Formation of the Planets and Asteroids: Some Difficulties of Accretion Theories

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Figure 1 is a plot of spin angular momentum, $J$, against mass, $m$, for some of the main sequence stars, the planets and the asteroids. As can be seen from the figure, the points for the majority of these objects, which extend over more than about 20 orders of magnitude in mass and 30 in spin angular momentum, can be well fitted by a line having the equation

$$ J \propto m^{5/3} $$

This striking result has a simple explanation in terms of Newtonian mechanics, as will be shown. The purpose of this note is to indicate that the explanation poses some interesting problems associated with current ideas about the formation of the planets and the asteroids.

More than 20 years ago McDonald (1963) noted that there is an approximate power-law relationship between the spin...
angular momenta and masses for six of the planets. Subsequently Fish (1967) and Hartman and Larson (1967) showed that a \((\text{mass})^{5/3}\) power law fitted not only most of the planetary data but also the asteroid data. More recently Burns (1975) confirmed the result, drawing on a sample of about 70 asteroids, which was larger than the sample available to the earlier authors. Figure 1 incorporates data for about 260 asteroids from a recent tabulation by Harris and Young (1983) and indicates few substantial departures from the \((\text{mass})^{5/3}\) law. Hartman and Larson, Fish and Burns pointed out that the \((\text{mass})^{5/3}\) relationship reflects a fact previously noted by Alfven (1964), which is that the spin periods of most of the planets and asteroids are very approximately equal—around \(3 \times 10^4\) s.

The way in which this comes about and its bearing on current ideas concerning the formation of the planets and asteroids is dealt with in the following discussion.

For a body spinning about an axis of symmetry, 
\[
J = \varepsilon m r^2 \omega, \quad (1)
\]
where \(r\) is the equatorial radius, \(\omega\) is the angular velocity, and the constant \(\varepsilon\) depends on the configuration, having the value 2/5 for homogeneous spheres, and smaller values for centrally concentrated objects such as stars and planets. After a little algebra we may rewrite (1) as
\[
J = \left( \frac{3}{4\pi} \right)^{1/6} \frac{G m^{5/3}}{(\bar{\rho})^{1/6}}. \quad (2)
\]
Here the mean density \(\bar{\rho}\) is given by
\[
\bar{\rho} = \frac{5m}{4\pi r^3},
\]
and the constant \(\alpha\) is the ratio of the actual angular velocity to the limiting angular velocity \(\omega_g\) for rotational stability. If \(\omega > \omega_g\), gravitational force alone cannot hold the body together. Thus
\[
\alpha = \frac{\omega}{\omega_g},
\]
\[
\omega_g = \left( \frac{G m}{r^3} \right)^{1/2} = \left( \frac{4\pi G \bar{\rho}}{3} \right)^{1/2}.
\]

It is noteworthy that \(\omega_g\) depends only on the mean density \(\bar{\rho}\). This result and the following discussion shed some light on Alfven's (1964) observation that most of the planets and asteroids have roughly the same spin period. It appears that this 'isochronism' extends also to the early-type main sequence stars.

Equation (2) indicates that if the ratio \(\alpha (\bar{\rho})^{1/6}\) is constant, the \(5/3\) power law follows immediately. Now \(\varepsilon\) varies very little over the range of objects considered, and \(\bar{\rho}\) varies by about two-and-a-half orders of magnitude, but because of the \(1/6\) power law this variation has very little effect.

Most remarkable however is the fact that for most of these bodies, \(\alpha\), the ratio of the actual angular velocity to the limiting angular velocity for rotational stability, lies between about 1/10 and 1/3. Thus an increase of angular velocity by a factor between 3 and 10 would make most of these bodies rotationally unstable.

This result is not particularly remarkable in the case of the stars, if in fact these were formed by gravitational collapse of primordial gas clouds. However, there are reasons for believing that the terrestrial planets were not formed in this way. The arguments have been summarized by Wetherill (1980), and they apply with even greater force to the asteroids.

An alternative explanation of planetary formation envisages a primordial solar system in which 'planetary embryos', in heliocentric orbits, grow by gravitational accretion of much smaller 'grains'. Various attempts have been made to explain planetary spins on the basis of this model. One version that seems to have survived was due originally to Giuli (1968) and was later developed by Harris (1977) and Harris and Ward (1982).

Giuli (1968) used a numerical integration technique to calculate the spin which would have been imparted to the Earth if it had been formed in this way. He first found that if the Earth embryo and the grains were originally in circular heliocentric orbits, the grains would have imparted spin angular momentum having the opposite sense to the orbital angular momentum. That is, the grains would have imparted retrograde spin rather than the prograde spin which is observed in the case of all of the planets except Venus. Giuli's numerical analysis shows that while an individual grain might impart prograde spin, a stream of grains would be expected to give retrograde spin.

However, Giuli (1968) then went on to show that if the embryos were in circular orbits but the grains were in elliptical orbits of small eccentricity, the effect of impact could be to impart prograde spin. He derived an expected spin period of about 15 hours for the Earth. While this is roughly the right order of magnitude, it is about two-and-a-half times too slow, if, as many people believe, the Earth had a period of about six hours before much of its angular momentum was transferred to the Moon (see e.g. Gerstenkorn 1967).

Harris (1977), and Harris and Ward (1982) have produced an approximate analytic approach, based on Giuli's (1968) ideas. This leads to an expected period for a planet given by
\[
P \approx 3 \frac{v_e}{v_g} (T, T_g)^{1/2}.
\]

Here \(T\) is the planet's orbital period around the Sun, \(T_g\) is the period of a satellite orbiting the planet at its surface, \(v_e\) is the surface escape velocity from the planet, and \(v_g\) is the root mean square 'encounter velocity'. The encounter velocity is the velocity of a grain with respect to the planet before it has been appreciably affected by the planet's gravity. We do not know the encounter velocities because they depend on the orbital eccentricities of the impacting grains. Harris therefore turns the problem around and uses the observed spin periods to estimate encounter velocities, and thus orbital eccentricities, of the
impacting grains. We can criticize this argument on the grounds that it constitutes a hypothesis that cannot be proved false. From our point of view there is the further difficulty that it provides no reason for expecting the actual spin velocities to be so close to the instability values. It is difficult to see how the orbital eccentricities, and hence the encounter velocities, should be so adjusted that for the primordial Earth, for Jupiter, Saturn, Uranus and Neptune, the ratios of actual spin rate to the limiting rate for rotational stability all lie between about 1/3 and 1/6.

The problems are even greater in the case of the asteroids, for 10 of which the spin sense has been determined. Two techniques have been used for the determination of spin, namely visual photometry and infrared radiometry. Both techniques have been used in the case of 2 Pallas, 4 Vesta and 433 Eros, and each technique indicates prograde spin for all three asteroids (Schroll et al. 1976; Taylor 1973; Dunlap 1976; Hansen 1977; Morrison 1976). Prograde spins have also been measured for 1 Ceres and 19 Fortuna by infrared radiometry (Hansen 1977) and for 6 Hebe, 624 Hektor and 1685 Toro by visual photometry (Gehrels and Taylor 1977; Dunlap and Gehrels 1969; Dunlap et al. 1973). Retrograde spins have been determined for 5 Astraea and 1620 Geographos by visual photometry (Taylor 1978; Dunlap 1974). Thus 8 of these 10 asteroids are believed to have prograde spin. Furthermore, all those asteroids of diameter greater than about 200 km for which the spin sense has been determined—namely 1 Ceres, 2 Pallas, 4 Vesta, 6 Hebe, 19 Fortuna and 624 Hektor—spin in the prograde sense. Tedesco and Zappala (1980) suggest that asteroids of diameter greater than about 200 km may be primordial ‘…possibly even in the sense of showing primordial spin rates’. In order to account for prograde spin the theory of Giuliani (1968) and Harris (1977) requires the embryo to be in an almost circular orbit. In fact, of course, asteroid orbits have substantial eccentricities, though it is possible that they may have originally been circular. This, the fact that almost all asteroids spin close to the instability limit must itself pose a difficulty for accretion theory.

A recent version of the accretion theory which goes some way towards meeting these objections is due to Prentice (1980). Prentice’s theory takes into account the effect on the impacting particle of drag due to gas and predicts prograde spin. It is clear from the author’s comments however that more work needs to be done on this theory before it can be regarded as definitive.

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Titan and the Dispersal of the Proto-Saturnian Nebula

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Introduction

The NASA Voyager spacecraft missions to the outer planets have provided a wealth of observational data with regard to the physical and chemical properties of the attendant satellite systems. Although the gaseous nebula discs from which these satellites are suspected to have formed are no longer present, these data provide at least some basis from which the characteristics of individual nebula can be deduced.

Coupled with the observations, two fairly recent theoretical developments have resulted in a far better understanding of nebula evolution and disc/satellite interactions. These are the application of viscous accretion disc theory to protoplanetary and protosolar nebula (Lin and Papaloizou 1980) and the application of density wave theory to gravitational interactions between a gaseous disc and a satellite (Goldreich and Tremaine 1980). The latter theory has been successfully applied to the formation of divisions in Saturn’s rings (Goldreich and Tremaine 1979). Here, we examine some of the ramifications that arise when these theories are applied to the proto-Saturnian nebula. In particular, we are interested in exploring whether the presence of Titan in a viscously evolving nebula leads to any restrictions being placed on the timescales of nebula dispersal and satellite accretion.

The Proto-Saturnian Nebula

The structure of the proto-Saturnian nebula may be estimated using the chemical and physical details of the presently observed satellite system. The total mass of the Saturnian satellites is approximately \(1.5 \times 10^{26}\) g, consisting in general of an ice/rock

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