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Size Distributions of Asteroidal Dust: Possible Constraints on Impact Strengths

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Abstract. We present results of a numerical collisional model which shows that the slope index of the equilibrium size distribution is dependent upon the size-strength scaling properties of the colliding bodies. This implies that individual asteroid families or distinct taxonomic classes within the mainbelt asteroid population may evolve different equilibrium size distributions. Well constrained observations of the size distribution over particular size ranges may allow constraints to be placed on the impact strengths of particles much larger or smaller than are capable of being measured in laboratory impact experiments.

1. Introduction

The Dohnanyi (1969) result that the size distribution of a collection of particles in collisional equilibrium can be described by a power law with a slope index of $p = 3.5$ was obtained analytically by assuming that all the particles have the same, size-independent impact strength. Among larger, asteroid size particles, however, strain-rate effects and gravitational self-compression can lead to size-dependent impact strengths. We have developed a numerical collisional model which verifies the Dohnanyi results and shows that size-dependent impact strengths change the slope index of the equilibrium size distribution.

An important distinction must be made between the impact strength of a body composed of a particular material and the more familiar static crushing or tensile strengths measured in the laboratory. Fragmentation in impacts occurs at strain rates far higher than are found in most engineering or geologic processes. In static fragmentation, at very low tensile loading rates, material failure is dominated by the growth of a single, weakest flaw. At very high strain rates the growth of a single flaw is not sufficient to relieve the strain and a distribution of new flaws are rapidly activated. This is the dynamic fragmentation regime, in which the tensile strength of the material increases with increasing strain rate. For an excellent description of dynamic fragmentation in impact processes see Melosh et al. (1992).

Housen & Holsapple (1990) put forth a plausible physical model showing how a strain-rate dependent strength may manifest itself as a size-dependent impact strength. When a body is impacted, a compressive wave propagates through the body and is reflected as a tensile wave upon reaching a free surface. The cracks and flaws naturally inherent in the material begin to grow and coalesce when subjected to the tension. Since the larger cracks are activated at lower stresses, they are the first to grow as the stress pulse rises. At low stress loading rates, failure is dominated by the larger cracks and fragmentation occurs at low stress levels. Since collisions between larger bodies are characterized by lower strain rates, their impact strengths are correspondingly low. In this way, strain-rate dependent strengths will translate to size-dependent impact strengths, with smaller bodies having greater impact strengths than larger ones composed of the same material. Housen & Holsapple show that the impact strength, S, is proportional to $D^{\mu'}$, where D is the diameter of the body and μ' is a constant dependent upon several material properties of the target (for natural rocky materials $\mu' \approx -0.24$.) We will show that the equilibrium slope of the size distribution is linearly dependent on μ'.

2. The Collisional Model

In our numerical collisional model an initial distribution of colliding particles is distributed among approximately 60 logarithmic size bins with diameters ranging from the largest asteroids down to roughly 1 meter. All particles are assumed to be spherical and have the same density ($\rho = 2.7$ g cm^{-3}). The characteristic size of the particles in each size bin is determined from the total mass and number of particles per bin. This size is used along with the assumed material properties, intrinsic collision rate, and projectile population to associate a mean collision lifetime with each size bin. The total number of projectiles capable of fragmenting and dispersing a particle of a given size is calculated at each timestep in the model by counting the total number of relevant projectiles from the appropriate sizebins. In this way the projectile population is determined in a self-consistent manner. When a particle of a given size is collisionally disrupted, its fragments are distributed into smaller size bins in a power-law size distribution given by $dN = Br^{-p}dr$, where the constant B is determined by conservation of mass. The exponent p is usually assumed to be somewhat larger than the equilibrium value of 3.5 in accord with laboratory impact experiments (Fujiwara et al. 1977) and observations of Hirayama asteroid families (Cellino et al. 1991).

A series of verification runs demonstrated that our numerical code properly reproduces the results of Dohnanyi (1969). With size-independent impact strengths our model produces an evolved equilibrium size distribution with a slope index of $p = 3.5$ (differential mass slope index $q = 1.83$), independent of the model bin size, initial size distribution, or fragmentation power law.

3. Model Results and Discussion

In order to examine the effects of size-dependent impact strengths, we created a number of hypothetical size-strength scaling laws, with several values of μ' ranging from -0.2 to 0.2 over the size range 10 km to 1 meter. The slope index of the resulting model size distribution was calculated over the size range 1 to 100

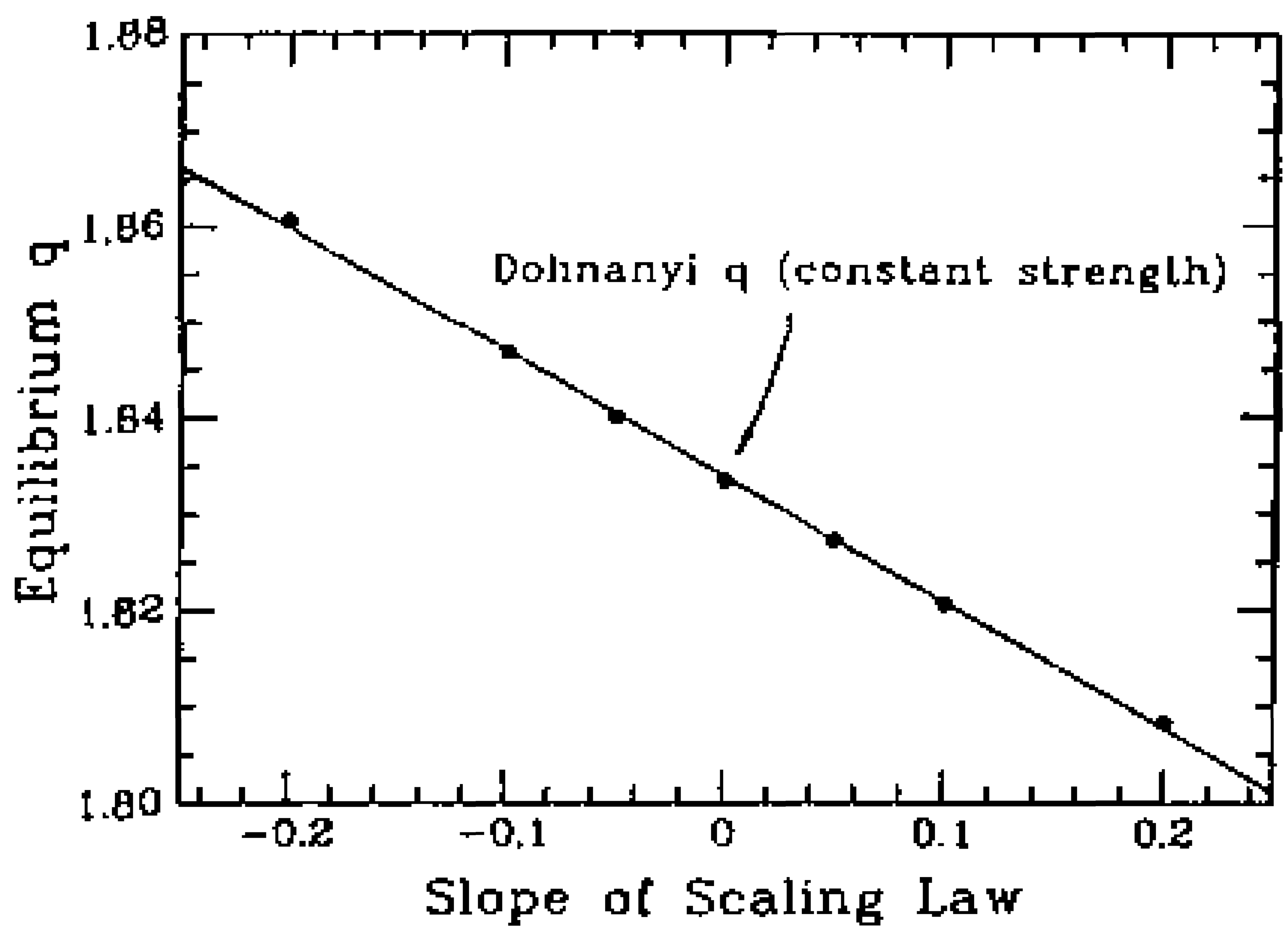

Figure 1. Equilibrium slope parameter as a function of the slope of the size-strength scaling law.

meters and the equilibrium value at 4.5 billion years recorded. Figure 1 shows that the slope of the equilibrium size distribution is linearly dependent upon the value of μ', with a slope of approximately -0.13. When $\mu' = 0$, corresponding to size independent impact strengths, the Dohnanyi equilibrium slope is obtained. When $\mu' < 0$, as is the case for strain-rate dependent strengths, the equilibrium slope is greater than the Dohnanyi value. The slope index is dependent only upon the size-strength scaling properties of the colliding particles, not upon the size distribution of the impactor population. The deviations from the nominal Dohnanyi slope are large enough that well constrained observations of a particle size distribution over a particular size range should allow constraints to be placed on the impact strengths of those particles.

These results imply that populations of asteroids with different compositions and, therefore, possibly different impact strengths, can evolve significantly different equilibrium size distributions. This might apply to the members of an asteroid family of a unique taxonomic class or to taxonomic sub-populations within the entire mainbelt, such as S-type or C-type asteroids. To illustrate this behavior we present in Figure 2 the results of a model following the collisional evolution of two asteroid families. Family 1 has the same arbitrary size-strength scaling law as the background population of impacting projectiles ($\mu' < 0$), while the scaling law for family 2 has $\mu' > 0$. Family 2 evolves an equilibrium slope significantly lower than that of family 1 or the background population, even though it is the background projectiles which are solely responsible for fragmenting members of the family.

The simple approximation of a power-law size distribution of flaws within a body, which leads to uniformly varying size-dependent strengths in dynamic fragmentation as outlined in Section 1, will break down when the particle is no longer homogeneous over the scale of fragmentation. Particles the size of IDP's are composed of assemblages of ~ 0.1 μm particles or have structures dominated by intergrown mineral crystals and may have impact strengths dictated by the failure of flaws with a narrow range of critical tensile strengths. We speculate

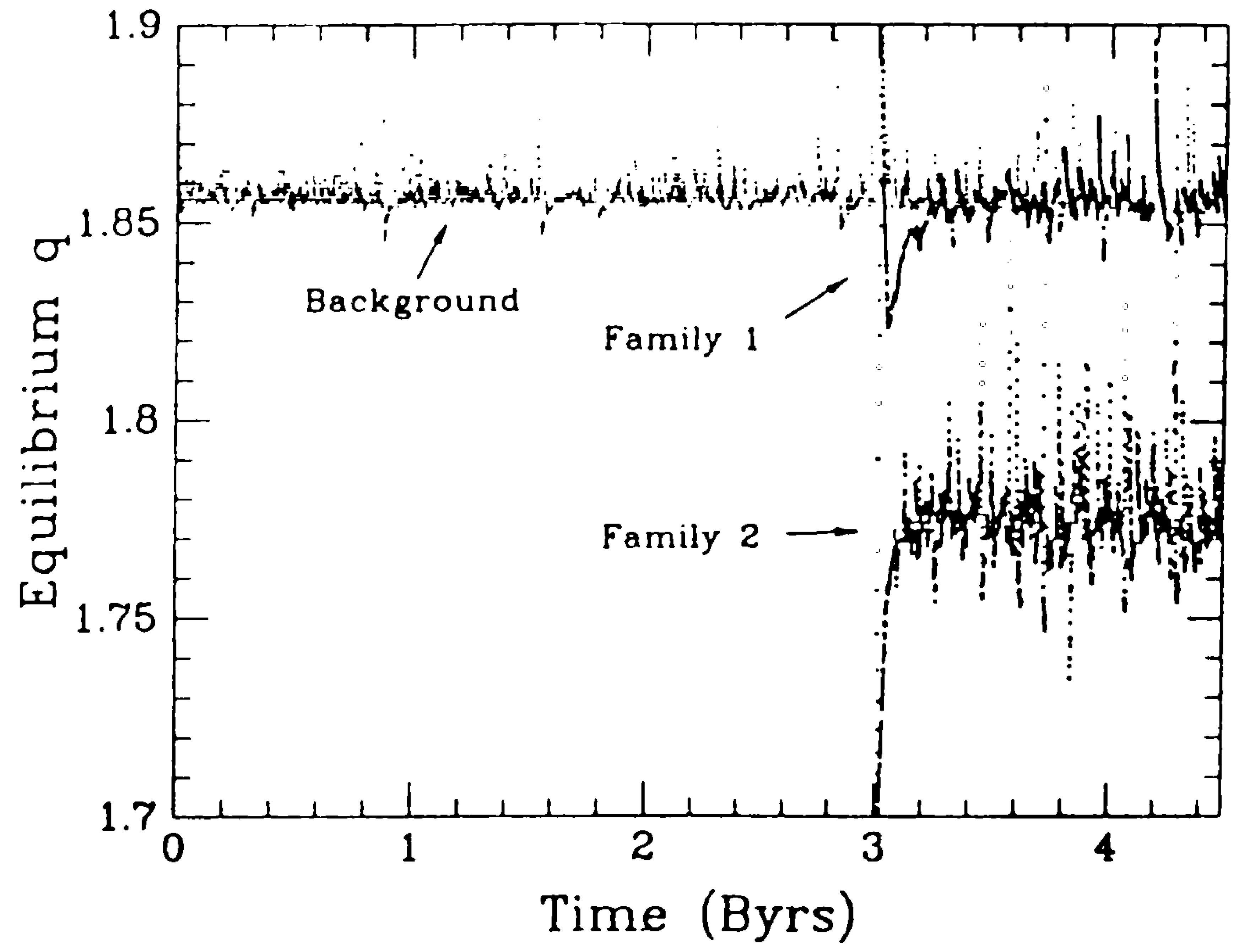

Figure 2. Difference in the equilibrium slope parameters for families with different strength properties. 'Spikes' in the slope parameter above the equilibrium value are due to the stochastic fragmentation of the larger asteroids in the population.

that impact strengths may still depend upon size, but may change discontinuously at specific sizes and/or have different size dependencies in different size ranges. We have shown, however, that such variations in impact strengths will have definite observational consequences in the evolved size distributions of particles in these size ranges. Since the total dust area associated with a population of debris is sensitively dependent upon the slope of the size distribution, it could be possible, if the small debris in the asteroid families resposible for the solar system dust bands has reached collisional equilibrium, to make use of IRAS and COBE observations of the effective areas of the bands to constrain the average impact strengths of asteroidal dust particles.

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