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BOOK REVIEWS

BELTRAMETTI, M. C. and SOMMESE, A. J. *The adjunction theory of complex projective varieties* (de Gruyter Expositions in Mathematics Vol. 16, de Gruyter, Berlin, New York 1995) xxi+398 pp., 3 11 014355 0, about £100.

Let $L \to X$ be a line-bundle over an n-dimensional complex variety and denote the space of sections of L by $H^0(L)$. Then L is said to be *spanned* if for all $x \in X$ there exists an $s \in H^0(L)$ with $s(x) \neq 0$. If L is spanned then there is a natural map

$$\phi_L: X \to \mathbb{P}^N$$
$$x \mapsto \{s \in H^0(L): s(x) = 0\},$$

where $N+1$ is the dimension of $H^0(L)$ and $\mathbb{P}^N$ is thought of as the space of codimension-1 subspaces of $H^0(L)$.

If the map ϕ_L is an embedding then L is said to be *very ample*. Conversely, if X is a projective variety then the restriction of the hyperplane section bundle $\mathcal{O}_{\mathbb{P}^N}(1)$ to X is very ample. Thus the existence of a very ample line bundle is characteristic of projective varieties. If tL is very ample for some $t \in \mathbb{N}$ then L is said to be *ample*.

Adjunction theory concerns the interplay between the intrinsic geometry of a projective variety and the extrinsic properties, defined via such an embedding ϕ_L.

On a smooth surface S, with $L \to S$ ample and spanned, one considers the *adjoint bundle* $K_S + L$, where K_S is the canonical bundle of S. The main result of classical adjunction theory is that *if L is ample and spanned then $K_S + L$ is spanned* (with certain simple exceptions). The associated mapping ϕ_{K_S+L} is called the *adjunction mapping*. Let $C \subset S$ be a smooth curve in the complete linear system $|L|$. Since the restriction of $K_S + L$ to C is K_C, the canonical bundle of C, the adjunction mapping restricts to the associated *canonical mapping of* C. Thus the adjunction mapping is closely tied to the geometry of $|L|$.

The existence of the adjunction mapping has strong implications for the classification of surfaces. With certain well-understood exceptions, ϕ_{K_S+L} has 2-dimensional image and Remmert–Stein factorisation $\phi_{K_S+L} = \phi_{K_{S'}+L'} \circ \psi$ where

$$\psi: S \twoheadrightarrow S' \quad \text{and} \quad \phi_{K_{S'}+L'}: S' \hookrightarrow \mathbb{P}^N$$

($N+1$ is the dimension of $H^0(K_S+L)$). Here $\psi: S \to S'$ expresses S as the blow-up of a smooth surface S' at a finite set of points, $L' = (\psi_* L)^{**}$ and $K_{S'} + L'$ is very ample. The pair (S', L') is the *relative minimal model* of the pair (S, L).

The book under review tackles the subject in great depth. Singular varieties, dimensions greater than two and various generalised adjunction processes are all treated. The authors provide much historical discussion, while bringing the reader up-to-date with the latest research developments and paving the way for future investigations. At many points results from the literature have been generalised and their proofs improved. There is a comprehensive (661 item) bibliography but the index is somewhat cursory.

Great efforts have been made to make the book self-contained—there is plenty of background material in chapters 1–4—but it is not an easy read: non-specialists may struggle with some of the technical requirements and the ordering of topics is occasionally confusing. The material on 'classical adjunction theory' (chapters 8–11) is probably the most accessible.

This is an authoritative book on a specialised subject: experts will relish it.

E. B. NASATYR