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Fast Acceptance by Common Experience: FACE-recognition in Schelling's model of neighborhood segregation

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Abstract

Schelling (1969, 1971a,b, 1978) observed that macro-level patterns do not necessarily reflect micro-level intentions, desires or goals. In his classic model on neighborhood segregation which initiated a large and influential literature, individuals with no desire to be segregated from those who belong to other social groups nevertheless wind up clustering with their own type. Most extensions of Schelling's model have replicated this result. There is an important mismatch, however, between theory and observation, which has received relatively little attention. Whereas Schelling-inspired models typically predict large degrees of segregation starting from virtually any initial condition, the empirical literature documents considerable heterogeneity in measured levels of segregation. This paper introduces a mechanism that can produce significantly higher levels of integration and, therefore, brings predicted distributions of segregation more in line with real-world observation. As in the classic Schelling model, agents in a simulated world want to stay or move to a new location depending on the proportion of neighbors they judge to be acceptable. In contrast to the classic model, agents' classifications of their neighbors as acceptable or not depend lexicographically on recognition first and group type (e.g., ethnic stereotyping) second. The FACE-recognition model nests classic Schelling: When agents have no recognition memory, judgments about the acceptability of a prospective neighbor rely solely on his or her group type (as in the Schelling model). A very small amount of recognition memory, however, eventually leads to different classifications that, in turn, produce dramatic macro-level effects resulting in significantly higher levels of integration. A novel implication of the FACE-recognition model concerns the large potential impact of policy interventions that generate modest numbers of face-to-face encounters with members of other social groups.

Keywords: ethnic, discrimination, lexicographic, non-compensatory, heuristic, urban economics, institutional design.

1 Introduction

Based on his counterintuitive observation concerning neighborhood segregation, Nobel Laureate Thomas Schelling (1969, 1971a,b, 1978) established what would become a large and influential literature connecting various subfields of the social sciences. Schelling's observations was this: even in the absence of intrinsic aversion to those who belong to other groups, and without anyone explicitly aiming to locate themselves in a segregated community, high levels of segregation could nevertheless result from a modest desire to avoid being too much of a relative minority. When one observes the sharp ethnic segregation that exists in a regrettably large number of US cities, Schelling argued we ought not conclude that this is necessarily the result of anti-ethnic sentiment among either majority or minority group members. Schelling's classic segregation model shows, for example, that when people are happy with any location at which up to half their neighbors belong to a different ethnic group, one should nevertheless predict dramatic segregation into nearly homogeneous ethnic blocs that no individual explicitly sought or wished for. The incongruity of macro consequences that do not reflect individual objectives is the overarching theme referred to in the title of Schelling's (1978) *Micromotives and Macrobehavior*.

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Of particular relevance to judgment and decision making researchers, we hope, is this link — or lack of link as was Schelling's argument — between individual-level decision-making process and macro spatial dynamics. One might dismiss the relevance of Schelling's very simple model to the complexity of real-world neighborhoods and other social communities, such as academic departments, where methodological splits into subgroups sometimes lead to conflict and segregation (e.g., empirical versus theoretical divides which are common in economics departments, or social psychologists interacting quite separately from other sub-disciplines within psychology departments). Yet Schelling's model *is* widely used to inform analyses of policies (at virtually all levels of local, state and federal government, as well as among private firms and non-profits such as universities) dealing with segregation of many kinds.

Although Schelling's neighborhood segregation model gave rise to a substantial new literature that remains active to this day, there is an important mismatch between theory and empirical observation that has received relatively little attention. Schelling's model predicts high levels of segregation starting from virtually anywhere within a very large set of initial conditions and parameter values. Yet empirical studies documenting various forms of segregation (e.g., ethnic types among cities, gender types among work places, or methodological types among academic institutions) reveal considerable variation in the extent to which social groups are observed to engage in inter-group mixing. Whereas the world presents observers with a rich variety of heterogeneous segregation outcomes, Schelling's theory does not easily account for this variation as a systematic function of variables or parameters within the model, which raises interesting questions. Can the Schelling model be squared with real world data? Are there extensions of the Schelling model that come closer to reality by predicting heterogeneous segregation levels that vary systematically with observable factors in the environment?

This paper presents such an extension. We augment Schelling's classic model by endowing agents with recognition memory. This capacity enables simulated agents to apply the FACE-recognition heuristic. FACE refers to an evolved capacity that is key for our model, namely recording faces into recognition memory. At the same time, the acronym FACE (for *Fast Acceptance by Common Experience)* refers to the insight that shared local experience can facilitate rapid formation of relationships and, thus, transform assessments of others' underlying quality in a process by which a recognized face, and the quality of its associated memory (i.e., positive or negative), absolutely over-rules the inference that would have been made by stereotyping based on group identity.

According to this definition, Fast Acceptance by Common Experience refers to rapidly formed recognition-based classifications of others' quality (e.g., an "acceptable" versus "unacceptable" neighbor) without regard to group identity, when classifying those with whom face-to-face experience has taken place in the past. When classifying those whose faces are unrecognized, classification continues strictly according to group identity (i.e., ethnic stereotyping). When an unrecognized other person is to be classified, the FACE-recognition heuristic reduces to stereotyping based solely on group identity, exactly the same as in the classic Schelling model. However, when there is even a small amount of shared experience, the quality of that shared experience from the past determines how other people are classified. Classifications based on recognition memory lexicographically over-rule group identity, which is the basis for classification of unrecognized agents in both FACE and classic Schelling models. Given the plausibility of the assumption that context-specific experience from the past can influence the classification of others, it came as a surprise to us that we could not find any previous attempts to extend the Schelling model in this direction.

The model shows that when agents possess face-recognition that lasts as short as a single period (encoding a maximum of only 8 individual faces out of a substantially larger population), this alone is enough to produce significantly higher levels of integration. The key comparison investigated in this paper concerns this variable degree of recognition memory (e.g., no recognition memory as in the classic Schelling model versus any positive number of periods for which the faces of those one encounters remains coded in memory). By introducing variable recognition memory as a representation of heterogeneity in real-world environments (which sometimes have few, sometimes many, opportunities for random face-to-face encounters with other-type agents), the model investigates a novel source of systematic variation into the otherwise classic model of segregation.

The motivation for studying the effect of recognition classification on segregation is to better understand why some real-world environments succeed at achieving sustained levels of cross-group interaction (i.e., high levels of integration) while others seem to be locked into a stubbornly unchanging pattern of segregation. The model is intended to contribute substantively and constructively to policy analysis with a simple message, namely, that we can, relatively cheaply, design institutions that produce modest opportunities for face-to-face encounters with members of other groups. Then, to the extent that people use an acceptance rule based partially on recognition, random face-to-face inter-group mixing could potentially generate large and stable levels of integration that are too pessimistically ruled out by the vast majority of studies based on Schelling's model.

The paper is structured as follows. We outline the classic Schelling model of neighborhood segregation, review previous research related to our extension of this paradigm, and present the limitations of the classic Schelling model (mismatch between its predictions and real world segregation data) that motivate our extension. We then introduce the FACE-recognition heuristic and specify the recognition-augmented Schelling model, an encompassing model that nests the classic Schelling model as a special case. Subsequently, we present a series of agent-based simulations demonstrating the effect of agents' recognition memory and decision rules (the micro-level) on their spatial distribution in the environment (the macro-level).

2 Neighborhood segregation: Schelling's classic model

Neighborhood segregation continues to be a relevant public policy issue (Alesina et al., 1999; Baughman, 2004; Brender, 2005; Musterd et al., 1999; Nechyba, 2003), and recent work in economics, sociology and related social sciences (Fossett 2006; Pancs & Vriend, 2007; Vinković & Kirman, 2006; Zhang 2004a, 2004b, in press) indicates that Schelling's ingenious model continues to play an influential role today.[1] Schelling's neighborhood segregation model consisted of a thought experiment showing that, even when no individual has a preference for segregation (i.e., an aversion to living near members of a different ethnic group), high levels of unintended segregation are very likely to occur. This basic result has been confirmed by many researchers working with theoretical extensions that add new features to Schelling's model. Before turning to these extensions, however, we describe the classic model.

Consider a G-x-G square lattice with a total number of G^2 locations that can be inhabited by up to that many agents. If there are only two groups, a majority and a minority, and if each agent belongs to only one group, then the total number of agents, N, is the sum of the number of majority agents, N_{MAJ}, and the number of minority agents, N_{MIN} (with $N = N_{MAJ} + N_{MIN}$ and $N_{MAJ} \geq N_{MIN}$). In each period, each agent has to make a binary decision: to stay at the current location or try to move somewhere else. To make this decision meaningful, there must be unoccupied locations available for agents who want to move, which implies strictly more locations than total number of agents ($G^2 > N$). Whether an agent considers his or her current location acceptable (i.e., wants to stay or move) is assumed to depend on the proportion of same-type agents in the immediate neighborhood. Schelling defined an agent's neighborhood as the locations directly proximal, or surrounding, an agent's location. Thus, for an agent on the interior of the lattice representing city or society, the neighborhood consists of the eight locations that form a small box around his or her location.[2] Agents located along edges have smaller neighborhoods.[3] An agent considers his current location acceptable as long as the proportion of same-type agents in the neighborhood is above the *acceptability threshold* τ, which is a preference parameter of sorts indicating the minimum acceptable fraction of neighbors who belong to the same group type as the agent. Larger values of this threshold impose more stringent homogeneity requirements in order to classify locations as acceptable, implying that τ can be interpreted as a measure of intolerance.

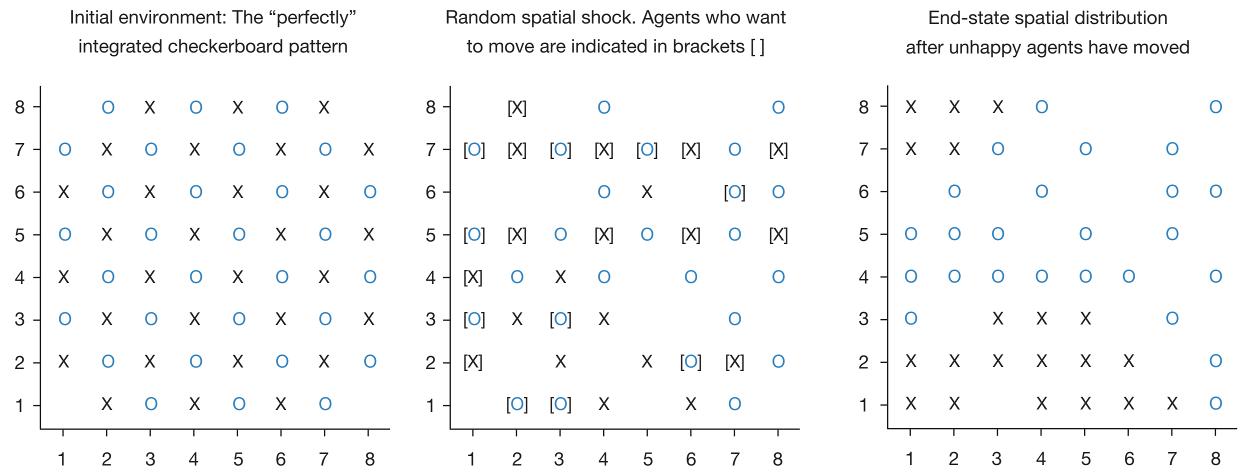
A sequential process then unfolds by which unhappy agents move from unacceptable to acceptable locations, with movers picked at random from the list of all unhappy agents and then moving to the nearest acceptable location. Whenever an agent moves, it changes the spatial distribution of types in other agents' neighborhoods. This, in turn, causes other agents to transition from happy to unhappy, or the reverse. This feedback loop — in which individual decisions (to stay or change locations) and the spatial geography of the environment are jointly causal — is a primary reason why this simple model has generated such enduring interest. Changes in the spatial distribution of types affect individuals' decisions about whether to move, and individual decisions about whether to move affect the spatial distribution of types. The distribution of types reaches a terminal state, which completes a single run of the Schelling model, when one of the following three conditions is met: (1) All agents are happy and thus nobody wants to move; (2) Some agents are unhappy, but no improving moves are possible because none of the un-

[1] In addition to the nonlinear dynamics that lead to counterintuitive mappings from individual behavioral rules into macro structure, which is the focus of Schelling's work and of this paper, multiple factors have been identified as jointly causing persistent segregation (Fossett, 2006), which include differences in income (Bayer et al., 2004), housing discrimination (Nyden et al., 1998), and related forms of social disorder (Musterd et al., 1999).

[2] Alternative definitions of neighborhoods have appeared in this literature and are not generally thought to strongly influence the basic results of the Schelling model. The square-shaped neighborhood definition given above is sometimes referred to as a Moore neighborhood, following Edward F. Moore's work in cellular automata theory, which is distinct from so-called von Neumann neighborhoods, which consist only of adjacent locations that share an edge and therefore resemble a diamond shape.

[3] Some researchers eliminate the effect of edges by defining neighborhoods and distance in a way that measures opposite edges as adjacent. This is something like walking on a globe, where one can never bump into an edge (or walk off the face of the earth). For cities and other physical spaces where integration is a real concern, edges seem to be an important real-world feature that we intentionally preserve in all models presented in this paper. In light of Fossett and Warren's (2005) finding of no boundary effects in the classic Schelling model, it is unclear whether this feature plays much of a role in the FACE-recognition model.

Figure 1: A single run of the classic Schelling model: Integrated checkerboard (left), random shock in which 20 agents disappear and 5 reappear (center), and end-state environment whose integration has unraveled to a high degree of segregation (right).



occupied locations are acceptable from the points of view of the unhappy agents; or (3) The maximum number of iterations is reached, indicating either very slow convergence or the presence of a cycle that will never converge to a terminal state, which we refer to as an indeterminate ending.[4]

For a population with two group types and equal numbers of each type, one can intuitively see that maximal integration is achieved by a perfect checkerboard pattern like the one depicted in the left panel of Figure 1.[5] In that panel, the neighborhood grid is 8x8 (with corner locations unoccupied), implying a total of 60 possible locations, occupied by 30 X-type and 30 O-type agents. Each agent not located on an edge has an equal number of neighbors of each type. Now imagine this perfectly integrated grid undergoes a random spatial shock.[6] In the simulation, this shock is implemented by selecting 20 of the 60 agents at random, chosen uniformly from all occupied locations without regard to type, and removing them from the board. Then five new agents of random type appear at randomly chosen locations, drawn uniformly from among the 24 unoccupied locations (4 unoccupied corners plus 20 newly unoccupied locations after the disappearance of 20 agents). This run of the Schelling model continues by forming a list of agents who want to move. A single unhappy agent is selected at random from this list and then moves to the nearest location that he or she considers acceptable. If there are two or more acceptable locations with the same minimum distance, then one is chosen at random, and the list of unhappy agents is then updated. The number of unhappy agents, although generally decreasing, is not monotonically decreasing, because one agent's move can make one or more other agents unhappy. This process of picking unhappy agents one at a time continues until a terminal state is reached as described above.

Figure 1 displays three (nonconsecutive) periods from a typical run of the classic Schelling model: initial checkerboard, subsequent spatial shock in which 11 X-types and 9 O-types disappeared and 1 X-type and 4 O-types appeared, and end-state spatial distribution. Following the initial shock, the first period in a single round begins with decisions made by each agent about whether he wants to move. The middle panel of Figure 1 indicates with brackets the agents who are unhappy and want to move. With both types' acceptability thresholds set to ½, not all agents were happy in the initial state, although the post-shock spatial distribution has a much larger number of those who want to move: 22 (marked with brackets in the middle panel of Figure 1) of the 45 agents. The right panel of Figure 1 shows the classic result of segregation, with X-types and O-types clustered in distinct areas and each having few or no other-type agents as neighbors. Note that this segregated end-state distribution occurred

[4]Imagine agent A moves, which makes happy agent B transition to unhappy; B in turn moves, which makes the newly happy A transition back to unhappy; but when A moves to make himself happy again, it makes happy B transition back to unhappy, etc.

[5]Schelling (1971b, 1978) begins with a perfectly integrated checkerboard as the initial state, whereas Schelling (1971a) begins with a random spatial distribution as the initial state.

[6]Real-world equivalences of such shocks are any events that effect ethnic composition of cities and neighborhoods. Examples include: (1) a meat packing company opens in a small Kansas town and hires 200 Latino workers; (2) housing prices in the south fall relative to the north, attracting a disproportionate influx of non-white (i.e., lower income) Americans; (3) affirmative action policy is changed at a university or department, and the ethnic composition of the group begins to change; (4) Hurricane Katrina displaces mostly black residents from New Orleans because of the random locational strike of the hurricane.

despite the willingness of all agents to live in a neighborhood that is 50 percent different from themselves, and despite of the fact that the initial state was near-perfect integration.

The mathematical social sciences are rich with studies that build on and modify Schelling's spatial proximity model.[7] Before discussing some of them in more detail, we give an overview of parameters that have been investigated in previous studies:

Size and shape of the spatial environment (measured by edge length G in the case of a square lattice, or by the number of possible locations G^2):

- Distribution of types (either given in frequencies, N_{MAJ} and N_{MIN}, or, equivalently, as total population size, $N = N_{MAJ} + N_{MIN}$, plus a minority rate, N_{MIN}/N);
- Density, or fullness, of the environment (sometimes referred to as occupancy rate) N/G^2;
- Acceptability, or intolerance, thresholds (i.e., the minimum fraction of same-type agents required to classify a location as acceptable) τ_{MIN} and τ_{MAJ}, for minority and majority agents, respectively;
- Procedure for generating the initial distribution;
- Definition of a neighborhood;
- Other parameters needed to implement agent-based simulation, e.g., number of runs, and maximum number of moves allowed before a single run of the model terminates.

One surprising finding is that the Schelling model's basic prediction — high levels of segregation starting from virtually any initial condition — is incredibly robust over a very large set of parameter configurations and modifications to the model (see, for example Epstein & Axtell, 1996). This robustness is noteworthy, given that most agent-based simulations dealing with problems other than segregation typically report numerous sensitivities between parameters and the resulting phenomena of interest.

Previous studies also introduced new elements that attempt to bring the model closer to real cities. Modifications include alternate definitions of the spatial environment, neighborhoods, rules for moving (e.g., simultaneous versus sequential), numbers of and overlap among group types, noise, and vacancy rates. For example, Flache and Hegselmann (2001) studied different shapes and definitions of neighborhoods. Fossett (2006) and Gilbert (2002) added information about the cost of residing at a particular location and reproduced the prediction of large degrees of segregation. Gilbert also considered models where neighborhood characteristics depend on recent histories, allowing agents to switch group membership (e.g., switch ethnic identity), leading again to high levels of segregation. Omer (2005) analyzed what happens when group divisions are organized hierarchically, re-producing the qualitative Schelling prediction of segregation. Scope of vision (i.e., how agents view the boundaries of their own neighborhoods) was analyzed in Fossett and Waren (2005) and Laurie and Jaggi (2003), leading again to segregation. A rather large literature has investigated different utility functions (Bøg 2005, 2006; Bruch & Mare, 2003; Pancs & Vriend, 2007), almost always reinforcing Schelling's prediction of high levels of segregation. Other notable extensions include Vinković and Kirman (2006) who draw on techniques borrowed from physics; attempts at analytic rather than agent-based simulation characterizations of the Schelling model's dynamics using the equilibrium concept of stochastic stability (Bøg 2005, 2006; Young, 1998, 2001; Zhang, 2004a, 2004b, 2008); and the continuum models of Yizhaq et al. (2004). The vast majority of these extensions generalizes or reinforces the original prediction of highly segregated end-state geographies that are unintended and produced by individuals who have no intrinsic preference for segregation.

Parallel to these studies, a growing body of literature relates Schelling's model to real-world data (Bruch & Mare, 2003; Clark, 1991; Fossett, 2006; Portugali et al., 1994), revealing an interesting clash between models and reality. In contrast to overwhelming agreement in the theoretical literature concerning the prediction of high levels of segregation from virtually any starting condition, empirical measures of segregation in cross-sectional studies of cities, countries and other social groupings show considerable heterogeneity (Huttman et al., 1991). For instance, Ellen (1998) examined data from the 1970, 1980, and 1990 decennial censuses and showed that racial integration in the US is not only possible but can also be stable. She defined integrated neighborhoods as those whose black residents constituted between 10 and 50% of the total population and found that in 1990 almost 20% of neighborhoods included in her study fell into this category. Half of these integrated neighborhoods were

[7] See Aydinonat (2007), pages 440–445, for a detailed review of Schelling segregation models and the many generalizations and extensions, sometimes integrating real world data, that mostly (but not always) confirm the prediction of stark segregation. Interestingly, psychologists appear rather less influenced by Schelling's segregation model than sociologists, economists, geographers and physicists (who are cited extensively in Aydinonat, 2007). For example, Colman's (2006) introduction to a special issue of *Journal of Economic Psychology* titled "Thomas C. Schelling's psychological decision theory" in which "segregation" appears as a keyword contains no contributed article on segregation. Colman (2006, p. 606) acknowledges this fact with the (psychology-specific) assertion: "relatively few researchers have developed Schelling's approach to the study of segregation", which means that relatively few *psychologists* have chosen to work in this area.

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also classified as stable, since their non-Hispanic white population did not change by more than 10% between 1980 and 1990. Glaeser and Vigdor (2001) analyzed U.S. 2000 Census data focusing on black/non-black segregation. They reported that segregation is currently at its lowest point since 1920, primarily due to formerly homogeneous white neighborhoods attracting non-white residents. The authors noted considerable geographical variation in this process, with the Western and Southern US achieving greatest levels of integration while the Northeast and Midwest remained segregated. Sethi and Somanathan, (2004) modeled preferences with both neighborhood affluence and ethnic composition in the utility function, showing that segregation may increase even as income levels converge, due to the nonmonotonic relationship between income gaps and geographic segregation. It should be acknowledged that, although relatively rare in the segregation literature, a handful of studies present conditions under which higher rates of integration are indeed possible, although still not likely or predicted in any strong sense (Sethi & Somanathan, 2004).

This disparity between the Schelling model's predictions and wide variation in real world integration is frequently overlooked. Instead, based on considerable evidence that discriminatory forces in contemporary societies, including Obama-era US society, continue to play a regrettable role in many cities with entrenched ethnic segregation, the literature tends to focus on social problems stemming from segregation — and with good justification, given the seriousness of these problems. Examples include long-term joblessness, single parenthood, school drop-outs (Cutler & Glaeser, 1997; Nechyba, 2003), problems in tax collection (Brender, 2005), and reduced chances of positive economic outcomes among the poor together with alienation among the well-off (Atkinson & Flint, 2004). Given the practical importance of attenuating segregation's detrimental effects on social cohesiveness, it is understandable that the Schelling model, which predicts the segregation that these policies aim to assuage, plays a prominent role in this literature. From a theoretical point of view, however, it is nevertheless unsatisfying that the model cannot provide convincing explanations for why some places are relatively integrated while others remain starkly segregated. We now propose our extension of the classic Schelling model that aims to make a step in exactly this direction.

3 Extending the classic Schelling model by FACE-recognition

If there were no constraints in terms of time and available information, we could form our attitudes towards others by collecting all possible information about them and their past behavior. But time and information-processing constraints typically allow us to use only a very limited set of information, categorizing others at least in part using stereotypes (Dovidio, Glick, & Rudman, 2005). Some suggest that cognitive mechanisms underlying stereotyping also produce beneficial results in certain contexts (e.g., Schneider, 2004).

The fundamental distinction between "us" and "them" (i.e., between in-group and out-group members) is documented in rich and often disturbing detail (Esses et al., 2005; Sherif et al., 1961; Tajfel et al., 1971; Tajfel & Turner, 1979; Turner et al., 1987). There is, however, abundant evidence demonstrating that people are not locked into their prejudices and stereotypical thinking.[8] Berg, Abramczuk and Hoffrage (in press) provide examples from history and literature illustrating that people frequently make exceptions, in the form of positive assessments of some out-group members, without changing or modifying their negative stereotypes about the out-group as a whole. The large literature on acceptance (e.g., Brewer & Miller, 1988; Hewstone, 1996, Miller, 2002; Rothbart & John, 1985) provides relevant background for our extension of the Schelling model. On the one hand, people can be deeply prejudiced and show blanket disdain of other groups, ranging from avoidance to full-blown hatred (e.g., of blacks, whites, Jews, Muslims, poor people, homosexuals, etc.). On the other hand, the same people can be willing, and even enthusiastic, to build friendships across these very same group boundaries.

Drawing on previous work documenting the important role of recognition in a variety of inferential and decision contexts (Berg & Faria, 2008; Bruce & Young, 1986; Semenza & Sgaramella, 1993; Semenza & Zettin, 1989; Schweinberger et al., 2002), we conducted a series of computer simulations aimed at demonstrating how the FACE-recognition heuristic can produce a much wider variety of spatial patterns of integration and segregation that are systematically linked to parameters in the model.

[8] One of the predecessors of the FACE-recognition heuristic is contact theory (Allport, 1954), describing how face-to-face interactions between members of different groups holding negative stereotypes of each other can limit prejudice. A key difference between contact theory and the FACE-heuristic is that the contact theory literature generally seeks institutions that categorically shift beliefs about the other group (i.e., reduce the general level of prejudice) well beyond particular situations or person-specific relationships. Critics of the goal of designing integration-promoting institutions questioned whether the requirements of Allport's contact theory could possibly be implemented as real-world institutions (Dixon et al., 2005), and considerable work has been devoted to this institutional analysis within the contact theory paradigm (see Pettigrew, 1998; Pettigrew & Tropp, 2006, and references therein). There is fairly widespread agreement that moderate contact can increase tolerance, measured variously in the related literatures. One interesting finding is that repetitive exposure to people (and abstract symbols, too!) appears to increase favorable sentiment (e.g., Homans, 1950; Zajonc, 1968).

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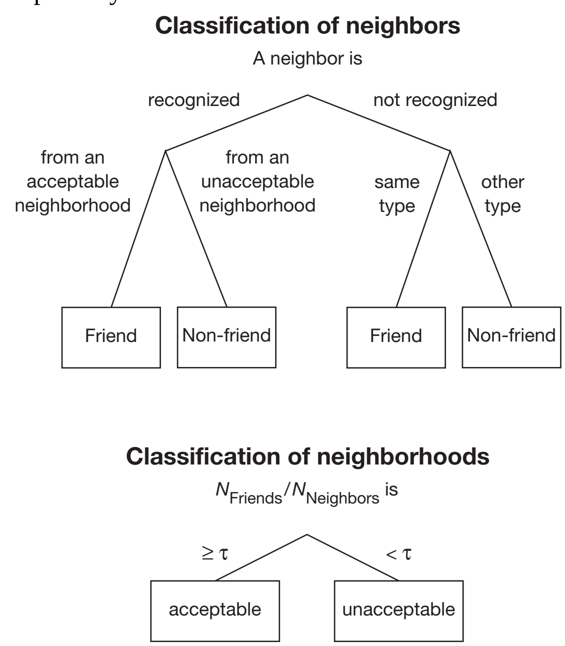
To simplify notation, we use friend as a short-form synonym for desirable neighbor. Thus, in the classic Schelling model, friends of agent i consist of all co-ethnics (i.e., other agents who belong to the same group as agent i.) A location is acceptable if and only if the proportion of friends in the neighborhood, $N_{friends}/N_{neighbors}$, weakly exceeds that agent's acceptability threshold τ, and unacceptable otherwise.

In the FACE-recognition extension of the model, the same threshold rule for determining whether a location is acceptable provides the crucial behavioral element, but with one important modification concerning how agents classify friends and nonfriends, which is depicted in the flow-chart in Figure 2. The recognition-augmented Schelling model assumes that agents are endowed with a small amount of memory about other agents they recently encountered. Each agent's memory stores information about these agents who are recognized from the K most recent neighborhoods and also records whether a particular agent is most recently recognized from an acceptable or unacceptable neighborhood. Agents who are recognized from an acceptable neighborhood in the past are counted as friends, no matter whether they are same-type or other-type. Agents who are recognized from an unacceptable neighborhood in the past are counted as nonfriends, no matter whether they are same-type or other-type.

Note that the strict ordering of the FACE-recognition variant of the model, in which previous experience with an individual agent trumps this other agent's group membership, is akin to simple heuristics that implement one-reason decision making (Gigerenzer, Todd, & the ABC Research Group, 1999) and consistent with the theoretical finding that ignoring information can be beneficial (Berg & Hoffrage, 2008). Further note that the FACE-recognition model can lead to states of happiness that differ from those in the classic Schelling model. Consider an agent who is surrounded by a majority of other-type agents whom he knows from a neighborhood in which he was happy. This agent would be unhappy in the classic model but happy in the recognition-augmented extension. Similarly, an agent who is surrounded by a majority of same-type agents is always happy in the classic Schelling model, but may very well want to move away in the recognition-augmented extension if those co-ethnics are remembered from a neighborhood in which he was previously unhappy.[9] It is important to note that this new way for agents to be happy about a particular location does not trivially lead to more happiness and therefore more integration — simply because there is also a new way to be unhappy. A priori, the recognition step in the classification of locations could just as well lead to more unhappiness and segregation.

Figure 2: Recognition heuristic for classifying neighbors as friends or nonfriends (upper panel), and classifying locations as acceptable or unacceptable (lower panel). How a potential or actual neighbor is classified depends critically on recognition; if recognized, classification as friend or nonfriend depends on acceptability of the neighborhood from which that agent is most recently recognized; if not recognized, classification depends on group identity just as in classic Schelling. To determine whether a potential or actual neighborhood is acceptable, the proportion of friends among all neighbors is compared to the acceptability threshold τ.



When encountering unrecognized agents, the FACE-recognition model reduces to the classic Schelling model. Cases of remembering another agent from two previous neighborhoods in the past K periods are extremely rare, but in the event that an agent is recognized from both good and bad neighborhoods, only the quality of the most recent memory matters for classification. This nested structure of the two models can be formalized using a memory span parameter, which specifies how many previous periods are stored into each agent's memory. The classic Schelling model is then recovered from the FACE-recognition model if this memory span parameter is set to zero, which implies that each agent recognizes no other agents and, consequently, all friend/nonfriend classifications are based solely on group identity.

When evaluating the acceptability of neighborhoods with one or more recognized agents, the changes that take place are few in number and mostly very local. The results below, however, show that these small, local

[9] For detailed illustrations of both these cases from an actual simulated run depicted in a variant of Figure 1, see Berg et al. (in press).

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changes lead to surprisingly large macro-level changes in the spatial geography of the environment. The micro and macro levels are connected by a jointly causal loop (Coleman, 1994) that generates co-evolution of individual behavior and the external environment. In other words, the macro pattern in the neighborhood influences the level of happiness experienced at the micro level. In turn, the happiness experienced at the micro level influences decisions to move, which completes the co-evolutionary loop by reformulating the composition of neighborhoods that constitute the macro level.

4 Pitting the FACE-recognition model against the classic Schelling model: Simulation results

4.1 Measures of end-state integration

The three periods of a single run depicted in Figure 1 show a stark contrast between initial and end-state spatial distributions. To make sure that such contrasts are systematic and not the result of mere chance occurrences, we repeat these simulations and report empirical distributions for two different measures of end-state integration across many runs. Each run includes (as a control condition) the classic Schelling model with no recognition memory and (as a treatment condition) the FACE-recognition model with at least one period of memory. In every run, the two conditions begin with the same integrated checkerboard and are then subjected to the same random spatial shock. In other words, exactly the same spatial shock is used to initiate both control and all treatment conditions for each run, enabling comparison of macro-level consequences of the FACE-recognition heuristic starting from exactly the same initial world.

To describe such end-state spatial distributions, Pancs and Vriend (2007) use six segregation measures, recognizing that they are highly correlated while emphasizing different aspects of inter-group mixing in the lattice environment. We turned their segregation measures into integration measures, that is, our coding is such that high values indicate high integration rather than high segregation. We focus on two of these measures: Other-Type Exposure and Contact with at least One Other.[10]

Other-Type exposure (*OT*) is the mean fraction of other-type agents as neighbors, averaged over agents. To compute *OT* on a spatial distribution, one computes for each agent *i* the number of other-type agents in the neighborhood, $N_{OT,i}$, and the total number of neighbors, N_i. Agent *i*'s fraction of other-type agents in his neighborhood is simply $N_{OT,i}/N_i$, and *OT* is computed as the average across agents: $\Sigma_i(N_{OT,i}/N_i)/N$.

Contact with at least One Other (*COO*) measures the fraction of agents whose neighborhood includes at least one other-type agent. To calculate *COO*, let $COO_i = 1$ if $N_{OT,i} > 0$, and $COO_i = 0$ otherwise. Thus, COO_i is an indicator variable coded as 1 if agent *i* has one or more other-type neighbors and zero otherwise. Then $COO = \Sigma_i COO_i /N$. The complement, 1-*COO*, is the fraction of agents who are absolutely segregated, that is, live entirely isolated from other-type agents.

For a given parameterization of the model, there is considerable variability in these integration measures, due to two sources of random variation: spatial shocks that generate the initial condition for each run, and random selection of agents from the list of those who want to move. Once the terminal state is reached in control and treatment runs, a single observation of the two integration measures is recorded for the control and each treatment run. Thus, after 100 runs, two sets of histograms are available, one displaying histograms of end-state *OT* for control and treatment conditions, and one displaying *COO* for control and treatment conditions.

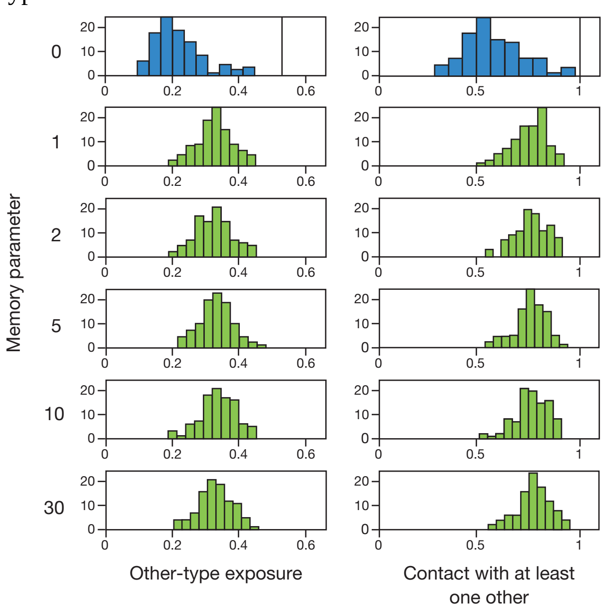
4.2 End-state integration as a function of memory size

We start our investigations with the question of how different quantities of recognition memory affect end-state integration. We implemented six memory treatments, starting with zero memory (which corresponds to the classic Schelling model), followed by the first treatment condition (FACE-recognition with a memory span of one period), ranging upward through a memory span of 30 periods. A memory span is the number of periods an agent is able to look back to determine whether or not a current neighbor was already a neighbor in the past, and if so, whether this memory occurred in an acceptable or unacceptable neighborhood.

Figure 3 shows histograms of end-state integration for the six memory conditions. Large differences between control (classic Schelling, when the memory span is set to 0) and treatment runs (FACE-recognition, when the memory span parameter ≥ 1) are visible, indicating a large effect that is both statistically and substantially significant. Another striking feature of Figure 3 is that a little memory (e.g., the capacity to remember one or two periods into the past) has almost the same effect as lots of memory. Thus, introducing a small amount of recognition memory leads immediately to a discontinuously large, or "quan-

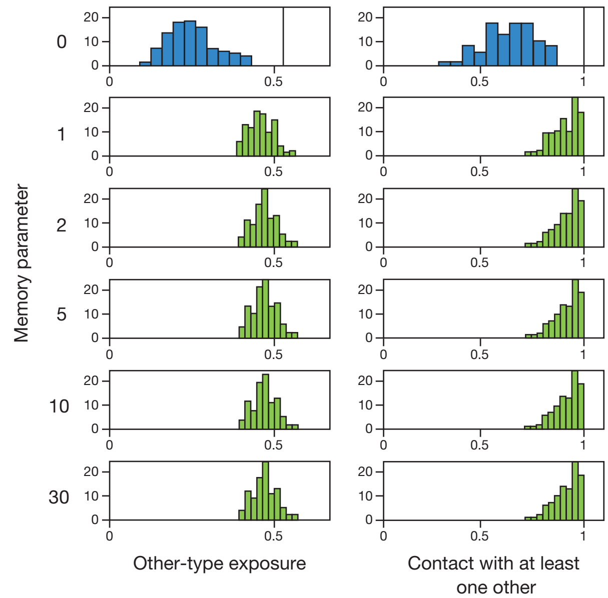
[10] In Berg et al. (in press), we also report another end-state integration measure referred to as the *switch rate*, defined as the average number of switches of type encountered in a 360-degree panoramic scan of each agent's adjacent locations, normalized to a range between 0 and 1 and averaged across agents. We omit this variable here owing to space considerations. The conclusions based on switch rate are basically the same as for the *OT* and *COO*.

Figure 3: Histograms of end-state integration as captured by two dependent variables (Other-type exposure and Contact with at least one other) in six memory treatments. When the memory span is set to zero, the FACE-recognition model reduces to the special case of the classic Schelling model. Memory spans of the previous 1, 2, 5, 10, or 30 rounds are variants of the FACE-recognition model. Unless otherwise stated, the parameter values here and in the following figures are: 8x8 grid, 30 of each type in the initial checkerboard, 20 randomly disappearing, 5 re-appearing, and acceptability thresholds for both types of τ = 1/2.



tum", change in the end-state spatial distribution's level of integration. Additional amounts of memory have very limited effects on integration. When interpreting Figure 3, it is useful to be clear about the benchmarks. The vertical lines in the first row of histograms, at 0.53 and 1.00, respectively, show the levels of integration in the "perfectly integrated" checkerboard neighborhood before the random shock (the same applies to Figures 4, 5, and 7). In the post-shock neighborhood, these upper bounds are not always attainable because the number of agents has typically changed. Better benchmark therefore are the starting levels of integration directly following the initial shocks — these had ranges of 40 to 55% (with mean of 48%) for Other-Type exposure, and 82 to 100% (with mean of 94%) for Contact with at least One Other. For each of the two dependent variables, other-type exposure and contact with at least one other, the median of the FACE-recognition extension falls about in the middle between the median of the classic Schelling and the respective benchmark.

Figure 4: Histograms of end-state integration when agents have a lower acceptability threshold τ. Parameter values are the same as in Figure 3, except for τ which is set here to 2/5.



4.3 End-state integration as a function of the acceptability threshold

We now turn to the question whether small, local changes in the classification of locations (as acceptable or not) generate sizable changes in end-state integration relative to the control runs. As Figure 4 shows, a slight relaxation of all agents' acceptability thresholds from 0.5 to 0.4 has enormous effects. With this slightly more tolerant threshold, the classic Schelling model's end-state integration shifts very slightly upward (compared to Figure 3), continuing to reflect the "unraveling" from perfect integration to unintended segregation. In contrast, the FACE-recognition model shows much greater sensitivity to reductions in intolerance, which shift the integration distributions shown in the histograms to near maximal levels, with large clusters concentrating around (and sometimes scattering above!) the initial-state levels of integration. As in Figure 3, these initial-state levels of integration, which can be regarded as benchmarks for maximal post-shock integration, are indicated by vertical bars in the first row of the histograms. They are never achieved as levels of end-state integration in the classic Schelling model, but are regularly achieved, and sometimes even surpassed (for Other-type-exposure), by the FACE-recognition model.

Next, we introduce differences between majority and minority agents' acceptability thresholds. Because we have already established that any increase in memory

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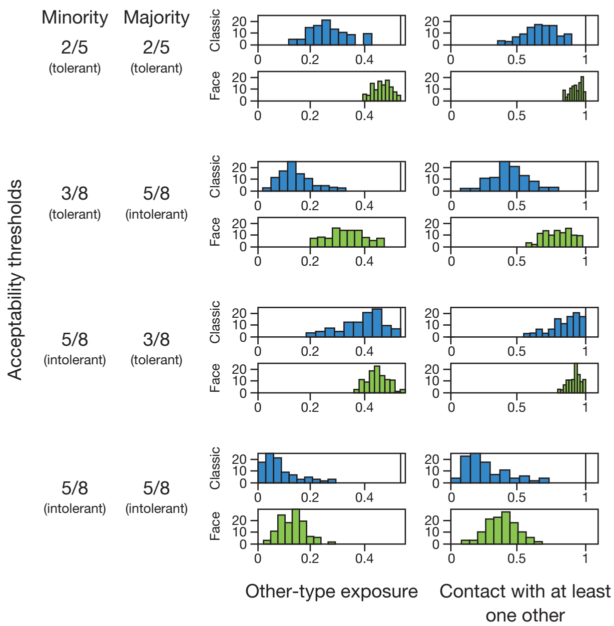
Figure 5: Histograms of end-state integration measures as a function of acceptability thresholds (memory span = 5).



size beyond 1 has little effect, we explore the effect of such differences between majority and minority agents only for memory sizes of zero (the classic model) and five (for the FACE-recognition model). Figure 5 presents four configurations of acceptability thresholds. In the first configuration (τ_{MIN} = τ_{MAJ}=2/5), both minority and majority agents are more tolerant (than the τ = ½ benchmark case), which produces a large difference between control and recognition treatments. In the second configuration (τ_{MIN} = 3/8 and τ_{MAJ} = 5/8), minority types are more tolerant and majority types less tolerant, which produces another large treatment effect (even larger than the first configuration in many runs), but with slightly lower levels of end-state integration in both cases. In the third configuration (τ_{MIN} = 5/8 and τ_{MAJ} = 3/8), minority agents are less tolerant and majority types are more tolerant. Because, by definition, most agents are majority types, and because they are more tolerant in this third configuration, the control runs have much higher levels of end-state integration and therefore produce smaller treatment effects (measured as the horizontal difference between distributions, or end-state integration in recognition runs minus end-state integration in control runs). In the fourth configuration (τ_{MIN} = τ_{MAJ} = 5/8), both types are less tolerant, which produces lower levels of end-state integration in all cases, but a still noticeable treatment effect.

We measured treatment effects in a variety of other configurations of acceptability thresholds, which reinforced two key findings visible in Figure 5. As soon as there is enough intolerance to produce unraveling of integration to segregation in the classic Schelling model, the effect of memory on end-state integration is large, decreasing steadily as all agents become less tolerant (i.e., holding both types' τ thresholds equal and increasing them toward 1). The second interesting result is the asymmetric effect of heterogeneous intolerance or acceptability thresholds. When minority agents are more tolerant and majority agents are less tolerant,[11] the treatment-control difference is much larger than if the acceptability parameters are switched between types (so that minorities are less tolerant and majorities are more tolerant). One reason why the treatment-control difference is small when only majorities are more tolerant is that tolerant majorities push the control-treatment levels of integration higher, thereby reducing the difference due to floor effects. Another reason is that most available locations tend to be majority-type heavy, by definition of there being more majority types. Therefore, when minority agents are less tolerant, more moves are required to find acceptable neighborhoods for all agents, and greater spatial concentrations of minorities are produced than would be the case for the same sized decrease in tolerance among majority types.

4.4 Dispersion and time to reach convergence as a function of memory size

Another interesting feature visible in Figures 3, 4, and 5 is that, in every single treatment-control comparison and for each of our integration measures, the runs with recognition memory produce dramatically less dispersed distributions. In many cases, the classic Schelling model's end-state integration distributions are more than twice as dispersed as the treatment distributions. This reduction in dispersion in the FACE-recognition model is important because it tightens the link between model parameters and the dependent variables. In other words, the FACE-recognition model provides a much higher signal-to-noise ratio, where "signal" is interpreted as a change in the model's parameters and "noise" is the dispersion in end-state integration due to random effects such as the random spatial shock, random ordering of when unhappy agents get to move, and random choice of locations when a mover has more than one minimum-distance acceptable location.

Related to the reduction of dispersion in the variables measuring end-state integration, the introduction of recognition memory in the model also leads to a dramatic reduction of the dispersion of the number of iterations needed to reach convergence. Fewer moves are needed to

[11] It is interesting to note that the first of these two schemes (i.e., majority type being less tolerant than minority types) is what Clark (1991), for example, suggests is found in real-world settings.

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Figure 6: Histograms of number of moves to reach convergence, by memory and acceptability threshold τ (memory span = 1).

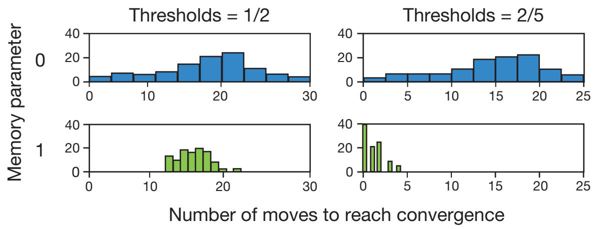
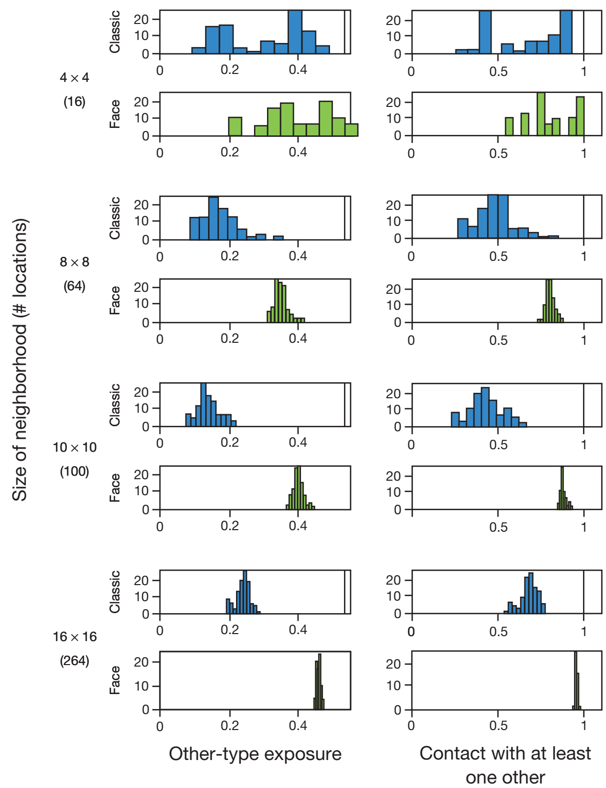


Figure 7: Histograms of end-state integration showing that the effect of recognition memory on end-state integration increases with neighborhood size (memory span = 5).



reach convergence in the FACE-recognition model, and the distribution of number of moves to convergence is far less dispersed than in the classic Schelling model. As can be seen in the first column in Figure 6, the range of number-of-moves-to-reach-convergence shrinks from roughly the interval [0, 30] to [10, 20]. That 2/3 reduction in range coincides with a clear reduction in the modal number of moves — from more than 20 in the classic Schelling model to somewhere around 15 or 16 once recognition memory is introduced.

Comparing the two histograms within the first row of Figure 6, one sees that reducing the acceptability threshold reduces the number of moves needed to reach convergence in the classic Schelling model by roughly 5. However, this reduction in moves needed to reach convergence is modest when compared to the dramatic decrease when the memory span parameter moves from 0 to 1 (or any positive integer, which produces a nearly identical reduction in moves). Thus, recognition memory increases end-state integration, reduces dispersion of integration, and dramatically reduces the number of moves to reach convergence.

Recall that the dynamics come to a terminal state in one of three ways: (1) happy convergence; (2) unhappy convergence; and (3) the maximum number of iterations allowed by the program reached without achieving convergence. An important difference between control and treatment runs is the relative frequency of happy versus unhappy convergent outcomes.[12] In the classic Schelling model 10 to 90 percent of runs end in unhappy convergences depending on acceptability thresholds and neighborhood density, typically where minorities cannot find any available locations with enough minority neighbors. In the recognition treatments (i.e., memory span parameter > 0), unhappy convergence occurred 1 to 3 out of a total of 100 runs across all parameterizations reported in these tables.

[12] Nonconvergent cycles are a theoretical possibility for some parameterizations of the Schelling model, but we never observed any in the parameterizations reported here. Using 40 as the maximum number of iterations allowable for the 8x8 neighborhood (and higher limits for larger neighborhoods discussed later), not a single indeterminate case was observed in any of the runs reported in this paper. This is reflected in the fact that the entire histogram of "number of moves to reach convergence" lies to the left of 40.

4.5 Integration as a function of the number of locations

Skeptics might worry that face-recognition is more important in small places because the fraction of all residents that are recognized is higher. As the number of locations (i.e., city size) increases, the fraction of all agents that any one particular agent can recognize approaches zero. One might therefore question whether recognition effects could withstand the test of scaling up to larger and larger sized environments. Figure 7, however, shows counterintuitively that recognition has a more dramatic effect, the larger the grid is. This figure was constructed as follows. Grid-size took on the values 4, 8, 10 and 16, resulting in numbers of locations of 16, 64, 100 and 256. The numbers of agents who randomly disappear and reappear in creating initial spatial shocks are, in all cases, proportional to the benchmark of Schelling's 8x8 setup with 20 of 60 (33%) disappearing, 5 of 60 (8%) reap-

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pearing, and finally arriving at a total number of agents equal to 45 of the original 60 (or 75%) of the cornerless checkerboard population. Thus, as the grid size ranges over 4, 8, 10 and 16, the parameter indicating the post-shock number of agents takes on the values 9 [= $0.75(4^2-4)$], 45 [= $0.75(8^2-4)$], 72 [= $0.75(10^2-4)$], and 189 [= $0.75(16^2-4)$].

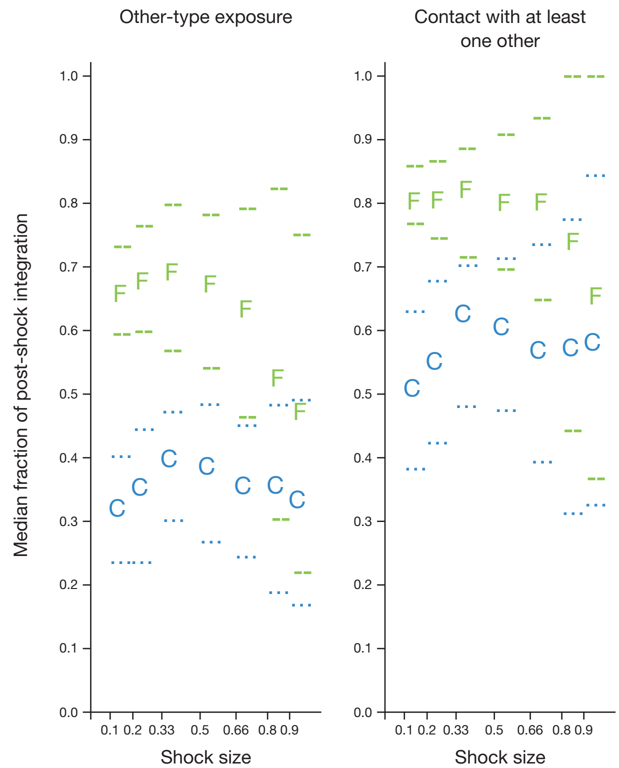
The resulting histograms show large, persistent, and ever-more precise (i.e., less dispersed) differences in end-state integration. Less dispersion implies that horizontal differences between histograms become less and less the result of noise from randomization steps in the simulation. Thus, large recognition effects reported earlier should not be dismissed as mere small-world phenomena and, instead, can be viewed as broadly applicable to groups of varying sizes — quite possibly including large metropolitan cities.

4.6 End-state/initial-state preservation of integration as a function of shock size

The results presented so far share one important feature, which is the magnitude of the spatial shock (1/3 of the agents randomly moved at the initial stage). In this section, we examine the sensitivity of our reported treatment-control differences with respect to shock size. Schelling emphasized that even very small shock sizes could produce dramatic unraveling from the integrated checkerboard to stark segregation. At the other extreme, as the shock size approaches 100 percent, the post-shock spatial distribution becomes increasingly close to a uniform distribution in which agents are placed in random locations without regard to group type.

Figure 8 shows the fraction of post-shock integration that is preserved in end-state integration as a function of shock size. The x-axis shows shock sizes of 10, 20, 33, 50, 66, 80 and 90 percent, ranging from near perfect integration to near random initial conditions. The y-axis shows end-state integration divided by post-shock integration, which measures the percentage of integration preserved in the process of moving to a convergent end-state spatial distribution. The median value of the percentage of integration preserved is indicated by “F” for the FACE-recognition treatment and “C” for the classic Schelling, or control, treatment. In addition, Figure 8 shows 80 percent confidence bands representing the 10^{th} and 90^{th} percentiles for each set of 100 runs. In each set of 100 runs, the C and F treatments begin with the same spatial shocks but evolve according to classic-Schelling or recognition-augmented rules for classifying locations as acceptable or not. For the 10 percent shock, the distributions of preserved integration are far apart, with entirely non-overlapping 80-percent confidence bands in all three integration measures.

Figure 8: Median fraction of post-shock integration preserved in the end-state, indicated by "F" for FACE recognition treatment and "C" for Classic Schelling Model, with 80 percent sample-distribution intervals (memory span = 5). Shock size on the x-axis represents the fraction of the population perturbed away from their respective beginning positions in the perfectly integrated checkerboard.



As shock size increases, two countervailing effects are noteworthy. First, because the post-shock (initial) distribution gets further away from perfect integration, end-state integration must be further away from perfect integration as well. All else equal, this would reduce the level of end-state integration. But because post-shock integration (directly following the shock and not at the end-state) is the denominator of the ratio depicted on the y-axis of Figure 8 and is negatively affected by shock size as well, this would increase the values plotted on the y-axis, all else equal. As shock size approaches 100 percent and the initial post-shock distribution becomes completely random, the treatment effect disappears, as intuition would suggest, indicated by increasing overlap between control and treatment distributions. It is rather remarkable, however, that large treatment-control differences persist for very large magnitude shocks, affecting 50 percent or more of the population with an involuntary move.

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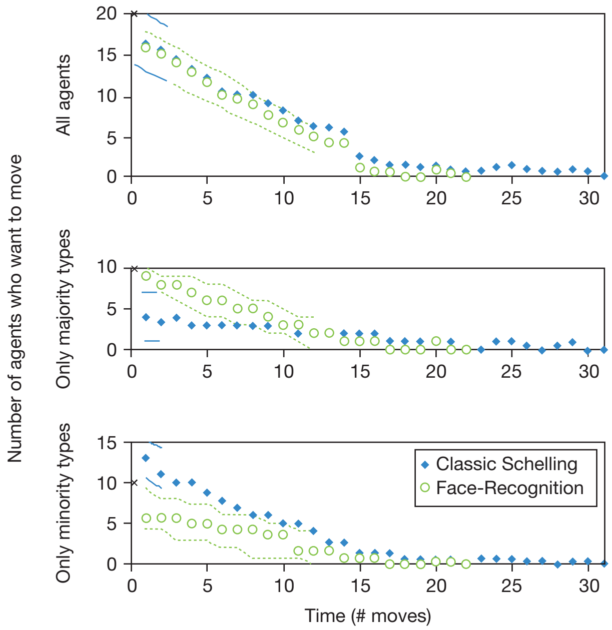


Figure 9: Median number of agents who are unhappy and thus want to move as a function of time (memory span = 5).

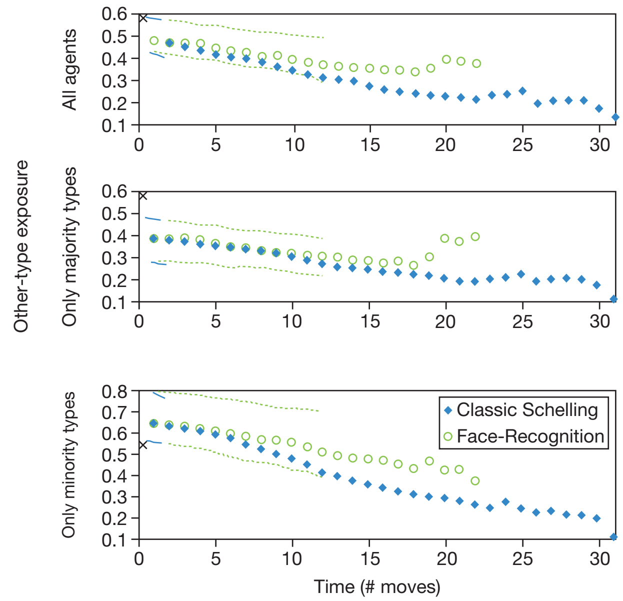


Figure 10: Median other-type exposure as a function of time (memory span = 5).

4.7 Effects of FACE recognition on the micro level

Until this point, we have adopted a macro-perspective and analyzed spatial distributions in the environment summarized by integration measures. In this section, we adopt the micro-perspective of an individual agent to see what the effects of recognition memory are on acceptance by individuals of their locations. Grid-size is once again 8, with 45 post-shock and acceptability thresholds set to 1/2.

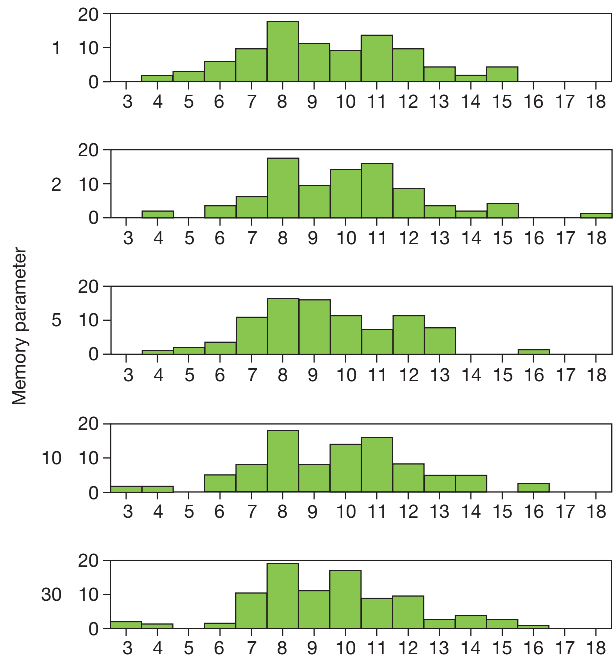
Figure 9 shows the time path of the number of agents who want to move, averaged period by period over 100 control and treatment runs, respectively. The 80-percent confidence bands appear only until the period at which the very earliest run among the 100 runs converged. At that point, the sample size (of runs at a particular period on the way toward convergence) changes because fewer and fewer of the 100 runs produce observations as the period number increases. The median number of agents who want to move is plotted for every number of periods at which there was at least one observation. The median reported at each period is computed among only those runs that reached that number of periods. Were we to continue counting converged runs at their end-state values (e.g., continuing to count the number who want to move as zero where happy convergences have already occurred), then the median number would approach zero more quickly than depicted in Figure 9.

Figure 9 shows an interesting asymmetry between minority and majority movers. When counted together in the upper time series, there is little within-period difference in the numbers of movers between control and treatment, except that the number of periods to reach convergence is smaller in FACE-recognition treatments. The middle and lower time paths in Figure 9, however, reveal significant within-period control-treatment differences with respect to the numbers of minority and majority agents, respectively, who want to move. Once recognition memory is introduced, there are significantly more unhappy majority agents in early rounds (because there are more negative shifts from friend to nonfriend among same-type neighbors) and significantly fewer unhappy minority agents (because there are more ways to be a happy minority agent as the result of nonfriend-to-friend shifts among other-type agents, thanks to recognition memory). The number of unhappy majority agents decreases rapidly in treatment runs, however, resulting in faster convergences and an increased rate of happy convergences. This period-by-period view along the path toward end-state convergence reveals, once again, that a small amount of memory which generates very few changes in terms of individual-level classifications nevertheless produces large effects on individuals' happiness with their locations.

Figure 10 shows similar period-by-period distributions in control and treatment runs for integration as measured by *OT*. The upper-most time series shows how integration changes along the time path. The second and third panels of Figure 10 break out integration by majority and minority, computing integration strictly among majorities and minorities, respectively. One observation from Figure 10 is that treatment-control differences in integration are nonlinear with respect to time, diverging sharply in the final few periods before convergence, especially for majority types. These time-path data indicate that later movers — especially unhappy majority types — make late-period moves that are responsible for a disproportionately large share of the increase in integration along that variable's time path. Meanwhile, minority types' level of integration reduces at a slower rate, steadily making larger and larger contributions to aggregate integration. The picture that emerges is one of gradual mixing by minorities but with rather more chaotic reductions followed by subsequent increases in majority types' level of integration.

Figure 11 provides one final piece of evidence regarding the effect of adding the first unit of memory contrasted with null effects of subsequent additions to memory. Figure 11 shows the number of agents who, in their end-state locations, would have wanted to move in the classic Schelling model but are made happy thanks to recognition memory. Across the five memory span parameters that we implemented, the number of such agents is about 9 out of 45 (i.e., about 20%). As the memory span increases from 1 to 30, there is a slight reduction in the distribution's dispersion, but no apparent change in its position, indicating that this number is largely independent of the amount of memory with which agents are endowed.

Figure 11: Made happy by memory: Histograms of end-state number of agents who would have wanted to move in the classic Schelling model but, by using the FACE-recognition heuristic, consider their current neighborhood acceptable.



5 Discussion

In this paper, we introduced the FACE-recognition model (Fast-Acceptance-by-Common-Experience), which extends the classic Schelling model of neighborhood segregation by giving agents a small amount of FACE-recognition memory. In this extension, agents classify neighborhoods the same way as in the classic Schelling model, by computing the fraction of all acceptable neighbors and comparing this with an acceptability threshold. Moreover, like in the Schelling model, unrecognized neighbors are classified as acceptable if they are same-type agents, and as unacceptable if they are other-type agents. Unlike in the Schelling model, however, recognition-augmented agents are able to recognize agents who were neighbors in previous periods and classify them as acceptable if they were neighbors in acceptable neighborhoods and as unacceptable if they were neighbors in unacceptable neighborhoods. This classification of recognized agents lexicographically overrules classification based on their group identity. Even though this extension of the classic Schelling model leads to only a small number of reclassifications of nearby agents in which group identity is overruled, it nevertheless results in large-scale shifts in end-state spatial distributions. These end-state distributions feature much higher levels of inter-group mixing as measured by two quantitative measures of integration that are standard in the segregation literature, faster convergence to stable states, and higher signal-to-noise ratio in terms of the influence of changes in model parameters versus noise from randomization steps in the sequence of moves. The effects persist across various acceptability thresholds, grid sizes, and shock sizes.

The FACE-recognition heuristic is similar but not identical to the recognition heuristic studied in Goldstein and Gigerenzer (1999, 2002). Goldstein and Gigerenzer proposed that recognition is an evolved capacity that can be used to make accurate rankings among pairs of objects whenever there is correlation between recognition and a criterion with respect to which objects are to be ranked. Reasoning according to the recognition heuristic is a one step process: if one of the two objects is recognized and the other is not, the one that is recognized is judged to have larger value. An important feature of the recognition heuristic is the fact that it is non-compensatory. The moment one object is recognized and the other not, the decision, or choice, or classification is determined. No other information enters the decision process and therefore no further information needs to be weighted, or has the potential for overruling the recognition-based decision. We use the same non-compensatory, or lexicographic, mechanism in the FACE-recognition model, which can be represented with a non-compensatory decision tree as in Figure 2. The difference is that for the FACE-recognition heuristic, mere recognition does not necessarily lead to a positive classification; instead, the attitude toward others is modified according to whether shared common experience had a positive or a negative flavor.

The FACE-heuristic also relates to important work on the forefront of game theory called inductive game theory, which models agents playing games that they do not fully know or understand (Hanaki, Ishikawa, & Akiyama, 2009; Kaneko & Kline, 2008). In our model, agents are endowed with a uniform ethnic preference parameter and identical recognition-based decision process, and yet different play-path histories among agents lead them to have different views of their location and the surrounding environment.

The FACE-recognition heuristic relates significantly to Aktipis' (2006) evolutionary game theory model in which agents repeatedly play Prisoner's Dilemma while using different decision rules for choosing with whom to play in each round. Numerous strategies in such population games have been studied in an attempt to explain the real-world observation that people, even in anonymous one-shot games, often play non-Nash strategies to achieve greater cooperation than is predicted by standard game theory (see, for example, Bowles & Gintis, 2004; Nakamaru & Kawata, 2002; Sudgen, 1986). Aktipis considers two simple strategies, D-mem and C-mem, that rely on recognition to choose with whom to play the game. The D-mem strategy for accepting partners is to always accept an unrecognized individual as a playing partner and then cooperate. Whenever a partner defects, D-mem records that individual's name on the defector list, thereby excluding this individual as a partner in the future. Once the agent's memory limit is reached, D-mem removes the oldest defector from the list to record new ones. The second strategy in Aktipis (2006) is C-mem, which remembers only the names of recent cooperators, and once its memory capacity is full, it accepts playing partners only from the names of cooperators on that list. A key similarity between our model and Aktipis' is that attitudes towards others are determined by simple memories based only on recognition and outcome of previous encounters. Moreover, for both models it can be shown that their very modest memory requirements and very simple decision rules lead to large-magnitude population-level effects.

Finally, the FACE-recognition heuristic shares some commonalities with decision processes studied by Yamagishi et al. (1999) and Yamagishi and Kiyonari (2000) in the context of in-group boasting. In a series of experiments (without face-to-face interaction, though), they show that in-group favoritism occurs only when subjects expect favorable treatment from in-group members. When other reliable information about partners' benevolence is available, group membership is ignored, as it is in the FACE-recognition model. In both empirical and theoretical forms, the decision processes are lexicographic heuristics.

A key result of our simulations is that a very small amount of recognition memory can produce surprisingly durable levels of integration. Thus, when comparing environments of agents who have opportunities to recognize even a handful of other-type neighbors with environments whose agents do not have this opportunity, our model identifies a new variable capable of explaining observed differences in levels of segregation. Beyond this more realistic range of predictions that offer a new explanation for low versus high degrees of integration, the FACE-recognition model implies that institutions which promote face-to-face mixing can have large effects on long-run integration. This stands in marked contrast to the classic Schelling model's rather pessimistic and unconditional prediction that all, or most, groups will unravel to unintended and high levels of segregation.

Regarding the literature concerning policy tools aimed at fostering integration, the extended Schelling model studied in this paper suggests a new theoretical account for explaining why cities and other social spheres of interaction differ in terms of inter-group mixing. The model generates the hypothesis that locations whose histories created above-average levels of inter-group face-to-face interaction in the past — by historical accident or by intentional institutional design — should have above-average levels of integration in the present.

A large fraction of any achieved level of integration can be maintained in the FACE-recognition model by fostering very modest quantities of face-to-face recognition across social groups. Small amounts of recognition robustly maintain integration in the face of significant spatial shocks. This finding lends theoretical support also to designed institutions in smaller-scale settings whose aim is to maintain integration in the face of continual shocks to group membership. One example is the prosaic-sounding coffee-and-cake institution discussed in Gigerenzer (2006), which is one part of a designed institution that attempts to generate a high frequency of chance face-to-face encounters within large and interdisciplinary research teams. Parks, bars, restaurants and road systems with an unavoidable central meeting location that generate unusually high levels of inter-group face-to-face experience doing normal, mundane things provide other examples of environments designed to facilitate the meeting of different group members on a regular basis. Nyden et al. (1998) note that the existence of such places is a regular characteristic of integrated neighborhoods. Recent evidence on social networks using GSS data document that Americans are not as segregated as some of the gloomiest estimates have suggested, although these results motivate the study's authors to call for institutions (perhaps those that promote opportunities for face recognition to develop would function well in this regard) and new attitudes that assuage the documented tendency to separate from those who differ on race, politics and religion (DiPrete, Gelman, McCormick, Teitler & Zheng, 2010).

There are several ideas that are natural to consider for modifying or extending the FACE-recognition model. We mention a few of them here without further analysis. In the real world, intergroup dynamics are affected not by a single shock, but by a sequence of occasional shocks. These occur when institutions change or other large-magnitude shifts in the environment take place. For example, the moves people make are sometimes caused by changing family structure, changes in school quality, or job changes. It would therefore seem worthwhile to investigate whether the large-magnitude effects of recognition memory on end-state integration are attenuated or accentuated by repeated shocks after specifying a reasonable stochastic process to model repeated but occasional shocks.

Another simplification in the FACE-recognition model that might be relaxed to better map onto real-world group dynamics is the friend-making process. In fact, the spatial channels through which the friend-making process unfolds could be entirely separate from the location choice decision and subject to its own set of institutional variables, while preserving the fundamental dependence of classification of locations on personal lists of friends and nonfriends. One might replace the binary friend-making process with a probabilistic spatial structure in which close-by agents are more likely to become friends. Such stochastic variants would extend the geographic range of effects of friend and nonfriend lists beyond immediately surrounding locations, although the large macro effects of small local shifts in lists of friends and nonfriends are already impressive.

A third extension of the FACE-recognition model concerns the question of designing institutions that promote integration and their often unintended consequences. One thinks of school busing programs in post-Civil-Rights America, and the prospects of embedding more specific geographic and institutional structure in the model to analyze the consequences of introducing new institutions aimed at modulating levels of inter-group. One might investigate the degree to which institutions introduced in the real world, after being introduced in the model, could produce simulated differences in integration that match observed differences, say, among regions in the US or within cities in the American South (e.g., the very different urban geographies of Dallas and Atlanta compared with that of Memphis and Jacksonville). Deeper differences in spatial mixing can be observed in countries like Israel, where cosmopolitan cities such as Haifa and Hadar have modest amounts of Arab-Jewish mixing in contrast to nearly all-Jewish cities, such as Lod and Ramle, and all-Arab cities, such as Nazareth and Shfa Amer.

Finally, FACE-recognition's positive effects on integration are likely also to be observable in other macro-systems, such as markets. The economic relevance of face-to-face encounters in cultivating near-instantaneous sympathy and its connections to the functioning of markets was already discussed by Adam Smith (1759/2008). Smith can be interpreted as hypothesizing that markets may fail to function well as they become globalized or administered in a way such that transactions become detached from ongoing face-to-face relationships (Harpham, 2004; Berg & Maital, 2007). Interestingly, online auction platforms such as eBay seem to function well only because they institutionalized a procedure to build reputation of agents, allowing participants to share personal categorizations of their trade partners as trustworthy versus non-trustworthy (Bolton, Katok, & Ockenfels, 2004).

6 Interpretation and implications for institutional design

Social scientists from numerous disciplines too often treat the Schelling model as an argument that unintended divisions between groups occur almost inevitably (i.e., without animosity between groups as a pre-condition). Ellen (1998), for example, writes against this pessimistic interpretation of inevitability, describing her finding that well integrated communities exist and thrive as "running counter to the popular, and often self-fulfilling, view that integration is unviable." For examples of the Schelling model's role in the rhetoric of authors espousing this view of inevitability (which, it is important to note, does not include Schelling himself), see, for example, the empirical studies and policy advice contained in those studies cited in Aydinonat (2007, p. 441).

In light of the inspiring mixes of institutions found in places with high levels of integration and our model that demonstrates a simple mechanism (of course, not the only one) by which this can occur, we believe that this is premature. As a proof-of-concept demonstration of what is possible, we can look in environments with desirable levels of integration to discover social institutions that play a beneficial role in supporting interactions among heterogeneous types that might be applied elsewhere. Based on the FACE-recognition model, we think

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that institutional interventions that promote a moderate amount of random inter-group mixing can, by the very simple mechanism of recognition, help facilitate surprisingly large and durable levels of integration. In fact, the simplicity of the recognition mechanism and what turns out to be its surprising power to cement durably integrated communities starting from chance face-to-face meetings gives us optimism that even modest institutional and policy changes can provide surprisingly salutary effects in terms of reducing ethnic segregation.

Private enterprise invests a tremendous amount on business travel and face-to-face meetings in situations where standard contract theory would surely suggest that firms should save money by sending the contracts straight-away to in-house lawyers rather than cementing relationships with face-to-face contact between the members of two firms, which provides little if any actionable legal protection in the event that the other party fails to perform on a contractual obligation. While this intense investment in face-to-face contact finds very little explanation in the social science literature, the fact that it is common-practice in business seems to reflect the belief and the experience of firms that such investment pays — comparable to the observation that extending the Schelling model with FACE-recognition makes agents more happy and the neighborhoods they create more integrated.

We observe the mismatch between the original Schelling model's prediction of absolute segregation and the reality of many different levels of integration and segregation as an important motivation for new models of segregation that better accord with heterogeneous segregation empirics. This raises the question of how the model could be changed to account for social outcomes observed in the world. The cognitive architecture that uses recognition to accomplish so many important inferential tasks also solves important tasks of social coordination across group-type. That is our claim, and we think the FACE-recognition model clearly demonstrates a mechanism by which this can occur.

There is a point of view that concerns the more general methodological issue of models and their relationship with the complexity of the real world. One might say that Schelling's concern was so much to model reality as to illustrate the possibility of surprising divergence between (micro-) intentions and (macro-) outcomes. Even as a pure thought experiment, we believe that the FACE-recognition model says something interesting and relevant to multiple literatures in which the Schelling model continues to loom large. Whereas most modifications of the classic Schelling model proposed in this literature reproduce the apparent robustness of Schelling's conclusions (i.e., from virtually all starting conditions and with almost all parameter settings, dramatically large levels of segregation result), the introduction of FACE-recognition shows that chance inter-group mixing can be durable and resilient in the face of exogenous shocks. If individual agents are endowed with just enough memory to encode face recognition for one period, then the divergence between intentions and outcomes is dramatically attenuated. This result intrigues us especially in terms of institutional design. It suggests that institutions not explicitly designed to facilitate cross-group face-to-face contacts can nevertheless have large effects on observed levels of integrations. For example, the soccer stadium hosting the ethnically integrated team of Marseille, France, is regarded as a model of successful inter-ethnic and religious comity. Similarly, educational institutions that provide a space for cross-ethnic face-to-face contacts appear to influence the urban geography of US college towns with dramatically higher-than-average levels of integration over multiple decades. Such institutions coordinate patterns of movement through physical space in a way that provides a modest increase in one's chances of randomly bumping into other-type agents, coded in memory as happy, safe, or satisfying experiences. The mechanism that we have identified provides theoretical justification for attributing to these institutions some portion of the salutary effects of ordinary human contact that helps bring us together rather than drive us apart.

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