Thermal convection at high Rayleigh number

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The linear stability analysis, essentially developed by Chandrasekhar, relating to the onset of thermal convection in a horizontal layer of viscous fluid establishes the background and motivating interest for the work undertaken. The problems considered are, in the main, nonlinear extensions of Chandrasekhar's linear theory. In particular the solutions have been derived for high Rayleigh number values with applications to astrophysics in mind. However, one aspect of the linear theory, not considered by Chandrasekhar, has been studied and the results compared with the available experimental values.

Fundamental nonlinear differential equations governing hydromagnetic convection in a rotating Boussinesq fluid layer heated from below have been established and a systematic study of these equations for stationary cellular convection (usually the mean field approximation) is undertaken. The main objective in each case is to obtain from the solutions an understanding of how the total convective heat transport across the layer at high Rayleigh number depends upon the horizontal wave number and to determine the convective cell size that maximizes this heat flux. Additional features of mathematical interest, over the ranges of the relevant parameters that support convection, are also included.

The influence of a magnetic field and rotation acting separately and in conjunction on the convective processes is also closely investigated. Although these externally applied constraints have a strong inhibiting effect on the total convective heat transport, it is found that the general

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properties of cellular convection still persist. Solutions are found over a wide range of conditions for free boundaries by a combination of numerical and analytic techniques. Good agreement is established between the results obtained from these methods, particularly at high Rayleigh number and wave number of order one. They show that the maximum value of the Nusselt number is shifted to a higher wave number due to the application of a magnetic field or rotation. Moreover, it is shown that at high rotational speed an increase in the magnetic field strength can, under certain circumstances, increase the convective heat transport across the layer. Conditions under which the perturbed magnetic field can be concentrated at the boundaries of the fluid layer by the convective motions are also examined.

Asymptotic and numerical solutions of the mean field equations for large values of the Rayleigh number and horizontal wave number are also considered and it is found that the character of these solutions is quite different from the asymptotic solutions obtained when the horizontal wave number is of order one.

Reference

 [1] S. Chandrasekhar, Hydrodynamic and hydromagnetic stability (Clarendon Press, Oxford, 1961).

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