CHAPTER 7

Freight network equilibrium: a review of the state of the art

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The calculation of an equilibrium is fundamental to the positive analysis of any economic system. In this sense it is no surprise that one would wish to calculate an equilibrium among spatially separated markets connected by a freight transportation system, for that problem is evidently basic to regional and national economic forecasting. When the spatially separated markets of interest are represented as nodes of a network, the freight system infrastructure as links, together with some additional nodes to model modal or carrier junctions and transfer points, and some attempt to capture the complex hierarchy of decisions inherent in freight transportation is made, we refer to this equilibrium problem as the "freight network equilibrium problem." What is a surprise to the uninitiated is that a theoretically rigorous representation of such an equilibrium and its efficient computation can be quite difficult, and that these are, to some extent, unsolved problems. In this chapter we endeavor to make this last point clear, to review some of the recent advances that have been made and to suggest future research necessary to a complete understanding of freight network equilibrium.

At first glance the freight network equilibrium problem, as we have described it so far, seems essentially the same as the spatial price equilibrium problem discussed in the seminal words of Samuelson (1952) and Takayama and Judge (1971). However, as will become apparent, there are many aspects of decision making in the transport of freight that are not at all addressed by the conventional spatial price equilibrium model. The introduction of such considerations very much complicates the problem of finding an equilibrium and leads to models sufficiently distinct from the usual spatial price equilibrium model that a new name is warranted – hence our reference to such models as freight network equilibrium models.

We shall distinguish between two key aspects of freight equilibrium. The first is the usual concept of market clearing, which ensures that commodity supply equals commodity demand or, when working with the transportation market alone, that transportation supply equals transportation demand. The second aspect is the equilibrium among freight transportation decisionmakers, wherein no decisionmaker has an incentive to alter his strategy. As we shall see, this later aspect influences supply and demand magnitudes, but is otherwise distinct from market clearing.

https://doi.org/10.1017/CBO9780511895913.008 Published online by Cambridge University Press

The point of departure for modeling equilibrium among freight transportation decisionmakers is the dichotomous perspective on traffic equilibrium originally put forward by Wardrop (1952). Wardrop described two principles of equilibrium for transportation systems which we may summarize as follows (see Fernandez and Friesz, 1983, for a review of these):

Wardrop's first principle. Each user noncooperatively seeks to minimize his cost of transportation. A network flow pattern consistent with this principle is called a "user-optimized equilibrium." Specifically, a user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action.

Wardrop's second principle. The total cost of transportation in the system is minimized. A network flow pattern consistent with this principle is called a "system-optimized equilibrium" and requires that users cooperate fully or that a central authority controls the transportation system. Specifically, a system-optimized equilibrium is reached when the marginal total costs of transportation alternatives are equal.

The essential difficulty in applying these concepts of equilibrium to freight systems lies in the fact that freight network flow patterns are determined by the decisions of more than just users. In fact freight flow patterns, unlike urban automobile flow patterns, are determined by the transport system users *and* by owner operators. The users and owner operators are generally considered to be synonymous in the automobile case, since network infrastructure is held fixed. However, even when network infrastructure is fixed, this luxury is not possible in the freight case, since the means of transport over the network is typically owned and operated by entities distinct from those originating goods shipments; that is, the owner operators are distinct from the users.

Conventional wisdom has tended to model freight systems by assuming that the owner operators or "carriers" (defined precisely in the next section) obey Wardrop's second principle. This leaves unanswered the questions of how to model the users or "shippers" (also defined precisely in the next section) and of the interrelationship between shippers and carriers. This point is developed more fully in the next and subsequent sections. For convenience, we refer to any freight model that enforces market clearing and determines an equilibrium among any subset of relevant decisionmakers as a "freight equilibrium model." Clearly, however, many of the models discussed subsequently do not calculate a true equilibrium in the freight transportation market since the behavior of shippers and carriers is not determined simultaneously and consistently.

1. Description of existing predictive freight network models

Descriptions of the key predictive freight network models developed since the 1960s are given in this section in order to better understand advances in the

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state of the art as well as the remaining shortcomings of freight network equilibrium models. Not all the models discussed in this section are *true* equilibrium models. Throughout our discussion in this and subsequent sections, shippers will be thought of as those decision-making entities desiring a particular commodity at a particular destination, and carriers as those decision-making entities that actually transport the commodities, thereby satisfying the transportation demands of shippers.

1.1. Harvard–Brookings

Without question, the first significant multimodal predictive freight network model was that developed by Roberts (1966) and extended by Kresge and Roberts (1971). This work has become known in the literature as the Harvard–Brookings model. In this model, applied to the transport network of a developing country (Colombia), links correspond to transport routes (not the actual physical transport links) and nodes to cities or regions. The model is explicitly multimodal–including highway, rail, air, and water modes; it is also explicitly a multicommodity model. Shippers' modal choices and general routings are determined from shortest path calculations for the intermodal network, with arc impedances measured as constant unit origin-to-destination (path) perceived shipping costs; these are used in a standard Koppmans–Hitchcock transportation submodel to determine commodity-specific flows between origin–destination (OD) pairs (see, for example, Wagner, 1975, for a definition of the Koopmans–Hitchcock transportation problem). The commodity production and consumption numbers needed to specify the constraints of the Koopmans–Hitchcock trip-distribution submodel are obtained from a separate macroeconomic driver. For freight flows of highly aggregated heterogeneous commodity groups, a standard gravity submodel (see Isard, 1975, chap. 3) rather than a Hitchcock formulation was used to determined OD flows. Once the trip-distribution process is completed, flows are assigned to the intermodal network by using the same shortest paths found previously (those based on impedances that are shippers' perceived costs). The assignment of flows is completed by adding appropriate volumes to account for the necessary backhaul to secure another load; backhaul trips are assumed to be routed exactly as the forehaul trips.

1.2. CACI

As part of the National Energy Transportation Study (NETS), CACI, Inc., developed a multicommodity, multimodal freight network model referred to as the Transportation Network Model (TNM) (CACI, 1980; Bronzini, 1980a–c). Although the model is intended for rather general application, it did not, as originally conceived, attempt to predict the freight shipment OD pattern,

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transportation demand being fixed in the model. The original version did, however, account for all plausible freight modes and could handle any number of commodities. There are two basic behavioral assumptions of the model:

1. Freight routing results exclusively from the decisions of shippers seeking to find minimum-cost paths.
2. The cost on a path is a linear combination of dollar cost, time, and energy use.

These assumptions ignore any role that carriers play in the routing of freight shipments. In addition, the cost measure is unusual in that it combines concerns usually limited to shippers, such as time, and to carriers, such as cost, as distinct from price. The inclusion of energy costs, normally considered by carriers as only a portion of total monetary costs, is used to assess various energy-conservation scenarios.

Although it is claimed that the model is both a multicommodity and equilibrium assignment model, no version of the model can rigorously accomplish both aims. In an early version of the model, the "multimodal network model" (MNM), multiple commodities are treated by loading the network sequentially. Since arcs are given absolute capacity constraints, the final loading may depend on the particular sequence in which the commodities are loaded. In addition, this version only approximates an equilibrium solution through an iterated capacity constraint algorithm. A more recent version of the model, the "transportation network model" (TNM), while implementing an equilibrium assignment model for shippers, uses aggregate instead of commodity-specific cost functions; it is in essence a single-commodity model. Backhauling is treated indirectly through adjustments to link delay and cost functions. The most recent version of the model employs a separate submodel to determine carrier routing behavior (Bronzini and Sherman, 1983); this carrier submodel employs fixed impedances and a shortest path algorithm.

Some tests of the model have been reported, although most use only highly aggregated data. These tests indicate an ability to replicate aggregate modal split data. Total link loadings generated by the model have been compared against historical link usage with significantly poorer results.

1.3. Peterson's model

Peterson (see Peterson and Fullerton, 1975) has proposed a predictive rail network model that employs either Wardrop's first or second principle of equilibrium assignment to model carrier decisions, although he states that the second principle (system optimization) is preferable for modeling freight sys-

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tems. The model is in the form of a mathematical program whose objective function is constructed from arc delay measures that depend on aggregate flow volumes and is meant to be minimized. Presumably, these delay measures are obtained from one of the queueing models reported by Peterson and Fullerton (1975). The model assumes that transportation demand is fixed and determined exogenously. The constraints of the mathematical program are the usual flow conservation and nonnegativity constraints. The model does not explicitly treat multiple carriers or multiple commodities; backhauling is not addressed. Tests of predictive capability against known data are not available.

1.4. Lansdowne's model

Although the rail freight traffic assignment model developed by Lansdowne (1981) is not as general in its scope as the other models reviewed here, it is included because of its explicit attempt to treat shipper–carrier interactions in a manner that conforms to current rail industry practice. The model assumes as input a rail-specific trip matrix. It is thus a unimodal as well as a fixed-demand model. Its output is a set of rail paths that include the interline locations, where control of the freight shipment is transferred from one carrier to another. The route that a shipment follows on the rail network, including the number and location of interlining points where rail carriers transfer control of the shipments, is jointly determined by the involved shippers and carriers. The location of the interlining point may be specified by the shipper, but is more commonly specified by the originating railroad. The originating railroad, while seeking to maximize its revenue, must also offer a reasonable level of service to the shipper to attract its business. These principles can be summarized as follows:

1. The only routes used will be those that have a minimum number of interlining points.
2. Each carrier will use the shortest path within its subnetwork.
3. Of the eligible routes, the one that maximizes the originating carrier's share of the revenue is selected.
4. If there is more than one potential originating carrier, the shipment is divided among all the potential carriers according to some prespecified rule.

Even though each interline movement creates additional expense, delay, and uncertainty for the shipment and would generally be avoided, the first assumption ignores a number of very real operating concerns. The condition and layout of the track being used and the delays caused by line-haul and intermediate yard congestion, for example, would certainly influence a carrier's routing choice. The third and fourth assumptions, like the first, appear to be

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reasonable operating premises because the originating carrier supplies the cars and negotiates the rates and is, therefore, in a strong bargaining position with the other railroads. Also, in regional analyses, the shippers may actually be an aggregation of many shippers, and thus it can be expected that each available railroad will originate part of the shipment. Backhauling is not addressed, and no tests of predictive capability against known data have been reported.

1.5. Princeton's model

The Princeton railroad network model (Kornhauser et al., 1979) employs two submodels. The first, called the "intercarrier route generation model," utilizes shortest path techniques on a series of single-carrier rail networks; carrier arc costs are fixed and determined judgmentally to ensure that flow is primarily along main lines. The intracarrier routings, and thereby implicitly the fixed impedances, have been validated qualitatively through the presentation of graphic displays of routes to rail system managers who were asked whether their railroads routed in accordance with the displays. The second submodel, called the "intercarrier route generation model," is based on an intercarrier network that includes junction links between separate carriers known to interline. Very high impedances are assigned to such junction links to ensure that interlining along a path between a given OD pair does not occur with excessive frequency. Again, impedances are determined judgmentally and validation is through graphic displays that allow comparison with selected historical route information. Arc loadings predicted in the Princeton model are, however, apparently not checked against published Federal Railroad Administration (FRA) density codes. Backhauling is not addressed.

1.6. Pennsylvania–Argonne National Laboratory model

The University of Pennsylvania (Penn) has developed a predictive freight network model called the "freight network equilibrium model" (FNEM) (see Friesz et al., 1981). This model was constructed under a research program sponsored by the U.S. Department of Energy and involved staff of Argonne National Laboratory (ANL) as well as Penn. FNEM explicitly treats the decisions of both shippers and carriers for an intermodal freight network with nonlinear cost and delay functions that vary with commodity volumes to model congestion externalities.

FNEM treats shippers and carriers sequentially as depicted in Figure 7.1. In particular, shippers are assumed to be user optimizers trying to noncooperatively minimize the delivered price of commodities they ship, and therefore Wardrop's first principle is used to describe their behavior. The shipper submodel is an elastic transportation demand, user-optimized trip

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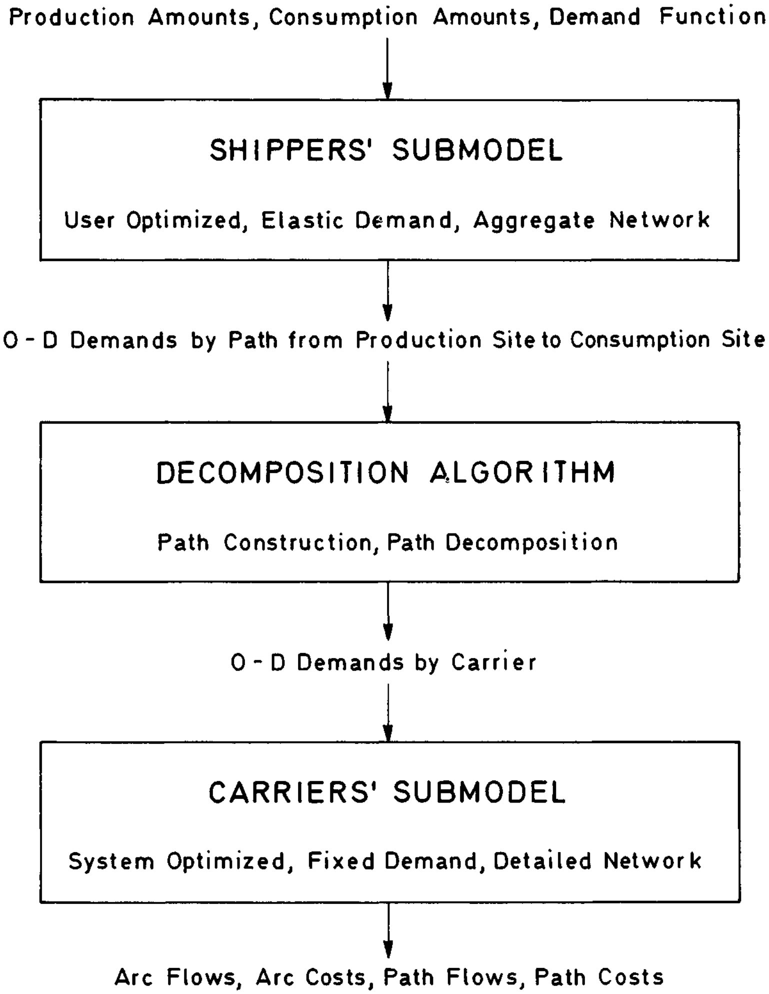

Figure 7.1. Flowchart of FNEM.

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assignment model expressed as a mathematical programming problem and solvable by the usual Frank–Wolfe-type algorithms for such problems (Gartner, 1977), with diagonalization methods used to treat modal or commodity interactions (see Fernandez and Friesz, 1983). The shipper submodel employs an aggregate, perceived network including only the OD pairs, transshipment nodes, interline nodes, other key nodes, and associated links actually considered by shippers in their decision process. Solution for flows over the aggregate shippers' network at user equilibrium leads to OD demand levels by commodity, shipper, mode, or mode combinations used, and carriers used. Following a detailed bookkeeping exercise identified as the "decomposition algorithm" in Figure 7.1, one obtains carrier-specific OD demands. The carrier demands are fixed numbers and recognize that carriers control specific subnetworks of the entire intermodal network whose origin and destination nodes include interline points with other carriers. The individual carrier networks represent the full physical detail of the actual freight system. Carriers are assumed to be individual profit maximizing firms. However, since the demands for the carriers' services are fixed in this sequential modeling framework, the revenue a carrier receives is constant and, thus, profit maximization is equivalent to cost minimization. Because the carriers are viewed as sequentially reacting to the transportation demands set up by the shippers, each carrier submodel is a fixed-travel-demand, system-optimized, trip-assignment model expressed as a mathematical programming problem. Each carrier-specific mathematical program is, like each shipper submodel, solved by Frank–Wolfe-type algorithms with diagonalization methods (Gartner, 1977; Fernandez and Friesz, 1983). Individual carrier flow patterns are combined to obtain flow patterns for the entire intermodal network. Backhauling is treated through adjustments to cost and delay functions. The model replicates FRA density codes at least 50 percent better than the CACI model described previously.

In fact, extensive validation tests of FNEM have been conducted (Friesz, Gottfried, and Morlok, 1983), and the results from these tests indicate that FNEM provides a substantial improvement in predictive capability over earlier models. Section 5 will describe these tests and certain applications of FNEM in greater detail.

It is important to realize that FNEM is not limited to strictly increasing cost and delay measures such as those encountered in urban passenger applications. When nonincreasing functions are employed, the objective functions in the shippers' and carriers' submodels may fail to be convex, giving rise to multiple equilibria. When this occurs, the Frank–Wolfe algorithm is still utilized to effect an efficient solution, but care is taken to select step sizes that guarantee convergence to local optimality; in this way FNEM may be relied upon to find one of the nonunique equilibria that may occur with nonincreas-

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ing functions. Avriel (1976) provides details of the step-size selection rules that guarantee convergence of the Frank–Wolfe algorithm to a local optimum when the objective function is nonconvex.

2. Typology of predictive freight network models

For effective comparison of the models presented in the previous section, it is useful to construct a typology of predictive freight network models. In this section, existing models are differentiated and evaluated according to the following criteria, which pertain primarily to routing and modal choice: (1) treatment of multiple modes, (2) treatment of multiple commodities, (3) sequential loading of commodities, (4) simultaneous loading of commodities, (5) treatment of congestion phenomena via nonlinear cost and delay functions, (6) inclusion of elastic transportation demand, (7) explicit treatment of shippers, (8) explicit treatment of carriers, (9) sequential solution of shipper and carrier submodels, (10) simultaneous solution of macroeconomic model and transportation network model, (11) sequential macroeconomic and network models, (12) simultaneous macroeconomic and network models, (13) solution employing nonmonotonic functions, (14) explicit treatment of backhauling, (15) explicit treatment of blocking strategies, and (16) inclusion of fleet constraints. The models described previously are evaluated in terms of these sixteen criteria in Table 7.1.

Criterion 1 recognizes that multiple modes are used to carry freight shipments. The data in Table 7.1 indicate that three of the six models address multimodal interactions whereas the remainder are unimodal (rail) models.

Criterion 2 incorporates the fact that freight transportation involves multiple commodities with distinct transportation-cost characteristics and different shipping time requirements that prevent meaningful treatment as a single commodity Criterion 3 refers to the fact that it is sometimes possible to rank commodities, assigning them individually to the network in order from highest to lowest shipment priority. Some commodity disaggregation schemes will lead, however, to commodities of identical shipment priority but with distinct unit-cost characteristics; for these commodities, a simultaneous loading procedure is required (criterion 4).

Criterion 5 recognizes the general variation of relevant costs and delays with flow volume due to congestion economies and diseconomies, and criterion 6 refers to the fact that demand for transportation will generally vary with transportation costs and delays. Two of the models incorporate elastic demand functions in the form of trip-distribution models to determine OD flow levels. The remainder of the models require fixed trip matrixes as input.

Criteria 7 and 8 address the fact that routing and modal choices in freight systems are the results of decisions of both shippers and carriers and that these

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Table 7.1. *Typology of predictive freight models with respect to routing and modal-choice characteristics*

	Criteria[a]															
Model	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Harvard–Brookings	Yes	Yes	Yes	Yes	No	Yes	Yes	No	NA	NA	Yes	No	No	No	No	No
CACI	Yes	Yes	Yes	No	Yes	No	Yes	No	NA	NA	Yes	No	No	No	No	No
Peterson	No	No	NA	NA	Yes	No	No	Yes	No	No	Yes	No	No	No	No	No
Lansdowne	No	Yes	Yes	No	No	No	Yes	Yes	Yes	No	Yes	No	No	No	No	No
Princeton	No	Yes	Yes	No	No	No	Yes	Yes	Yes	No	Yes	No	No	No	No	No
Penn–ANL (FNEM)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	Yes	No	No	No

[a]Criteria
1. Multiple modes
2. Multiple commodities
3. Sequential loading of commodities
4. Simultaneous loading of commodities
5. Congestion
6. Elastic transportation demand
7. Explicit treatment of shippers
8. Explicit treatment of carriers
9. Sequential shipper and carrier submodels
10. Simultaneous shipper and carrier submodels
11. Sequential macroeconomic and network models
12. Simultaneous macroeconomic and network models
13. Nonmonotonic functions
14. Explicit backhauling
15. Blocking strategy
16. Fleet constraints

NA = not applicable

groups obey distinct behavioral principles that may at times have conflicting goals. Three of the six models explicitly treat shippers and multiple carriers, and of these three only the Penn–ANL model is a multimodal model. Criteria 9 and 10 refer to whether one ascertains the decisions of shippers first and then the decisions of the carriers or determines both simultaneously. Only a simultaneous determination gives a true equilibrium; otherwise there exists the possibility of further adjustments by shippers whose perceptions of freight transportation levels of service differ from those actually provided by the carriers. None of the models discussed previously determines these decisions simultaneously.

Criteria 11 and 12 recognize that virtually all reported freight network models use as input fixed supplies and demands of individual commodities obtained from a separate regional or national economic model. Generally, such economic models employ assumptions about freight transportation costs and the question naturally arises of whether the network model outputs are consistent with those costs. Iteration between the economic model and the network model in an attempt to produce consistency is, of course, a heuristic device with no rigorous convergence properties; only a simultaneous solution of the model that generates the supplies and demands of individual commodities and the network model will always result in the desired consistency.

Criterion 13 refers to the ability of a given model to treat nonmonotonic functions, particularly nonmonotonic cost functions that are expected to occur as a result of average rail operating costs that initially decline as volume increases and then begin to increase as capacity is approached (see, for example, Morlok, 1978). When nonmonotonic functions are used, multiple equilibria may exist.

Criterion 14 recognizes that a large portion of traffic is made up of empty rolling stock, empty barges, and empty trucks that contribute to costs and congestion. Freight transportation's dependence on the availability of empty rolling stock necessitates considerable attention to backhauling operations if carriers are to be able to satisfy shippers' transportation demands. Criterion 15 recognizes that rail freight flows comprise trains of varying length, made up of different types of rail cars, which are frequently "blocked" into groups bound for common or similar destinations. This blocking has a significant impact on yard delays encountered by a shipment. Criterion 16 refers to the fact that there are generally restrictions on the supply of rolling stock and vehicles that cannot be violated in the short run; as such, this criterion is intimately related to criterion 14 dealing with backhauling.

The key unresolved issues with respect to freight network models are (1) the simultaneous treatment of shipper and carrier decisions, (2) the simultaneous solution of the economic model that generates supplies and demands and the network model itself, (3) the treatment of nonmonotonic functions,

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particularly nonmonotonic cost functions, (4) explicit treatment of backhauling operations, (5) explicit treatment of blocking strategies, and (6) fleet constraints. In a later section we present some recent advances in the development of models addressing the first two of these unresolved issues.

3. Simultaneity of shipper and carrier decisions

Perhaps the greatest lack of behavioral realism in the models treating both shippers and carriers is the sequential perspective on shipper and carrier decisions. Such a perspective, as Friesz and Morlok (1980) have pointed out, is not consistent with the calculation of true equilibrium flow values because it presupposes that carriers will provide commodity routings that give levels of service perfectly consistent with those levels of service perceived and anticipated by shippers. However, because the shippers' costs depend on the carriers' costs and routings, this can happen only if the shippers know beforehand the carriers' costs (or, more precisely, that portion passed through to each shipper from each carrier). But such perfect foresight is not possible because of congestion externalities that make the carriers' costs depend on routing patterns. Similarly, no carrier may establish its routes, and hence its costs, before knowing the shippers' demands. Since neither category of decisionmaker can finalize its decisions without knowing the decisions of the other category, the problem is one in which decisions by shippers and carriers must be modeled simultaneously.

It is possible to write a single mathematical program to describe simultaneous shipper–carrier decisionmaking, as Harker (1981) and Harker and Friesz (1982) have shown. This is accomplished by representing the shippers through the extended notion of Wardrop's first principle employing delivered price. The Harker–Friesz formulation, however, utilizes nonlinear, and unfortunately nonconvex, constraints patterned after Tan et al. (1979) to describe such shipper behavior. The behavior of carriers is modeled using an extended notion of Wardrop's second principle of system optimality; this perspective, when carrier networks are appropriately defined, yields a single objective function describing the carriers' operating costs that is minimized. The Tan et al. types of constraints are articulated in terms of path variables, thereby necessitating path enumeration as well as special numerical procedures to treat their nonlinear, nonconvex nature. Seemingly, these constraints may be replaced by a variational inequality, after Smith (1979) and Dafermos (1979, 1982), and the method of constraint accumulation suggested by Marcotte (1981) employed to overcome the need to enumerate paths fully and possibly to achieve some increased computational efficiency. Fisk and Boyce (1983) suggest a similar approach but represent the shippers' behavior as a nonlinear complementarity problem that appears as constraints in the

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carriers' extremal problem. Nonconvexity still remains in these approaches because the variational inequality or complementarity problem one employs to model Wardrop's first principle, like the constraints of Tan et al., is nonconvex.

The Harker–Friesz mathematical programming formulation, because of its innate nonconvexity, prevents investigations of the uniqueness of a shipper–carrier equilibrium; that nonconvexity also severely complicates proofs of existence of a combined equilibrium. Questions of existence and uniqueness are much more easily explored through nonlinear complementarity and pure variational inequality formulations of the problem. These questions are of theoretical interest and also affect computation in that one desires to know whether there is one equilibrium solution, more than one, or none.

Friesz, Viton, and Tobin (1985) have proposed a variational inequality formulation of the simultaneous shipper–carrier equilibrium problem that leads directly to equivalent optimization problems that may be solved quite efficiently using the usual Frank–Wolfe algorithm together with diagonalization as is done for urban network equilibrium problems. These solution methods are convergent for the simultaneous shipper–carrier equilibrium problem under appropriate conditions. Moreover, the variational inequality allows one to study the existence and uniqueness of solution of a simultaneous equilibrium. If the variational inequality is such that it admits multiple equilibria, the Frank–Wolfe diagonalization approach may be used to compute a nonunique equilibrium by utilizing appropriate step sizes, as discussed in Avriel (1976).

The variational inequality formulation of Friesz, Viton, and Tobin (1985) assumes marginal cost pricing on the carriers' part. Although this certainly is not the general case, the arguments leading to their model formulation are particularly easy to follow and are, hence, of pedagogic value, since the approach used to construct the variational inequality may be extended to other economic settings. For these reasons we go to some length in this section to present the single variational inequality for simultaneous shipper–carrier equilibrium with marginal-cost pricing.

3.1. Assumptions and notation for simultaneous shipper–carrier equilibrium

First assume that there are many shippers, each with equivalent information, who compete for limited transportation services as well as for certain homogeneous commodities needed for their individual economic activities (either production or consumption). Each shipper is assumed to be a profit-maximizing economic agent. Because of the assumed presence of many shippers, each shipper takes the supply prices (the prices at origin markets), the demand prices

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(the prices at the destination markets), and the effective transportation rates (which include the economic value of shipment delays as well as the actual rates that carriers charge shippers) as given when deciding upon its profit-maximizing pattern of origin–destination commodity flows. As we shall prove in the next section, this assumed profit-maximizing behavior leads to a situation wherein the shippers noncooperatively minimize the delivered price of each commodity. Because the delivered price measure reflects congestion externalities, the shippers may be viewed as the players of the noncooperative mathematical game with a Cournot–Nash equilibrium solution in quantities that is constrained by the assumptions to be made regarding the carriers' behavior.

Each freight carrier is assumed to be a profit maximizer facing an exogenous market price for its service. This assumption results in cost-minimizing behavior and marginal-cost pricing by the individual carriers. Perfect competition among freight carriers, of course, leads to both outcomes. But, interestingly, so do other assumptions about market structure, notably one in which two or more carriers produce a given transportation service (product) and cover costs. Such a configuration is termed ''sustainable'' by Baumol, Panzar, and Willig (1982; see especially their Proposition 11B5). The observed tendency of carriers to retain possession of a shipment and to avoid interlining with other carriers may be captured by including appropriately high penalty costs for interline movements.

Because these assumptions may be regarded as controversial, it is appropriate to discuss them in somewhat more detail. First it is generally accepted that cost functions in the rail industry exhibit economies of density; see, for example, Keeler (1983) for a review of the evidence. Under these conditions the sort of competitive equilibria discussed in this section will never obtain. That observation, however, is far from depriving them of interest, for the (loss-incurring) marginal-cost price solution, with appropriate subsidies, remains the optimal solution to which all other proposals (complete deregulation, Ramsey pricing, and so forth) should be compared. Certainly, it is unlikely that the sort of reregulation entailed by the marginal cost pricing with lump-sum payments will be enacted by Congress. Nonetheless, the criterion value of service-quality calculations based on competitive assumptions remains.

Second, it is equally well accepted that the trucking industry exhibits little or no economies of density. Marginal-cost pricing is therefore feasible, and our analysis is directly applicable to this sector of the freight transport industry. Moreover, when applied to this sector, observing that under deregulation barriers to entry will be small, our detailed analysis of freight flows over network paths will have a predictive function, as well as an evaluative one. That is, given empirical estimates of model parameters, one could compute

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the equilibrium situation in a contestable industry. More will be said concerning the marginal-cost pricing assumption in Section 3.4.

It is assumed that the extent of the freight network is given – in other words, that the question of optimal investment to influence network capacity does not arise. The behavior of the shippers and carriers are linked through flow-conservation constraints that, in addition to their usual role of ensuring that flow gains and losses do not occur except where intended, guarantee that carriers meet the transportation demands established by shippers. The concept of dual shipper and carrier networks put forward in Friesz and Morlok (1980), Friesz et al. (1981) and Friesz, Gottfried, and Morlok (1983) and mentioned in the discussion of FNEM in Section 1 is also used in this formulation.

That is, although each shipper has identical information about the transportation marketplace, the shippers are not generally cognizant of the entire set of routing options available; moreover these options are generally not within any shipper's purview, but rather are usually selected by the carrier originating a given shipment. For these reasons shippers are assumed to make their decisions with respect to an aggregate network representing the shippers' perceptions (all equivalent) of the freight transportation system. The set of nodes of this "perceived network" consists of all actual commodity origins and actual commodity destinations, plus interline locations (nodes at which shipments may be transferred between carriers) and transshipment locations (nodes at which shipments may be transferred between modes). Arcs of the perceived network are generally aggregations of actual arcs. Taken together, the perceived arcs represent only the very general and very restricted routing options within the control of shippers. By contrast each carrier has full routing control of some subnetwork of the actual detailed physical freight network and its behavior is modeled on such a subnetwork. The intersection of any two carrier subnetworks is taken to be empty, while the union of all carrier subnetworks is the detailed freight transportation network itself. In cases where two or more carriers share a particular transportation facility, this facility will be represented by two or more separate network elements (arcs or nodes) in the model. The effect of this assumption is merely that individual carriers control their own networks, and not that a given carrier is a local spatial monopolist. Clearly for consistency all the origins, destinations, interline locations, and transshipment locations of the shippers' network are represented in the carrier subnetworks, although generally the carrier subnetworks and, hence, the entire detailed freight network will include many more nodes to describe the junction of all physical arcs as they occur in the real world or some abstraction that will never be less detailed than the shipper's network.

The distinctions made among types of networks requires the careful use of certain terminology. In particular a true origin–destination (OD) pair differs, in general, from a carrier-specific origin–destination pair. A true OD pair

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consists of the actual origin and the actual final destination of some shipment. A carrier-specific OD pair consists of the node of the carrier's subnetwork at which the carrier first takes responsibility for a shipment and the node at which it relinquishes responsibility; as such, a carrier-specific OD pair may consist of one or two interlined nodes and not the true origin and/or not the true destination. The interdependency of shippers' and carriers' behavior is captured through constraints requiring that carrier OD travel demand equal the sum of shippers' flows that traverse the carrier OD pair in question.

A simple example may clarify these network distinctions. In Figure 7.2 we show a shippers' network, consisting of a single origin O and destination D; this is referred to as the true OD pair. The set of solid arcs are the arcs of the shippers' perceived network. From the shippers' point of view, this perceived network is all that matters; shipments must get from O to D with the possibility of transshipment/interline at I. In this example, shippers see two choices. But from the point of view of those who actually provide the transportation (the carriers) matters may be immeasurably more complex. Figure 7.2 also shows the assumed carriers' subnetworks, the union of which is the actual freight network. In this depiction broken lines of different types represent arcs of different subnetworks; that is, different carriers or different modes of some single carrier. It is readily apparent that the true origin and destination (on the shippers' network) are represented in the actual network as well as in the shippers' network. Further, the true OD pair may differ from a carrier-specific OD pair [see for example the carrier-specific OD pairs (O,I) and (I,D)], as the general structure previously described allows.

In the exposition that follows we employ the device of a multicopy network to simplify the notation. This concept allows us to suppress all indexes of variables and parameters, save those that identify the particular network element with which these quantities are associated and yet to treat multiple modes, multiple carriers, and multiple commodities. An example will help to clarify this concept. Consider the extremely simple network with two nodes (1,2) and one arc (a) depicted in Figure 7.3a. Imagine that two commodities are transported by a single mode from node 1 to node 2. Let $\phi_a^k(f_a^1, f_a^2)$, $k =$ (1,2), denote an arbitrary measure (usually some type of cost or delay) associated with arc a where f_a^k is the flow on arc a of commodity k. This situation may be represented with more compact notation using the multicopied network of Figure 7.3b. In that figure arc α corresponds to commodity 1 movements on the actual arc a, and arc β to commodity 2; thus $\phi_\alpha(.,.)$ and $\phi_\beta(.,.)$ are the respective commodity measures on the actual arc a; similarly f_α and f_β are the respective commodity flows on the actual arc a. Consequently we can think of any multicommodity network as having arc measures $\phi_a(\mathbf{f})$, where $\mathbf{f}$ is the *full* vector of network flows, with commodity identities carried by the arc names. Quite clearly multicopy networks can be layered on top of one

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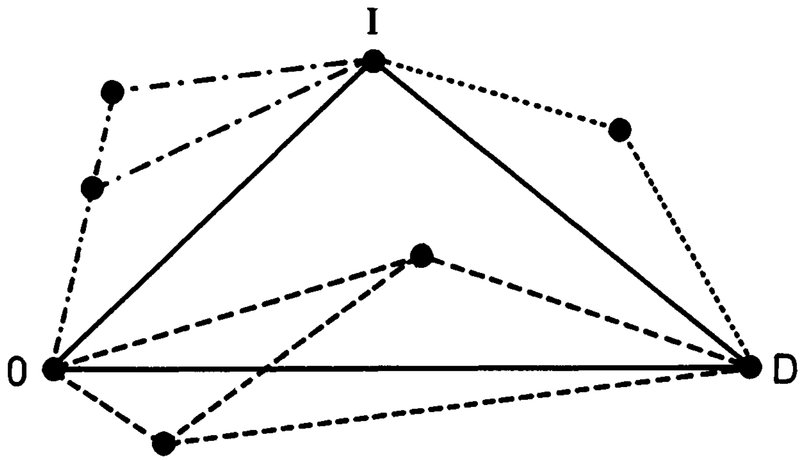

Figure 7.2. Shippers' network.

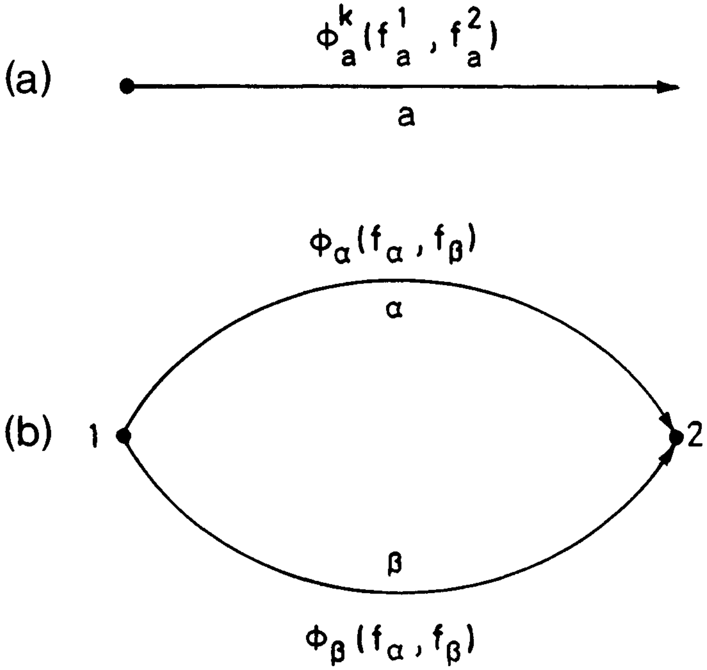

Figure 7.3. (a) and (b). Multicopy network.

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another so that multiple-mode and multiple-carrier, as well as multiple-commodity considerations may be included in arc names, provided one employs nonseparable functions for each arc. It is important to reemphasize that the multicopy network concept is only a notational convenience and does not imply any separability of the network itself with respect to modes, flows, or commodities.

We turn now to a description of the setting of the shipper and carrier problem. We shall consistently use the indexes a and b to denote arcs, the indexes p and q to identify paths and w or v to denote a particular OD pair (i,j) consisting of an origin i and destination j. In light of the previous remarks we must describe two distinct networks: one pertaining to the shippers, the other to the carriers. Once this is done, we specify how the two are linked.

First, the shippers. Their network is made up of a set M of nodes and A of arcs. The shippers' graph is denoted by $\mathbf{G}(M,A)$. Relative to $\mathbf{G}$, we have the set W of OD pairs and the set P of paths. For a particular OD pair $w \in W$, we let P_w stand for the set of paths $p \in P$ connecting i and j. We shall wish to know whether a particular arc $a \in A$ is part of the path $p \in P$ between the elements of w. Consequently, we define the element δ_{ap} of the shippers' arc-path incidence matrix to be unity if $p \in$ P contains $a \in A$; and zero otherwise.

Shippers' decisions result in a vector $\mathbf{f} = (\ldots, f_a, \ldots)$ of flows on arcs, with f_a the flow on arc a; similarly $\mathbf{h} = (\ldots, h_p, \ldots)$ is a vector of path flows. As previously discussed, a particular arc may be congested; this congestion is modeled by an average delay function t_a on arc a. Note that in this chapter delay is measured with respect to the instantaneous traversing of an arc, and so the "average delay on arc a" is really the unit time taken to traverse arc a and *not* the excess travel time over and above some reference value. Path time is assumed to be additive in arc times; hence the delay on path p is given by $t_p = \sum_{a \in A} \delta_{ap} t_a$.

Shipper economics are described by the assumption of price-taking behavior and a demand for transportation S_w between the OD pair $w \in W$. Each S_w is assumed to be a function of the full vector of minimum delivered prices of shipments between all OD pairs. We use the notation u_w to denote the minimum delivered price between OD pair w, and $\mathbf{u}$ to denote the full vector of such delivered prices. Hence, $S_w = S_w(\mathbf{u})$. If we assume a given technology of transportation, then knowledge of transport demands is equivalent to knowledge of commodity shipments. An inverse demand function is defined for each OD pair $w \in W$ and specifies the minimum delivered price u_w of a shipment between w as a function of the full vector of transport demands; we denote the inverse demand function for OD pair w by $\theta_w(\mathbf{S})$. The commodity transported (we may speak of a single commodity because we employ a multicopy network) has a given price π_i at the origin, and the shipper faces a

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rate (price) of r_p per unit of the commodity shipped on its perceived path $p \in P$. In addition, all shippers have a common value ϕ of the disutility of delay for this commodity. The price π_i is assumed to be a fixed value; this might be the case if commodity production at the origin is small relative to a large national market and if transport costs are small relative to other production costs. In this case calculated network equilibria at the (given) commodity prices π_i will be in equilibrium in both the transport and commodity markets. Alternatively, one could incorporate demand and supply for the transported commodities explicitly, and introduce additional equilibrium conditions. The latter course has been pursued by Harker (1983) and Harker and Friesz (1985a–c).

The carrier structure is similarly characterized. We let K denote the set of all carriers; and the index k an individual carrier. Each carrier k has a subnetwork described by the set N_k of nodes, B_k of arcs, and V_k of OD pairs. The graph of the kth carrier is $\mathbf{H}_k$ (N_k, B_k). For the whole set K of carriers we have the sets $N = \cup_k N_k$ of nodes, $B = \cup_k B_k$ of arcs, giving rise to the carrier-system graph $\mathbf{H}(N,B)$. Letting Q denote the set of paths on $\mathbf{H}(N,B)$, we define the carriers' arc-path incidence matrix by the generic element $\Delta_{bq} = 1$ if $q \in Q$ contains $b \in B$; and zero otherwise.

Carrier decisions also result in flows: an arc flow is e_b on $b \in B$, a path flow is g_q on $q \in Q$, and the corresponding vectors of all flows are $\mathbf{e}$ for arcs and $\mathbf{g}$ for paths. The structure of the problem makes each carrier a cost minimizer. We suppose that each carrier has a path cost function that is additive in arc costs. We denote by MC_b the marginal cost of transporting additional traffic on arc b, and by additivity, $\text{MC}_q = \sum_{b \in B} \Delta_{bq} \text{MC}_b$ is the marginal cost on path q. We use the notation y_v to denote the minimum marginal carrier path cost for OD pair v, and $\mathbf{y}$ to denote the full vector of such costs.

Finally, the shippers' and carriers' networks are linked by requiring that M, the set of shipper nodes be a subset of N, the set of carrier nodes. Relative to both networks, we define the element γ_{vp} of the carrier OD/shipper path incidence matrix; $\gamma_{vp} = 1$ if shipper path $p \in P$ contains the carrier OD pair $v \in V$, and zero otherwise.

In the notation given here it is important to reiterate that, in keeping with our earlier remarks, the shippers are assumed to route themselves over a perceived network that includes all origins, destinations, interline nodes, and transshipment nodes. Furthermore, the shippers' network is a multicopied, intermodal, multicommodity network; that is, each link label identifies both a commodity and a mode. Each carrier makes routing decisions for its own detailed subnetwork; the union of all carrier subnetworks is the actual physical intercarrier network. Each subnetwork is a multicopied, intermodal, multi-commodity network, since a carrier may control more than one mode within its

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subnetwork (although frequently a carrier will control only a single mode) and generally moves multiple commodities.

3.2. *Mathematical characterization of simultaneous shipper–carrier equilibrium*

The problem at hand is to characterize the equilibrium network flow pattern; that is, the utilized paths and flows on those paths that will be observed in equilibrium. If shippers are price takers in the final (destination) market and in the transportation factor market, and are also quality takers (where quality has a single dimension, delay), then the delivered price (DP) faced by the shippers desiring to ship commodities in the market w characterized by origin i and destination j, along path p may be expressed as

$$DP_p\,(\mathbf{r},\mathbf{h}) = \pi_i + r_p + \phi\, t_p(\mathbf{h}) \qquad \forall(w = (i,j) \in W, \quad p \in P_w). \tag{1}$$

Clearly (1) states that delivered price is the sum of origin price, π_i, the transportation rate on the path p of the shippers' perceived network, r_p, and the monetary value of delay experienced. Note that in (1) the path delay t_p is a function of the shippers' path flow vector **h**, expressing the presence of congestion in the shippers' network. Some may consider the dependence of delays on flows, leading to congestion phenomena, to be somewhat controversial in the case of trucking. See, for example the exchange between Boyer (1980) on the one hand, and DeVany and Saving (1980) on the other. See, additionally, on road congestion, Keeler and Small (1977). However, congestion cannot be neglected in network analysis, and so it is retained here. Each shipper is assumed to maximize the profit received from moving goods among the spatially separated markets. Defining W' as the set of OD pairs under a particular shipper's control and π_j as the fixed destination price (DP), the profit function for a particular shipper can be written as

$$\sum_{w=(i,j)\in W'} \sum_{p\in P_w} [\pi_j - \pi_i - r_p - \phi t_p(\mathbf{h})] h_p$$

$$= \sum_{w=(ij)\in w'} \sum_{p\in P_w} [\pi_j - DP_p(\mathbf{r},\mathbf{h})] h_p. \tag{2}$$

At equilibrium we require that the total commodity flow on paths connecting each OD pair w equals the demand for transportation between that OD pair and that flows be nonnegative; that is,

$$S_w(\mathbf{u}) = \sum_{p\in P_w} h_p = 0 \qquad \forall(w \in W') \tag{3}$$

$$\mathbf{h} \geq 0. \tag{4}$$

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Since each shipper is assumed to take transportation costs as given, the vector **u** (the vector of minimum delivered prices) is constant, and thus, so are the OD demands $S_w(\mathbf{u})$. Using (3), we can rewrite (2) as

$$\sum_{w=(i,j)\in W'} \left[\pi_j S_w - \sum_{p\in P_w} \mathrm{DP}_p(\mathbf{r},\mathbf{h}) h_p \right]. \tag{5}$$

Because π_j and S_w are constant, the first term in (5) can be ignored when maximizing (5), and carrier profit maximization can be expressed as

$$\text{minimize} \sum_{w=(i,j)\in W'} \sum_{p\in P_w} \mathrm{DP}_p(\mathbf{r},\mathbf{h}) h_p \tag{6}$$

subject to

$$S_w - \sum_{p\in P_w} h_p = 0 \qquad \forall (w \in W') \quad (u_w)$$

$$\mathbf{h} \geq 0,$$

where u_w is the Kuhn–Tucker multiplier for the flow conservation constraint. Taking the Kuhn–Tucker conditions for each shipper's profit-maximization problem (6), we arrive at the following set of first-order conditions:

$$\begin{aligned} &[\mathrm{DP}_p(\mathbf{r},\mathbf{h}) - u_w]\, h_p = 0 \qquad \forall (w \in W', \quad p \in P_w) \\ &\mathrm{DP}_p(\mathbf{r},\mathbf{h}) - u_w \geq 0 \qquad \forall (w \in W', \quad p \in P_w), \end{aligned} \tag{7}$$

where u_w is the minimum delivered price for OD pair w. In words, positive shipper flow on a path p implies that the carrier-quoted delivered price for that path is the minimum price; otherwise path p has no flow. This condition is Wardrop's first principle of user optimality (Fernandez and Friesz, 1983) expressed in terms of delivered price. Thus, under the assumption of fixed commodity prices, the profit-maximizing behavior of the shippers results in the minimization of delivered price between every OD pair.

It is assumed that there is perfect competition among the freight carriers and, hence, at an equilibrium, rates are equal to marginal costs. Because a particular path will be chosen only if it adds least to total cost, and because path costs are assumed to be additive in arc costs, then in terms of the previous notation this assumption may be expressed as

$$r_p = \sum_{v\in V} \gamma_{vp} y_v \qquad \forall p \in P. \tag{8}$$

The minimum marginal costs y_v depend implicitly on the carriers' flow **g** as will be seen shortly.

Because each carrier is assumed to be a profit maximizer facing an exogenous market price, its short-run behavior is described by the minimization

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of operating costs. Thus, additional traffic will be carried on that path only if it adds least to total cost. If y_v is the minimum marginal cost (MC) of all paths feasible for the OD pair v, we may express this assumption as

$$\begin{aligned} &(\mathrm{MC}_q(\mathbf{g}) - y_v)\, g_q = 0 \qquad \forall (k,\ v \in V_k,\ q \in Q_v) \\ &\mathrm{MC}_q(\mathbf{g}) - y_v \geq 0 \qquad \forall (k,\ v \in V_k,\ q \in Q_v), \end{aligned} \tag{9}$$

which says that, for all paths q that are feasible (in Q_v), positive commodity flow implies that costs on that path are at the minimum incremental cost; otherwise path q has no flow. Note that in (9) the marginal path cost MC_q is a function of the carriers' path flow vector $\mathbf{g}$, expressing the presence of congestion in the carriers' network. Because of the linkage between the shippers' and carriers' network, the carriers' flow conservation constraints take the form

$$\sum_{q \in Q_v} g_q - \sum_{p \in P} \gamma_{vp} h_p = 0 \qquad \forall v \in V. \tag{10}$$

In words, for each OD pair v, the sum of carrier flows g_q on the paths connecting v (the paths belonging to Q_v) must equal the sum of shippers' flows h_p that traverse v. Of course we require

$$\mathbf{g} \geq 0 \tag{11}$$

to ensure nonnegative carrier flows.

Expressions (1), (3), (4), (7)–(11) define the combined shipper–carrier equilibrium problem for freight networks. Moreover, (3), (4), (10), and (11) prompt us to define the following set of feasible solutions:

$$\begin{aligned} \Omega = \Bigg\{ \mathbf{f,e,S}:\ & \sum_{p \in P_w} h_p - S_w = 0 \qquad \forall w \in W, \\ & \sum_{q \in Q} g_q - \sum_{p \in P} \gamma_{vp} h_p = 0 \qquad \forall v \in V, \\ & f_a - \sum_{p \in P} \delta_{ap} h_p = 0 \qquad \forall a \in A, \\ & e_b - \sum_{q \in Q} \Delta_{bq} g_q = 0 \qquad \forall b \in B, \\ & \mathbf{h} \geq 0, \quad \mathbf{g} \geq 0 \Bigg\}. \end{aligned}$$

Using the foregoing notation, Friesz, Viton, and Tobin (1985) have shown that an equilibrium point of the combined shipper–carrier model is the solution of a single mathematical problem, as explained in the following theorem.

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Theorem 1
A shipper-carrier flow pattern $(\mathbf{f}^*,\mathbf{e}^*,\mathbf{S}^*) \in \Omega$ is a combined shipper-carrier network equilibrium if and only if

$$\phi \sum_{a\in A} t_a(\mathbf{f}^*)(f_a - f_a^*) + \sum_{b\in B} \mathrm{MC}_b(\mathbf{e}^*)(e_b - e_b^*) + \sum_{w\in W} [\pi_i - \theta_w(\mathbf{S}^*)](S_w - S_w^*) \geq 0 \qquad (12)$$

for all $(\mathbf{f}, \mathbf{e}, \mathbf{S}) \in \Omega$.

We will use the mathematical characterization (12), known as a "variational inequality," to prove the existence of a combined shipper–carrier equilibrium in the next section.

Assuming that equilibria for this problem exist, what economic properties will they have? As discussed in detail in the introduction, with price-taking behavior from all actors, an equilibrium will be first-best Pareto-optimal relative to the given infrastructure (in particular, the carriers' networks) and the given configuration of firms. In other words, the equilibrium will be a short-run phenomenon. It may or may not be a long-run competitive equilibrium, since the equilibrium here examined may result in negative economic profits for carriers if transport cost functions exhibit economies of scale or scope. Even in this eventuality, of course, the equilibrium is a valuable criterion in assessing networks, since the first-best solution to the requirement that profits be nonnegative (in other words that the industry be feasible in the sense of Baumol, Panzar, and Willig, 1982) is marginal-cost pricing with lump-sum taxation if necessary.

3.3. Existence and uniqueness of a simultaneous shipper–carrier equilibrium

In this section we give immediate proofs of the existence and uniqueness of equilibria in our model. The method of proof is based on the recognition that the equilibrium conditions stated in the last section define a "variational inequality"; we then apply available existence and uniqueness theorems for variational inequalities.

Before proceeding to the results, some general remarks on the technique used here are in order, because we believe that the applicability of the methodology, together with the efficient computational methods available makes these techniques widely useful in economic problems [for example, as an alternative to Scarf's (1973) method of calculating equilibria]. A variational inequality problem is

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$$\begin{aligned}&\text{find } \mathbf{x} \in D \text{ such that}\\&\mathbf{F}(\mathbf{x}) \cdot (\bar{\mathbf{x}} - \mathbf{x}) \geq 0\\&\text{for all } \bar{x} \in D\end{aligned} \tag{13}$$

where $\mathbf{x} = (x_1,\ldots,x_n)$, $\mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}),\ldots,F_l(\mathbf{x}),\ldots,F_n(\mathbf{x})]$, *and* $F_l: D \subseteq R^n \rightarrow R$ with n finite. The existence and uniqueness conditions for this problem are readily characterized. For present purposes we note that two other problems may be transformed into variational inequalities. Consider first the nonlinear complementarity problem, to find $\mathbf{x}$ such that

$$\mathbf{F}(\mathbf{x}) \cdot \mathbf{x} = 0 \qquad \mathbf{F}(\mathbf{x}) \geq 0 \qquad \mathbf{x} \geq 0. \tag{14}$$

This problem has immediate application to the computation of equilibria, and generalizes linear models to nonlinear formulations. The crucial result is stated for example, by Lemke (1980): The nonlinear complementary problem (14) is completely equivalent to the variational inequality (13) provided that $D = \{\mathbf{x}: \mathbf{x} \geq 0\}$. Also, consider the optimization problem

$$\text{Min } G(\mathbf{x}) \qquad \mathbf{x} \in D \tag{15}$$

where $\nabla G(x) = \mathbf{F}(\mathbf{x})$, or $G(\mathbf{x}) = \oint \mathbf{F}(\mathbf{x})dx$, provided appropriate integrability conditions hold. It is immediate (see Rockafellar, 1980) that the necessary conditions for a local solution to (15) are given by (13), and that if G is convex, the solution is global. In other words, the solution of the variational inequality (13) is useful in the characterization and solution of a large number of problems of interest to economists.

With this excursion we now return to the main argument. In the context of the variational inequality problem (13), it is well known that if $\mathbf{F}(\mathbf{x})$ is continuous and D is compact (closed and bounded) and convex, then (13) has a solution. This result was first proven by Hartman and Stampacchia (1966) and is reported by Kinderlehrer and Stampacchia (1980). It is also well known that if $\mathbf{F}(\mathbf{x})$ is strictly monotone on D, any solution of (13) is unique. Thus, we may state the following two theorems without detailed proofs.

Theorem 2

Suppose the functions $t_a(\mathbf{f})$, $MC_b(\mathbf{e})$, and $\theta_w(\mathbf{s})$ are continuous and the shipper transportation demand functions $S_w(\mathbf{u})$ are bounded from above. Then a combined shipper–carrier freight network equilibrium exists.

Proof

The boundedness of the $S_w(\mathbf{u})$ implies $\mathbf{g}$ and $\mathbf{h}$ are bounded from above. Hence, Ω is a bounded set. By inspection Ω is closed. It is composed of linear constraints and, therefore, convex. Hence, by Theorem 3.1 on page 12 of Kinderlehrer and Stampacchia (1980) a combined shipper–carrier equilibrium exists. Q.E.D.

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Theorem 3
A combined shipper–carrier freight network equilibrium exists and is unique if

i. the shipper travel delay functions $t_a(\mathbf{f})$ are continuous and strictly monotone increasing;
ii. the carrier marginal cost functions $MC_b(\mathbf{e})$ are continuous and strictly monotone increasing;
iii. the inverse shipper transportation demand functions $\theta_w(\mathbf{S})$ are continuous and strictly monotone decreasing; and
iv. the shipper transportation demand functions $S_w(\mathbf{u})$ are bounded from above.

Proof
Theorem 2 ensures existence. The strict monotonicity assumptions, by page 14 of Kinderlehrer and Stampacchia (1980), guarantee uniqueness. Q.E.D.

It is important to recognize that in some freight environments the assumption of strict monotonicity is unrealistic. For example, Morlok (1978) describes cases for which rail travel delay is a decreasing function for low flow volume and an increasing function for higher volume, and, thus, not monotonic at all. When strict monotonicity is not maintained, it is possible that multiple equilibria will occur.

3.4. Extensions of the simultaneous shipper–carrier model

The Friesz, Viton, and Tobin (1985) simultaneous shipper–carrier model makes the rather strong assumption that freight rates will equal marginal costs in all transportation markets. As discussed in Section 3.1, this assumption may not be unrealistic for the motor carrier industry. In analyzing the railroad, inland waterway, and deep-draft waterway systems, it may appear as if this assumption is completely unrealistic and hence useless. However, the marginal-cost pricing assumption does have merit even in these cases. First, from a pragmatic perspective, the mathematical models that are based upon the decentralization of each agents' decision are far easier to solve. In essentially all large-scale economic models the concept of marginal-cost pricing is used because it leads to a manageable fixed-point problem. When models are developed that attempt to capture the game-theoretic interactions between agents, they are typically difficult if not impossible to solve for large-scale problems. For example, the recent model by Fisk and Boyce (1983) is an attempt to formulate a Stackelberg model of freight system. The model is stated as a set of nonlinear, nonconvex mathematical programs with nonlinear, nonconvex constraints comprising the model of shippers' demand (these con-

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straints are in the form of a nonlinear complementarity problem). This mathematical form is very similar to the model by Harker (1981) (see also Harker and Friesz, 1982). Fisk and Boyce (1983) have suggested some methods of solution, but none has been tested. In the numerical studies done on the Harker (1981) model, it was found that even small test examples were very difficult to solve and that the use of this model for large-scale problems was impossible. One may immediately say that the solution of a model is not important as long as its theoretical underpinnings are correct; this we believe is false. No one theory is completely correct, and it is the purpose of modeling to implement theory on actual data and test its correlation with reality. If a model is stated that is insoluble, it cannot be tested against data. The marginal-cost pricing assumption does lend itself readily to empirical tests. If these tests show that this assumption is very poor, then it is time to expand the theoretical framework. If these tests show that the use of this assumption replicates the empirical data fairly well, then we can have some confidence in the use of this model while at the same time we strive to expand the theory. Therefore, the logical first step in modeling freight systems is to begin with the marginal-cost pricing assumption, test its validity, and use it as a basis to develop more sophisticated models.

The second argument in favor of beginning with the marginal-cost pricing assumption is that it does provide a basis for comparison with models based upon other economic assumptions. As Heaver and Nelson (1978, pp. 128–29) write: "the model of perfect, or pure, competition is usually regarded as a standard by which a firm's pricing can be judged, even when the industry under consideration does not have firms which exhibit all of the rather stringent assumptions, that theory requires of the truly competitive firm." Friedlaender and Spady (1981, p. 75) put this argument more succinctly: "an analysis of a competitive equilibrium provides a useful benchmark with respect to the characteristics of the outcome of the competitive process." Thus, the marginal-cost pricing model does have merit in its use as a basis to judge other solutions.

The last argument is that it may not matter very much if we use marginal-cost pricing or some model of imperfect competition for commodities for which there is strong intermodal competition. In their study of midwestern agricultural goods movements, Daughety and Inaba (1981) found that there was very little difference between the rail rates in a perfectly competitive market and when the railroads acted as one firm (perfect collusion, the polar extreme of perfect competition). Thus, for some commodities the use of the perfect-competition assumption may yield essentially the same results as a more sophisticated model.

We still would like to extend the theoretical framework of the simultaneous shipper–carrier model, however, to be able to deal with imperfectly competitive markets: Harker (1983) (see also Harker and Friesz, 1985a,b) has done this through the use of a concept called a "rate function." Before

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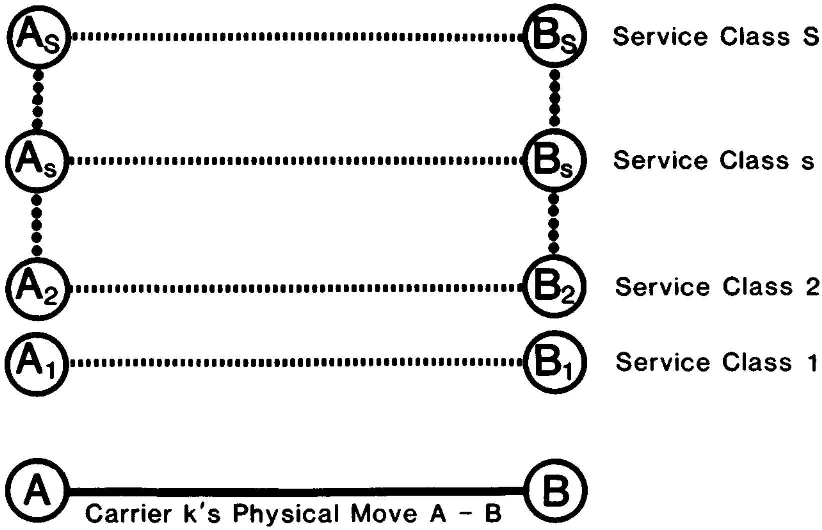

Figure 7.4. Service class network.

discussing this, however, let us first clarify one issue concerning the previous model.

At first glance, the Friesz, Viton, and Tobin model seems to assume that the only competitive factor is the freight rate. From transportation economics, however, we know that both price and level of service affect the choice of a transportation mode. One could include both a price variable and a level of service variable as part of the carrier's decision model as Fisk and Boyce (1983) have done, but their inclusion would make the model even more difficult to solve. The approach taken here is to define for each carrier OD move a set of level of service classes. In reality, there is a continuum of service classes that a carrier could offer a shipper; we are approximating this continuum by a discrete set of service classes. Figure 7.4 illustrates this concept. Each physical carrier OD pair is represented by a set of OD pairs, each having a particular service class associated with it. If carrier k supplies service between A_1 and B_1, it is actually providing this service in level of service class 1. Thus, we have expanded each carrier OD pair into a set of multiple outputs, each being represented as a particular OD pair in this extended network.

The previous discussion noted that the decentralization of each agent's decision allows one to formulate a solvable mathematical model. Harker (1983) recognized this fact when exploring the possibility of extending the simultaneous shipper–carrier model to include imperfect competition and developed the concept of a rate function. The rate function $R_v(\boldsymbol{\tau})$ is our a

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priori assumption concerning the rate a carrier will charge for service on its OD pair $v \in V$ given the vector of carrier OD flows $\boldsymbol{\tau} = (\ldots,\tau_v,\ldots)$:

$$\tau_v = \sum_{q \in Q_v} g_q = \sum_{p \in P} \gamma_{vp} h_p \tag{16}$$

Under the marginal-cost pricing assumption, the rate function would equal y_v, the minimum marginal cost between OD pair v. More generally, the rate function could be used under the assumption of cost-plus pricing (rate = some markup over average cost), or pricing up to a legal limit such as those imposed by the Stagger's Act. This approach does not assume that the carrier has knowledge of the shippers' transportation demand functions as would a Cournot–Nash model of this market. Thus, this approach is not in the strictest sense a noncooperative game-theoretic model of the freight system; the rate function is an approximation of this type of model. As Harker and Friesz (1985b) discuss, this approximation retains the decentralization of each agent's decisions that is present in the marginal-cost pricing model. However, this approach has merit beyond being an approximation to a game-theoretic model. Many economists, especially those studying industrial organization, do not believe that firms behave in the manner described by the Cournot–Nash model, but in fact do price according to some simple principle such as cost-plus pricing. In reality, the firm's pricing policy most likely lies between the extremes of having perfect information concerning the demand behavior of the shippers and pricing according to some simple formula. The rate function allows one to test various pricing policies in terms of their power in replicating historical data. Thus, the rate function can be considered a method of approximating more complex game-theoretic models and a method of testing alternative pricing policies that the carriers may follow in large-scale applications.

The model of the previous section can easily be expanded to include the rate function, as the following theorem shows:

Theorem 4

A shipper–carrier flow pattern $(\mathbf{f}^*, \mathbf{e}^*, \mathbf{S}^*) \in \Omega$ is a combined shipper–carrier network equilibrium when the freight rates on each carrier OD pair are described by the rate function $R_v(\boldsymbol{\tau})$ if and only if

$$\begin{aligned}\phi \sum_{a \in A} t_a(\mathbf{f}^*)(f_a - f_a^*) = \sum_{b \in B} \mathrm{MC}_b\mathbf{e}^*(e_b - e_b^*) \\ + \sum_{w \in W} [\pi_i - \theta_w(\mathbf{S}^*)](S_w - S_w^*) \\ + \sum_{v \in V} [R_v(\boldsymbol{\tau}^*) - y_v^*](\tau_v - \tau_v^*) \geq 0\end{aligned} \tag{17}$$

for all $(\mathbf{f},\mathbf{e},\mathbf{S}) \in \Omega$.

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Proof
This proof is a special case of the proof in Theorem 3.4 of Harker (1983) or Theorem 4 of Harker and Friesz (1985b).

Harker (1983) has also shown that this model can be cast in the form of a nonlinear complementarity problem, has used this form to prove existence and uniqueness, and has developed solution algorithms for this new model; the next section will discuss these results further. The point is that through the use of this rate-function concept, we need not rely solely on the marginal-cost pricing assumption in order to apply these models to problems of realistic size.

4. Simultaneity of macroeconomic and network models

As shown in Table 7.1, the freight models applied to date have all used exogenously determined commodity supplies and demands. These fixed freight trip productions and attractions have been forecast using various economic models that treat the freight transportation system in an aggregate fashion. Consequently, when such economic models are employed sequentially with a detailed freight network model, the question of consistency of the two models naturally arises. The legitimacy of this question was unintentionally demonstrated by the work of Charles River Associates (CRA) (1981). CRA iterated between a macroeconomic coal model (the National Coal Model) and a version of the CACI freight network model; they found this approach to be numerically unstable and nonconvergent, as well as computationally intensive.

Recent work by Friesz, Tobin, and Harker (1981), Friesz and Tobin (1981), Friesz et al. (1983), Tobin and Friesz (1983), and Friesz, Harker, and Tobin (1985) has shown that the "spatial price equilibrium problem" may be generalized to treat arbitrary networks with congestion externalities and demand and supply asymmetries. The spatial price equilibrium problem does not employ explicit transportation demand functions; instead it derives transportation demand from the production and consumption characteristics of spatially separated markets. As such, a spatial price equilibrium model can be used as a replacement for the Wardropian shipper model discussed previously; because such a model may employ elastic commodity supply and demand functions for each node, trip generation becomes endogenous and the problem of consistency we mentioned is avoided.

Gottfried (1983) has generalized the FNEM model to include trip generation by incorporating commodity supply and demand functions for each regional centroid, following the formulation of the spatial price equilibrium problem by Tobin and Friesz (1983). A demand-driven (supply functions only) version of this formulation has been programmed and applied to study

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U.S. coal movements (see Tobin et al. (1983). However, this new model suffers from the same inconsistency between the shipper and carrier submodels as is found in FNEM due to the sequential nature of FNEM.

Harker (1983) (see also Harker and Friesz 1985a,b) has recently stated a model called the "generalized spatial price equilibrium model" (GSPEM), which ties together the concepts of spatial price equilibrium and shipper–carrier equilibrium to predict simultaneously the production and consumption of goods, the shippers' routing of freight, the freight rates using the rate-function concept, and the carriers' routing of the freight traffic. We can briefly describe this model by using the notation of the previous sections and defining L to be the set of regions (actually the regional centroids) where production or consumption of goods occurs and

$$
\begin{aligned}
Q_l^S &= \text{the supply in region } \ell \in L, \\
Q_l^D &= \text{the demand in region } \ell \in L \\
\mathbf{Q}^S &= (\ldots, Q_\ell^S, \ldots), \\
\mathbf{Q}^D &= (\ldots, Q_\ell^D, \ldots), \\
\psi_\ell(\mathbf{Q}^S) &= \text{the inverse supply function for region } \ell \in L, \\
\Lambda_\ell(\mathbf{Q}^D) &= \text{the inverse demand function for region } \ell \in L.
\end{aligned}
$$

Therefore ψ_ℓ is the price a shipper would pay for the commodity at the production site ℓ and Λ_ℓ is the price for which this good could be sold in region ℓ. The conservation of flow in every region can be written as

$$Q_\ell^D - Q_\ell^S + \sum_{w=(\ell,j)\in W} T_w - \sum_{w=(i,\ell)\in W} T_w = 0 \qquad \forall \ell \in L \tag{18}$$

The spatial price-equilibrium concept states that if there is flow on a path between two regions, then the purchase price plus the transportation cost will equal the sale price at the destination in equilibrium. If the purchase price plus the transportation cost is less than the sale price, the shipper has an incentive to keep shipping goods until these terms equalize. If the purchase price plus the transportation cost is greater than the sale price, the shipper has no incentive to ship anything between these two regions since he would lose money in doing so; the flow on this path would thus be zero at equilibrium. To add these conditions to the simultaneous shipper–carrier model described in the previous section, let us first notice that the delivered price can be rewritten as

$$\mathrm{DP}_p(\mathbf{r},\mathbf{h}) = \psi_i(\mathbf{Q}^\mathbf{S}) + r_p + \phi t_p(\mathbf{h}). \ \forall\ (w = (i,j) \in W,\ p \in P_w) \tag{19}$$

The shipper equilibrium condition (7) is then replaced by the following:

$$\left.\begin{aligned}
&[\mathrm{DP}_p(\mathbf{r},\mathbf{h}) - \Lambda_j(\mathbf{Q}^\mathbf{D})]h_p = 0 && \forall\ (w = (i,j) \in W, p \in P_w) \\
&\mathrm{DP}_p(\mathbf{r},\mathbf{h}) - \Lambda_j(\mathbf{Q}^\mathbf{D}) \geq 0 && \forall\ (w = (i,j) \in W, p \in P_w)
\end{aligned}\right\} \tag{20}$$

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or

$$\left.\begin{aligned}[\psi_i(\mathbf{Q^S}) + (r_p + \phi t_p(\mathbf{h})) - \Lambda_j(\mathbf{Q^D})]h_p = 0 \ \forall \ (w = (i,j) \in W, p \in P_w) \\ \psi_i(\mathbf{Q^S}) + (r_p + \phi t_p(\mathbf{h})) - \Lambda_j(\mathbf{Q^D}) \geq 0.\end{aligned}\right\} \quad (21)$$

By redefining the feasible set Ω as

$$\begin{aligned}
\Omega = \{(&\mathbf{Q}^S,\mathbf{Q}^D,\mathbf{f},\mathbf{e},\mathbf{S}): \\
&Q_\ell^D - Q_\ell^S + \sum_{w=(\ell,j)\in W} T_w - \sum_{w=(i,\ell)\in W} T_w = 0 \qquad \forall \ell \in L; \\
&\sum_{q\in Q_v} g_q - \sum_{p\in P} \gamma_{vp} h_p = 0 \qquad \forall v \in V; \\
&f_a - \sum_{p\in P} \delta_{ap} h_p = 0 \qquad \forall a \in A; \\
&e_b - \sum_{q\in Q} \Delta_{bq} g_q = 0 \qquad \forall b \in B; \\
&\mathbf{Q}^S \geq 0, \quad \mathbf{Q}^D \geq 0, \quad \mathbf{h} \geq 0, \quad \mathbf{g} \geq 0\},
\end{aligned}$$

we can incorporate the spatial price–equilibrium concept into the model described in Theorem 4 as follows:

Theorem 5
The production-consumption-flow pattern

$$(\mathbf{Q}^{S*},\mathbf{Q}^{D*},\mathbf{f}^*,\mathbf{e}^*,\mathbf{S}^*) \in \Omega$$

is a combined spatial price-shipper-carrier equilibrium if and only if

$$\begin{aligned}
\sum_{\ell\in L} \psi_\ell(\mathbf{Q}^{S*})(Q_\ell^S - Q_\ell^{S*}) = &\sum_{\ell\in L} \Lambda_\ell(\mathbf{Q}^{D*})(Q_\ell^D - Q_\ell^{D*}) \\
&+ \phi \sum_{a\in A} t_a(\mathbf{f}^*)(\,f_a^* - f_a^*) \\
&+ \sum_{b\in B} \mathrm{MC}_b(\mathbf{e}^*)(e_b - e_b^*) \\
&+ \sum_{v\in V} [R_v(\boldsymbol{\tau}^*) - y_v^*](\tau_v - \tau_v^*) \geq 0 \quad (22)
\end{aligned}$$

for all $(\mathbf{Q}^S,\mathbf{Q}^D,\mathbf{f},\mathbf{e},\mathbf{S}) \in \Omega$.

Proof
See Harker (1983) or Harker and Friesz (1985b).

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Harker (1983) also proves that this model can be cast in the form of a nonlinear complementarity problem. Using this formulation and the recent existence results by Smith (1984) for this class of problem, Harker (1983, pp. 66–73) proves that a combined spatial price, shipper–carrier equilibrium will exist as long as the functions are continuous; r_p, t_a and MC_b are strictly positive; and in every region there exists a price for which the supply in that region will exceed the demand in that region for any other price above this level. Thus, GSPEM is capable of using U-shaped cost curves on the carrier arcs, which is a typical case when dealing with railroad networks. The assumption of monotone marginal arc costs (which implies that the average costs are never decreasing with increasing flow) is not necessary for this model to have a solution. However, the assumption that marginal costs are strictly increasing is necessary to insure the uniqueness of an equilibrium; the use of U-shaped average-cost functions implies the existence of multiple equilibria. What can be said at this point is that GSPEM is the most complete predictive model of the freight transportation system to date; it can use realistic arc cost functions, does not rely on the marginal-cost pricing assumption, and simultaneously includes the generation of freight trips and the equilibrium behavior of the shippers and carriers.

Harker (1983) (also see Harker and Friesz, 1985c), has also developed two solution algorithms for this model. The first algorithm is for use on the nonlinear complementarity problem formulation and its convergence properties are based upon the recent work by Pang and Chan (1982). With this algorithm, any form of the rate function can be used. However, this type of algorithm requires a fair amount of computer storage and thus it is limited to problems that are not excessively large. For very large problems, Harker (1983) has developed an algorithm for the variational inequality problem (22) which is based upon the assumption of marginal-cost pricing (this algorithm is a variant of the algorithm suggested by Friesz, Viton, and Tobin, 1985). In this case $R_v = y_v$ for all $v \in V$ and thus the last summation in (22) disappears. The resulting variational inequality is then solved by a diagonalization algorithm (see, for example, Pang and Chan, 1982 or Dafermos, 1983) in which each function in (22) is made to be separable (the function depends only upon one variable, and not a vector of variables) to form a sequence of separable variational inequalities. Of course, if the functions are separable to start with, the diagonalization algorithm collapses to the solution of one variational inequality problem. This algorithm is capable of solving very large problems, as evidenced by the application of this model that will be described in the sequel. Finally, both of these algorithms are capable of dealing with nonmonotonic cost functions; Harker (1983, ch. 5) presents proofs that if these algorithms converge, they will converge to a true equilibrium solution in this case.

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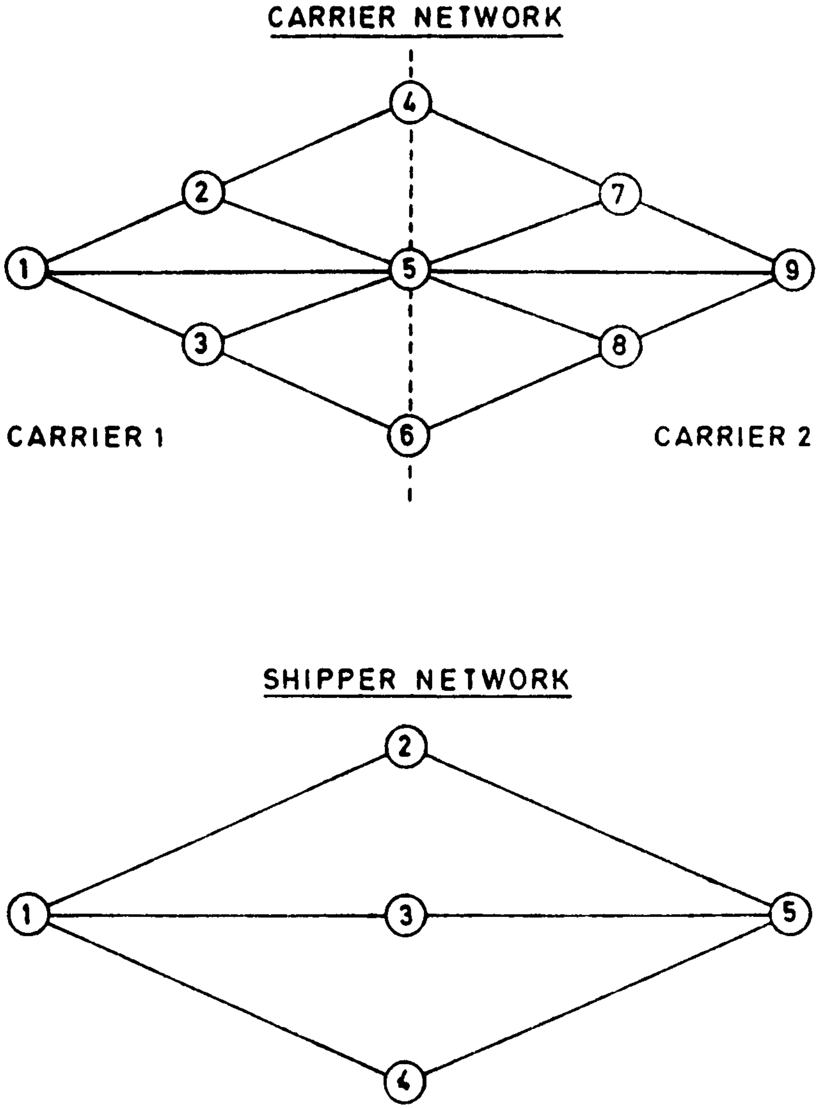

Figure 7.5. Test networks for example.

To illustrate the concepts discussed in this section, consider the networks depicted in Figure 7.5. The networks of the two carriers have been aggregated to form the shipper network as described in Section 3.1. The supply and demand function in each region is given by

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Table 7.2. *Data for example*

Carrier network data

Arc b	From node	To node	XH_b	XI_b	XJ_b
1	1	2	4.0	0.1	0.01
2	1	3	3.0	0.2	0.02
3	1	5	2.0	0.2	0.03
4	2	1	1.0	0.1	0.04
5	2	4	4.0	0.1	0.01
6	2	5	3.0	0.2	0.04
7	3	1	2.0	0.2	0.03
8	3	5	1.0	0.1	0.04
9	3	6	4.0	0.1	0.01
10	4	2	3.0	0.2	0.02
11	4	7	2.0	0.2	0.03
12	5	1	1.0	0.1	0.04
13	5	2	4.0	0.1	0.01
14	5	3	3.0	0.2	0.02
15	5	7	2.0	0.2	0.03
16	5	8	1.0	0.1	0.04
17	5	9	4.0	0.1	0.01
18	6	3	3.0	0.2	0.02
19	6	8	2.0	0.2	0.03
20	7	4	1.0	0.1	0.04
21	7	5	4.0	0.1	0.01
22	7	9	3.0	0.2	0.02
23	8	5	2.0	0.2	0.03
24	8	6	1.0	0.1	0.04
25	8	9	4.0	0.1	0.01
26	9	5	3.0	0.2	0.02
27	9	7	2.0	0.2	0.03
28	9	8	1.0	0.1	0.04

Shipper network data

Arc a	From node	To node	XK_a	XL_a	XM_a
1	1	2	1.0	0.01	0.001
2	1	3	2.0	0.02	0.002
3	1	4	1.0	0.09	0.002
4	2	1	2.0	0.01	0.001
5	2	5	3.0	0.02	0.002
6	3	1	1.0	0.03	0.002
7	3	5	1.0	0.01	0.001
8	4	1	2.0	0.01	0.004
9	4	5	1.0	0.01	0.001
10	5	2	3.0	0.02	0.001
11	5	3	6.0	0.05	0.002
12	5	4	1.0	0.03	0.005

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Table 7.2. (*Continued*)

Node ℓ	XA_ℓ	XB_ℓ	XC_ℓ	XE_ℓ	XF_ℓ	XG_ℓ
1	0.0	0.1	0.05	400.0	−2.0	−4.5
5	0.0	0.01	0.025	500.0	−1.0	−0.05

Note: ϕ = 1.0, the value of time.

$$Q_\ell^S = XA_\ell + XB_\ell \pi_\ell + XC_\ell \pi_\ell^2 \qquad \forall\, \ell \in L \tag{23}$$

$$Q_\ell^D = XE_\ell + XF_\ell \pi_\ell + XG_\ell \pi_\ell^2 \qquad \forall\, \ell \in L \tag{24}$$

where π_ℓ is the price in region ℓ and XA_ℓ, XB_ℓ, XC_ℓ, XE_ℓ, XF_ℓ, and XG_ℓ are constants. The inverse supply and demand functions are found by numerically inverting (23) and (24). The time-delay functions on the shipper arcs are given by

$$t_a = XK_a + XL_a f_a + XM_a f_a^2 \qquad \forall\, a \in A \tag{25}$$

where XK_a, XL_a, and XM_a are constants, and the marginal-cost functions for the carrier arcs are

$$\mathrm{MC_b} = XH_b + 2.0 XI_b e_b + 1.5 XJ_b e_b^2 \qquad \forall\, b \in B \tag{26}$$

where XH_b, XI_b, and XJ_b are constants. Table 7.2 lists the data for this example.

Let us first assume that $R_v = y_v \quad \forall\, v \in V$, that is, there is marginal-cost pricing on all carrier OD pairs. Table 7.3 presents the results of the model. As the reader can easily verify, all equilibrium conditions have been attained. Therefore, this small example illustrates that the solution of GSPEM does indeed yield a simultaneous solution of the spatial price–shipper–carrier equilibrium conditions. To illustrate the use of alternative rate-function specifications, let us assume that the rate for any carrier OD pair will be equal to \$10 plus one half the marginal cost, or that

$$R_v = 10 + 0.5\, y_v \qquad \forall\, v \in V \tag{27}$$

Thus, the rate is some fixed value plus a percentage of the marginal costs. Note that any rate specification could be used; (27) is used just for illustrative purposes. Table 7.4 presents the results of the model with this alternative rate specification. Again, all the necessary equilibrium conditions are met.

5. Applications of FNEM and GSPEM

FNEM, the sequential shipper–carrier freight network equilibrium model described in Section 1, has been extensively tested and used in various studies of

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Table 7.3. *Results of marginal-cost pricing example*

Shipper network results

Node	π_ℓ	Q_ℓ^S	Q_ℓ^D	$Q_\ell^S - Q_\ell^D$
1	27.624	40.917	1.358	39.560
5	71.908	129.990	169.550	−39.560

OD $w = (i,j)$	S_w	u_w	Path p	Traverses nodes	h_p	DP_p
1,5	39.560	44.280	1	1,4,5	8.029	71.904
			2	1,3,5	21.724	71.919
			3	1,2,5	9.807	71.907
5,1	0.000	3.000	1	5,4,1	0.000	74.908

Carrier network results[a]

OD $v = (i,j)$	T_v	y_v	Path q	Traverses nodes	g_p	MC_q
1,4	9.807	17.249	1	1,2,4	9.807	17.249
1,5	21.724	15.161	1	1,5	13.225	15.161
			2	1,2,5	4.362	15.161
			3	1,3,5	4.137	15.161
1,6	8.029	18.879	1	1,3,6	8.029	18.879
4,9	9.807	20.119	1	4,7,9	9.807	20.119
5,9	21.724	11.897	1	5,9	17.224	11.897
			2	5,7,9	0.062	11.897
			3	5,8,9	4.438	11.897
6,9	8.029	16.935	1	6,8,9	8.029	16.935

[a]Only those OD pairs and paths with flows are shown.

commodity movements in the United States. In terms of validation, Gottfried (1983) and Friesz, Gottfried, and Morlok (1983) report on tests conducted with three network data bases: (1) a highly detailed railway and waterway network data base of the northeastern United States with a single aggregate commodity and five rail carriers (the northeast validation), (2) a somewhat more aggregate railway network data base of the entire United States with fifteen commodities and a single aggregate rail carrier (the national rail validation), and (3) a combined railway and waterway network database of the entire United States with fifteen commodities and seventeen rail carriers (the "full" national validation). A summary of the characteristics of these validation exercises is given in Table 7.5. The network data bases used in these

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Table 7.4. *Results of nonmarginal-cost pricing example*

Shipper network results

Node	π_ℓ	Q_ℓ^S	Q_ℓ^D	$Q_\ell^S - Q_\ell^D$
1	27.488	40.529	4.999	35.530
5	72.249	131.222	166.752	−35.530

OD $w = (i,j)$	S_w	u_w	Path p	Traverses nodes	h_p	DP_p
1,5	35.530	44.662	1	1,4,5	7.810	72.150
			2	1,3,5	18.071	72.465
			3	1,2,5	9.649	72.217
5,1	0.000	23.000	1	5,4,1	0.000	95.249

Carrier network results[a]

OD $v = (i,j)$	T_v	y_v	Path q	Traverses nodes	g_p	MC_q
1,4	9.649	16.211	1	1,2,4	9.649	16.211
1,5	18.071	13.355	1	1,5	11.995	13.355
			2	1,2,5	2.890	13.355
			3	1,3,5	3.186	13.356
1,6	7.810	17.493	1	1,3,6	7.819	17.493
4,9	9.649	19.637	1	4,7,9	9.649	19.637
5,9	18.071	10.369	1	5,9	14.861	10.369
			2	5,7,9	0.000	11.624
			3	5,8,9	3.210	10.371
6,9	7.810	15.827	1	6,8,9	7.810	15.827

[a]Only those OD pairs and paths with flows are shown.

validation exercises were primarily assembled from sources in the the public domain: the U.S. Federal Railroad Administration (FRA) (1980a,b), CACI (1980), Roberts and Dewees (1971), the Interstate Commerce Commission (1975, 1981), and the Transportation Systems Center (1980). In addition commercial data services were employed as required: Data Resources (1981) and Reebie and Associates (1982). Some original surveys to determine arc and modal-cost functions were conducted as described in Friesz et al. (1981) and Gottfried (1983). Transportation demand functions were of the negative exponential or entropy maximization variety suggested by Wilson (1970).

The link loadings produced by the model have been compared against the historical usage of rail links for the base year 1978. The historical usage is

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Table 7.5. *Summary of FNEM validation exercises*

Characteristics	Validation name		
	Northeast	National rail	National
Study of region	Northeast	Nation	Nation
Number of modes	2[a]	1	2[a]
Number of commodities	1	15	15
Zone definition	Transportation zone[b]	TBEA[c]	BEA[d]
Number of zones	105	129	183
Number of OD pairs	10,920	29,225	42,087
Shippers' network	FRA[e]	TSC[f]	TSC[f]
Number of shipper nodes	2,258	1,594	3,072
Number of shipper arcs	9,799	7,341	14,589
Number of rail carriers	5	1	17
Carriers' network	FRA[e]	TSC[f]	FRA[e]
Solution time (CPU minutes)[g]	35	14	86

[a]Rail and barge.
[b]There are 105 U.S. Department of Transportation (DOT) zones in the northeastern United States.
[c]Traffic BEAs or TBEAs are defined by FRA (1980b).
[d]Bureau of Economic Analysis area.
[e]FRA.
[f]Transportation Systems Center of U.S. DOT.
[g]These correspond to time on an IBM 370/3033.

derived from the density codes assigned to rail links by the FRA (1980a). The FRA density codes are given in Table 7.6. Table 7.7 exhibits the cumulative frequency distribution of the differences between the FRA historical data and the densities computed by Friesz, Gottfried, and Morlok (1983) and Gottfried (1983) in different validation exercises involving FNEM and by another recent attempt to model large scale multicommodity freight movements, the multimodal network model (MNM) (Bronzini, 1980a,b). The results presented in Table 7.6 for the MNM are the only presently available quantitative accuracy tests of a large-scale freight-network model against which to compare FNEM. As can be seen, FNEM performed substantially better than MNM. Furthermore, in the national rail and the full national validations of FNEM, the availability of commodity-specific data about OD transportation demands allows analysis of how well the OD flows predicted by FNEM conform to known data. Since previous large-scale freight-network models have tended to be based on fixed transportation demands, there are no reported predictions of this type against which to compare the model. Smith and Hutchinson (1981) recommend the three goodness-of-fit measures described in Table 7.8 for the evaluation of doubly constrained gravity-demand models

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Table 7.6. *FRA density codes*

Code	Annual gross tons (millions)
1	0–1
2	1–5
3	5–10
4	10–20
5	20–30
6	>30

Table 7.7. *Differences between predicted railroad link traffic densities and FRA density codes*

Density code difference	MNM	Northeast FNEM	National rail FNEM	Full national FNEM
0	21	43	56	63
±1	55	74	76	80
±2	76	84	92	93
±3	90	92	97	97
±4	97	96	98	99
±5	99	100	100	100
±6	100	—	100	—

Table 7.8. *Definitions of goodness-of-fit statistics*

R^2 (coefficient of determination)	$1 - \dfrac{\sum_i \sum_j (T_{ij} - T_{ij}^*)^2}{\sum_i \sum_j (T_{ij} - \bar{T})^2}$
Φ (normalized)	$\sum_i \sum_j \dfrac{T_{ij}}{\bar{T}} \left\| \ln \dfrac{T_{ij}}{T_{ij}^*} \right\|$
M (normalized mean absolute error)	$\dfrac{1}{N} \sum_i \sum_j \left\| \dfrac{T_{ij} - T_{ij}^*}{\hat{T}} \right\|$

Legend: T_{ij} = observed flow from i to j.
T_{ij}^* = predicted flow from i to j.
$\bar{T}$ = mean flow of all OD pairs.
$\hat{T}$ = total flow for all OD pairs.
N = number of OD pairs.

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Table 7.9. *Goodness of fit of origin–destination flows for national rail validation*

Category	R^2	Φ	M
Grain	.53	1.199	.719
Iron ore	.62	.975	.679
Coal	.53	.832	.667
Stone, sand gravel	1.00	.146	.095
Nonmetallic minerals	.85	.939	.607
Grain mill products	1.00	.322	.237
Food products	.69	.984	.641
Forest products	.44	1.029	.813
Lumber and wood products	.36	.770	.658
Pulp and paper mill products	.36	.852	.701
Chemicals	.65	.677	.566
Cement, clay, and glass	.54	.974	.777
Primary metal products	.70	.737	.571
Transportation equipment	.81	.573	.476
All other	.40	1.226	.903
Overall	.89	.916	.637
Range	.36–1.00	.146–1.226	.095–.903
Mean	.65	.822	.609

like that employed in FNEM; these are the coefficient of determination (R^2), normalized phi (ϕ), and normalized mean absolute error (M). An ideal value of R^2 is 1, while the ideal value of ϕ or M is 0. The goodness-of-fit results by commodity for the national rail and the full national validations are presented, respectively, in Tables 7.9 and 7.10, which are drawn from Friesz, Gottfried, and Morlok (1983) and Gottfried (1983).

In addition to the three applications described, FNEM has been used in a series of regional transportation studies. For example, Tobin, Jastrow, and Meleski (1983) report on the use of FNEM in assessing the impacts of powerplants converting to coal in the state of Florida. In summary, FNEM has proven itself to be a useful tool for policy analysis.

In order to illustrate the applicability of GSPEM (the simultaneous spatial price-shipper-carrier model) to realistic problems, Harker (1983) describes the application of this model to the problem of predicting domestic coal production, consumption and interregional flows. In conjunction with Argonne National Laboratory, the marginal-cost pricing version of GSPEM and its associated solution algorithm were implemented for use with the Bureau of Economic Affairs (BEA) Economic Areas and Argonne's version of the Transportation Systems Center's rail and waterway networks; Figure 7.6 illus-

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Table 7.10. *Goodness of fit of origin–destination flows full national application*

Category	R^2	Φ	M
Grain	.71	.893	.582
Iron ore	.69	.884	.601
Coal	.74	.658	.553
Stone, sand gravel	.98	.228	.117
Nonmetallic minerals	.89	.825	.561
Grain mill products	.96	.339	.260
Food products	.65	.991	.666
Forest products	.55	.908	.751
Lumber and wood products	.49	.602	.581
Pulp and paper mill products	.52	.636	.596
Chemicals	.81	.490	.448
Cement, clay, and glass	.47	.992	.814
Primary metal products	.63	.801	.619
Transportation equipment	.83	.551	.449
All other	—	—	—
Overall	.91	.876	.613
Range	.47–.98	.228–.992	.117–.814
Mean	.71	.700	.543

trates the rail network. The final database used consisted of eighteen carriers, 2,577 nodes and 7,668 arcs in the carriers' network, and 960 nodes, 6,993 arcs and 1,238 origin–destination pairs in the shippers' network. Two runs of GSPEM were made. In the first run, the 1980 levels of coal exports from the ports of New Orleans, Hampton Roads, Baltimore, Philadelphia, New York, Toledo, and Mobile were taken as given and a base-year comparison of the model's results with the historical 1980 data was performed. In general, the model performed well. The total solution time for this run was 33.05 CPU minutes on an IBM 370/3033, which is very good considering the size of the problem and the amount of information which the model yields (predictions of regional prices, supplies and demands of coal, interregional coal flows and the routing of the coal traffic on the rail and water networks). In the second run, a comparative statics analysis was performed on the possibility of coal exports doubling in volume and the ports of Philadelphia, New York, and Mobile being closed to coal traffic. The results of this run when compared with the base-year results allowed analysis of the effects these events would have on the regional prices of coal, the interregional flows of coal, and the potential congestion of certain links in the transportation network. As expected, the domestic supply level rose and the domestic demand level dropped

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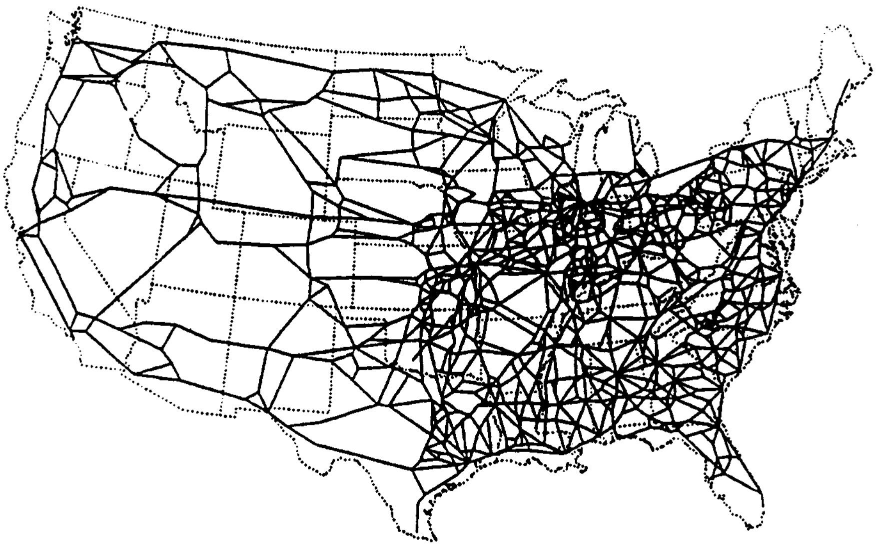

Figure 7.6. Rail network of the U.S. Department of Transportation's Transportation Systems Center.

because of this increase in foreign demand and the upward pressure this demand places upon the regional coal prices. However, the solution of this scenario exhibited some nonintuitive results. For example some of the remaining ports where coal traffic doubled saw a *decrease* in flow from certain BEA regions, whereas one would expect flows from all BEA regions to increase with increased exports. This nonintuitive result is due to the competition among the spatially separated regions and the interrelationships between the regions which are incorporated into GSPEM. The interested reader is referred to Harker (1983) or Harker and Friesz (1985c) for a more detailed discussion of this application. The point to be made here is that this application provided proof that GSPEM is applicable to very large-scale problems and that the results of this model are far more useful than results from any of the previous freight models since it captures more of the interrelations inherent in the freight system than any other model.

6. Conclusions

We have reviewed the key freight network equilibrium models proposed in the literature and found that several key considerations have been overlooked in the models reported heretofore. Foremost among these oversights have been (1) the simultaneous treatment of shippers and carriers, (2) the simultaneous solution of the macroeconomic model, which generates commodity

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supplies and demands and the network model itself, (3) explicit treatment of backhauling operations, (5) explicit treatment of blocking strategies, and (6) fleet constraints. We have seen how the first two of these issues may be resolved using single variational inequality and nonlinear complementarity formulations of the freight network equilibrium problem. We have also pointed out that nonmonotonic functions do not present any appreciable difficulty in the computation of equilibria, so long as one is content to deal with multiple equilibria. It remains an important research topic to establish what classes of nonmonotonic functions are consistent with a unique freight equilibrium and whether these functions can be used to model the economies of traffic density observed in freight transportation.

Backhauling, blocking, and fleet constraints remain essentially unaddressed; the prospects for their rigorous inclusion into a freight equilibrium model that is computationally tractable are bleak at the present owing to the fact that such considerations seem to imply the use of integer variables to model indivisibilities.

Nonetheless the results reported indicate that even a relatively simple model that includes both shippers and carriers leads to dramatic improvements in forecasting capability. This bodes well for the future of freight equilibrium models.

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