PRIORI ESTIMATES OF THE FK5 COVARIANCE FUNCTIONS

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ABSTRACT. Covariance functions of individual differences of positions and proper motions of stars in the FK5 and FK4 catalogues are given. The application of these functions for the study of the FK5 system and for its further improvement are discussed.

The individual differences $f(\alpha,\delta)=(\Delta\alpha\cos\delta,\ \Delta\delta,\ \Delta\mu\cos\delta,\ \Delta\mu')$ of positions and proper motions of common stars in the FK5 and FK4 catalogues are used for estimating the covariance function (Moritz,1980)

$$Q(\psi) = \frac{1}{8\pi^2} \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \int_0^{2\pi} f(\alpha,\delta) f(\alpha',\delta') \cos\delta\, d\delta\, d\alpha\, d\varphi. \tag{1}$$

Here (α,δ), (α',δ') are the equatorial coordinates of stars, ψ is the angular distance between two stars and φ is the relative azimuth.

Let us separate the differences $f(\alpha,\delta)$ into systematic and random components $f(\alpha,\delta)=s(\alpha,\delta)+r(\alpha,\delta)$. Providing that $s(\alpha,\delta)$ and $r(\alpha,\delta)$ are not correlated we can write

$$Q(\psi) = Q_s(\psi) + Q_r(\psi). \tag{2}$$

If the random component $r(\alpha,\delta)$ is a white noise, its covariance function $Q_r(\psi)$ will be δ-function and we will have

$$Q(\psi)_{\psi\neq 0} = Q_s(\psi), \qquad Q(0) = \sigma_s^2 + \sigma_r^2 , \tag{3}$$

where $\sigma_s^2=Q_s(0)$ and $\sigma_r^2=Q_r(0)$ are the variances of the systematic and random components of the differences $f(\alpha,\delta)$, respectively.

The covariance functions $Q_s(\psi)$ have been computed by Gubanov and Titov (1992) by formula (1) using the systematic components $s(\alpha,\delta)$ of the (FK5-FK4) differences. Theoreticaly the real covariance functions should be positive-definite. Therefore, the coefficients c_n of their analytical approximation by Legendre polynomials

I.I. Mueller and B. Kołaczek (eds.),
Developments in Astrometry and Their Impact on Astrophysics and Geodynamics, 397–401.

https://doi.org/10.1017/S0074180900173577 Published online by Cambridge University Press

$$Q_s(\psi) = \sigma_s^2 \sum_{n=1}^{\infty} c_n P_n(\cos\psi) \qquad (4)$$

should be not negative. These coefficients obtained by Gubanov and Titov (1992) are presented in Table 1. The functions $Q(\psi)$ are illustrated on Fig.1.

TABLE 1. Coefficients c_n and their RMS σ_n of the $Q_s(\psi)$ approximation by Legendre polynomials

n	$\Delta\alpha\cos\delta$	σ_n	$\Delta\delta$	σ_n	$\Delta\mu\cos\delta$	σ_n	$\Delta\mu'$	σ_n
1	.01	.05	.00	.01	.01	.05	.00	.06
2	.15	.12	.31	.10	.12	.11	.07	.10
3	.23	.19	.06	.08	.16	.18	.16	.08
4	.20	.08	.01	.15	.37	.17	.01	.16
5	.01	.14	.07	.10	.05	.16	.01	.11
6	.00	.14	.01	.12	.01	.17	.18	.18
7	.01	.08	.01	.06	.06	.10	.04	.11
8	.01	.17	.04	.08	.00	.17	.09	.17
9	.05	.14	.02	.04	.07	.16	.03	.09
10	.14	.22	.04	.08	.06	.10	.02	.11
11	.01	.06	.02	.04	.01	.04	.00	.06
12	.00	.06	.12	.13	.00	.04	.13	.16
13	.00	.06	.01	.06	.00	.03	.03	.06
14	.00	.07	.01	.11	.00	.04	.02	.12
15	.03	.05	.00	.02	.01	.02	.02	.06
16	.01	.05	.01	.11	.00	.02	.01	.05
17	.00	.05	.00	.03	.00	.02	.00	.03
18	.01	.04	.04	.08	.00	.01	.01	.03
19	.03	.05	.03	.07	.00	.01	.01	.02
20	.02	.04	.08	.13	.00	.02	.02	.03
21	.01	.03	.01	.02	.00	.02	.00	.03
22	.00	.03	.00	.06	.00	.02	.01	.04
23	.00	.02	.01	.02	.00	.01	.02	.04
24	.01	.03	.04	.04	.00	.01	.04	.05
25	.00	.02	.00	.02	.00	.01	.00	.02
26	.01	.02	.01	.02	.00	.02	.01	.01
27	.00	.02	.00	.01	.00	.01	.00	.01
28	.02	.03	.01	.02	.01	.02	.00	.02
29	.01	.01	.01	.01	.01	.02	.01	.01
30	.01	.01	.03	.02	.01	.02	.04	.02
31	.01	.01	.01	.01	.00	.01	.01	.01

The functions $Q(\psi)$ and $Q_s(\psi)$ are similar to each other at $\psi\neq0$ in accordance with (3). Their differences in $\psi=0$ give the variance of random components $\sigma_r^2=Q_r(0)=Q(0)-Q_s(0)$. The global estimates of the RMS σ_s and σ_r obtained from (3) and (4) are given in Table 2.

https://doi.org/10.1017/S0074180900173577 Published online by Cambridge University Press

TABLE 2. The RMS of the (FK5-FK4) differences in 0.01 arcsec

σ \ f	$\Delta\alpha\cos\delta$	$\Delta\delta$	$\Delta\mu\cos\delta$	$\Delta\mu'$
σ_s	2.2	1.8	9.1	6.1
σ_r	7.8	5.8	22.6	14.6

A more detailed analysis of the differences $f(\alpha,\delta)$ reveals those which are not stationary. First, the RMS σ_r of their random components depend on the declinations (see Fig.2), and second, their covariance functions $Q_s(\psi)$ are not strictly isotropic but depend on the azimuth φ. For the determination of non-isotropic two-dimentional covariance functions it is necessary to exclude the integration on the azimuth φ in formula (1). As an example, the normalized correlation function $R_s(\psi,\varphi)=Q_s(\psi,\varphi)/Q_s(0,0)$ of the (FK5-FK4) differences $\Delta\alpha\cos\delta$ taken from Gubanov and Titov (1992) is shown on Fig.3.

The applications this results are as follows:

1. According to Gubanov and Titov (1992) there are essential reasons to suppose that both the FK5 systematic errors and the (FK5-FK4) systematic differences have approximately similar statistical structure, which may be described by the correlation function $R_s(\psi)=Q_s(\psi)/Q_s(0)$.

Using the global estimates $(\sigma_s)_{FK5}$ of the FK5 systematic errors by Fricke, Lederle and Schwan (1988) we can find the a'priori estimates of the FK5 covariance functions by the formula

$$Q_s(\psi)_{FK5} = (\sigma_s^2)_{FK5} R_s(\psi)_{FK5-FK4}. \quad (5)$$

There is an analogous formula for the non-isotropic covariance function.

2. As the FK5 covariance function $Q_s(\psi)$ is a reproducing kernel in Hilbert space (Moritz (1980)), it is possible to calculate the a'priori weights of the zonal harmonics coefficients of the FK5 systematic errors representation by spherical functions by Gubanov and Titov (1992). These weights can be used for the regularization of the parametric adjustment process (Moritz(1980) and Gubanov(1991)) in the case of improvement of the FK5 system using new observational data.

3. In the introduction to the FK5 (Fricke, Lederle and Schwan (1988)) the estimates of the FK5 random errors are given. Therefore we have the variances $(\sigma_r^2)_{FK5}=Q_r(0)_{FK5}$. Using this information we can reconstruct the covariance function $Q(\psi)_{FK5}$ by formulas (3), (4) and (5). Then, in accordance with the least-squares collocation techniques [1] we can create the optimal filter for both the FK5 and other catalogues as an alternative to the analytical approximation process.

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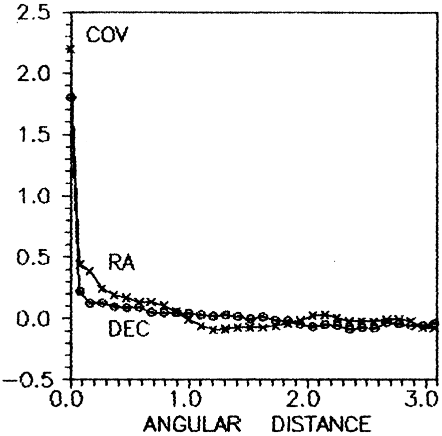

Fig.1a

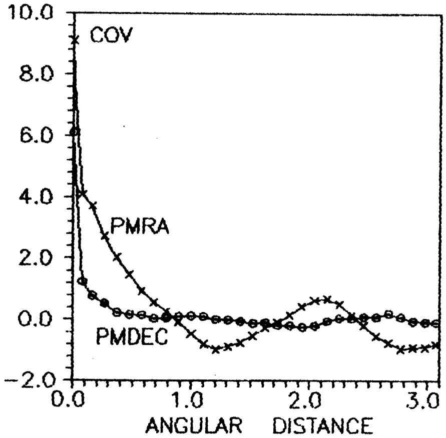

Fig.1b

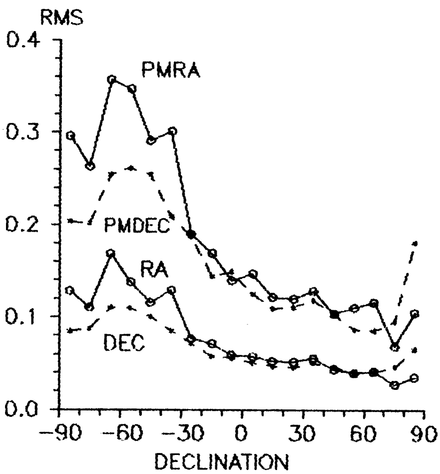

Fig.2

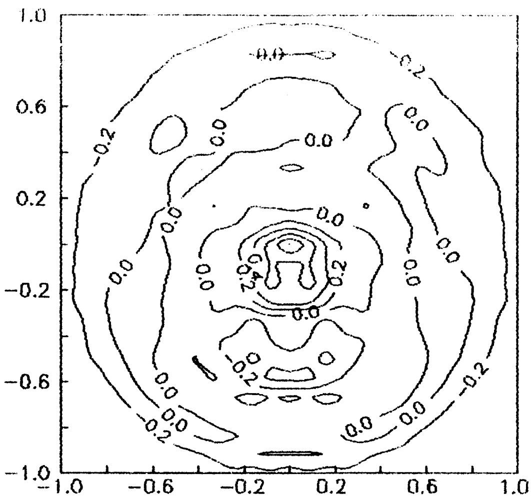

Fig. 3

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