, main sequence stars with solar chemical composition show a considerable acoustic energy output only in very narrow temperature range. It is worth noting that the situation is very different in stars whose outer layers consist mostly of helium (Nariai, 1969). Probably the most extreme objects in this respect are the white dwarfs with helium-rich outer layers (Böhm and Cassinelli, 1971) in which acoustic fluxes can be reached which are considerably larger than the radiative flux of the Sun. The situation is illustrated in Figure 8, which shows the completely different behavior of the mechanical flux as a function of the effective temperature for main sequence stars on the one hand side and for helium-rich white dwarfs on the other side. It is to be expected that these helium stars should have considerably denser and hotter coronae than main sequence stars.

Let us now look at the problem of the computation of coronal densities and temperatures for stars of different effective temperature and surface gravity.

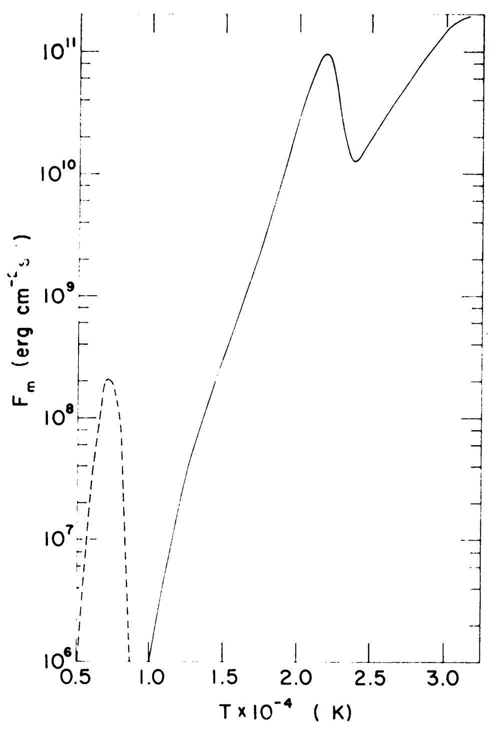

Fig. 8. Comparison of the acoustic flux F_m in main sequence stars (broken line) and in helium-rich white dwarfs (solid curve).

The basic theory has been outlined very clearly in the Ph.D. thesis of Kuperus (1965; see also de Jager and Kuperus, 1961). Very considerable improvements in the details of the physical theory have been made later (cf. Ulmschneider, 1967, 1971a, 1971b; Stein, 1968). However, we feel that the basic ideas can be most easily understood if we restrict ourselves mostly to the simple approach described in Kuperus (1965, 1966) work.

We have to start from the assumption that the mechanical flux, F_m, above the convection zone has already been calculated [Equation (26)]. One now makes the very plausible assumption that the (spatial) decrease of F_m due to shockwave dissipation can be described by a local 'absorption coefficient' $\tilde{\kappa}(z)$. One finds

$$F_m(2) = F_m(0) \times \phi(z) \times \exp\left(-\int_0^z \tilde{\kappa}(z)\right) \mathrm{d}z, \tag{27}$$

z is the geometrical height counted from the point where the shock wave dissipation starts, $\phi(z)$ is a correction factor which takes into account the losses in the radial mechanical energy flux due to the refraction and reflection of shock waves. After logarithmic differentiation (27) can be written as a differential equation for F_m:

$$\frac{\mathrm{d}F_m(z)}{\mathrm{d}z} = -\tilde{\kappa}(z)\,F_m(z) - \frac{F_m(z)}{\phi(z)}\frac{\mathrm{d}\phi(z)}{\mathrm{d}z}. \tag{28}$$

The local energy dissipation E_d, (available for the heating of the gas) is related to $\tilde{\kappa}$ and F_m according to the simple relation:

$$E_d(z) = \tilde{\kappa}(z) \times F_m(z). \tag{29}$$

Let us now call the radiative energy loss (per cm^2 and second) E_r. The increment $\mathrm{d}F_c$ in the conductive energy flux (going back towards the photosphere) will be equal to the difference between the radiative energy loss and the input of heat due to shock wave dissipation:

$$\mathrm{d}F_c(z) = \{E_r(z) - E_d(z)\}\,\mathrm{d}z. \tag{30}$$

Taking into account the well-known dependence of the heat conductivity of a fully ionized gas on the temperature we have the following relation between conductive flux and temperature

$$\frac{\mathrm{d}T}{\mathrm{d}z} \simeq 6 \times 10^5 \times T^{-5/2} \times F_c. \tag{31}$$

The Equations (28), (30) and (31) have to be supplemented by the hydrostatic equation which Kuperus (1965) writes as

$$\frac{\mathrm{d}n}{\mathrm{d}z} = -n\left\{\frac{1}{H} + \frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}z}\right\}, \tag{32}$$

with n = total particle density.

In the simplest type of problem one assumes that $\phi(z) \approx 1$ (i.e. refraction effects are unimportant). It turns out that in many cases F_m and $\tilde{\kappa}$ can be expressed in terms of the local Mach number, the local velocity of sound, the 'period' P of the shock waves and the density. For instance, if the Mach number does not get too large Kuperus finds (this is based in part on the work of Landau and Lifschitz, 1959):

$$F_m = \tfrac{4}{3}\varrho c_s^3 \frac{(M_*^2 - 1)^2}{(\gamma + 1)^2 M_*}, \tag{33}$$

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with $M_* =$ Mach number and $\gamma =$ ratio of specific heats. He also finds

$$\tilde{\kappa} = \frac{4(M_*^2 - 1)}{M_*^3 c_s P}. \tag{34}$$

The whole problem is defined by the four coupled ordinary differential Equations (28), (30), (31), and (32) and three algebraic Equations: (29), (33), and (34). Correspondingly, we have seven unknown functions, namely F_m, $\tilde{\kappa}$, E_d, F_c, M, T, and n. Note that E_r and c_s should not be counted because they can directly be calculated from T and n.

Obviously the system (28), (30), (31), (32) can be integrated (at least in principle) as an initial value problem if n, T, F_m and e.g. M_* at the bottom of the chromosphere were known (Ulmschneider, 1967). One can also start the trial integrations from the corona (de Jager and Kuperus, 1961; Kuperus, 1965) assuming that (dF_c/dz) in the corona is known.

In the actual calculations one encounters a number of difficulties of which we shall mention only two:

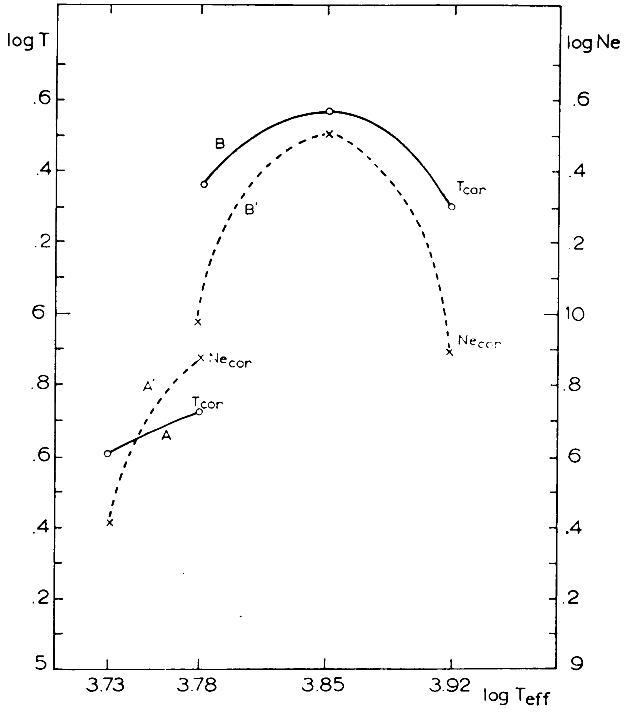

Fig. 9. Coronal temperatures (solid curves) and coronal electron densities (broken line) as a function of T_{eff}. The curves on the left side refer to $g = 10^5$ cm s^{-2} the lines in the middle of the diagram to $g = 10^4$ cm s^{-2}. After de Loore (1970). (By permission of Reidel Publ. Co.)

(1) It is difficult to describe the radiative losses E_r in the low temperature region (near the photosphere) correctly.

(2) It is important but very difficult to know what the energy distribution in the acoustic noise spectrum emerging from the convection zone will be like. [In the simple approach above this enters in the form of the factor P in the denominator of Equation (34)] Our knowledge of stellar convection as well as our understanding of noise generation by convection is still far from permitting reliable predictions in this respect. However, some very interesting studies of this problem have been made recently for the solar case. (Stein, 1968; Ulmschneider, 1971.)

The complexity of the problem is of course increased if the stellar wind influences the energy balance. (Kuperus, 1965; Shklovskii, 1965).

As an example of the type of information which one gets we reproduce some of the results by de Loore (1970) in Figure 9. The drawing shows the dependence of the coronal temperature and density on the effective temperature of the star in a range of surface gravities which are not too different from those for main sequence stars. One essentially finds that both the coronal temperature and density seem to be monotonic functions of the acoustic flux (at least in the range covered by these calculations).

It seems to me that in the field of stellar coronae a number of interesting developments are to be expected in the near future. Even a basically simple theory like that of Kuperus (1965) has not yet been applied to the objects with very high acoustic fluxes like helium stars and especially helium-rich white dwarfs. How high a coronal temperature would we expect in these objects? What kind of X-ray spectrum would we predict for these coronae? (Predictions of the X-ray emission of coronae of normal stars have been made by de Loore and de Jager 1970.) Even more drastic effects may be expected if one includes more exotic ways of coronal heating in such objects. (Strittmatter *et al.*, 1972).

Acknowledgement

This study has been supported by NSF Grant GP 28882 A, No. 1.

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