# PARTICLE PHASE TRANSITIONS CAN PREVENT AN INITIAL COSMOLOGICAL SINGULARITY

S.A. Bludman
Department of Physics, University of Pennsylvania
Philadelphia, Pennsylvania 19104

The present universe has small vacuum energy density (cosmological constant) and small spatial curvature. Therefore, before symmetry breaking, it had to have huge positive vacuum energy density, significant curvature, and low entropy. Because the energy positivity condition was not then satisfied, there need not have been an initial singularity (Hot Big Bang). If the universe is spatially open or the initial entropy high enough, then before the elementary particle phase transition, it was indeed an inflecting universe with an initial singularity. But if the universe is spatially closed and of low initial entropy, it needed to be a curvature-dominated bouncing universe, with a minimum size and maximum temperature (Tepid Little Bang). The closure of the present universe will therefore determine whether or not there was an initial singularity.

Motivated by the successful unification of electromagnetic and weak interactions, theoretical physicists have proposed various grand unification theories (GUTs) of electroweak and strong interactions at energies above $10^{14}$ Gev. In these theories generally, the Weinberg parameter is naturally fixed, neutrinos have a small mass, and domain walls and magnetic monopoles are created at elementary particle symmetry breaking.

In the present universe, elementary particle symmetry is hidden or seemingly "broken" by the presence of an order parameter $\sigma \sim 10^{14}$ Gev. In the early hot universe ($T > T_c$), this order vanished ($\sigma = 0$) so that the presently hidden symmetry was unbroken. Although some of the above implications of GUTs may be observed in relatively low-energy terrestrial laboratories, the most dramatic consequences appear in the hot and dense very early universe. This happens because as the universe expands and cools below $T_c$ the vacuum (elementary particle ground state) undergoes a transition from unbroken to broken symmetry with the release of a huge latent heat $\varepsilon_V(0) - \varepsilon_V(\sigma) \approx 1/8\ B\sigma^4$. The ordered phase has lower energy $\varepsilon_V(\sigma)$ than the disordered phase, $\varepsilon_V(0)$. This is why the ordered phase is the stable phase at $T < T_c$.

In the present universe, the energy density of empty space (cosmological constant $\Lambda = 8\pi G/3\ \varepsilon_V$) is observed to be very small or zero. Precisely because of this empirical fact, the early universe must



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have had a huge vacuum mass density $\rho_V \equiv \varepsilon_V(0)/c^2$. As a consequence of accepted particle and gravitational physics, there had to be a huge cosmological constant in the very early universe, precisely where its gravitational effects are most important.

THE PRESENT BROKEN-SYMMETRY UNIVERSE

The present universe has vanishing energy density $\varepsilon_{vac}(\sigma) = 0$ and is described, in the large, by the Friedman equation:

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{1}$$

where the constant space curvature is described by k = +1, 0, -1 (closed, critical density, open universe). The radiation-dominated Friedman equation depends on only one parameter:

$$a_R^2 \equiv \frac{8\pi G}{3}\varepsilon_R R^4 = 2\pi(3/4\ a_2')^{1/3} S^{4/3} m_{PL}^{-2} \tag{2}$$

where $\varepsilon_R = a_s T^4$ is the radiation energy density, $S = 4/3\ a_s T^3R^3$ $4/3\ a_s^{1/4}(\varepsilon_R R^4)^{3/4}$ is the total entropy, and $m_{PL}^{-1} \equiv G^{1/2} = 1.6 \times 10^{-33}$ is the Planck length. (We use units $h = c = k_B = 1$ so that the radiation constant $= \pi^2/30$ and, allowing for three flavors of two-component neutrinos, $a_s = [\pi^2/30]\ g_s = 0.64$). The total entropy is, in the absence of dissipations, the only strictly conserved quantity.

Ever since $R > R_x \equiv (\varepsilon_R R/\varepsilon_M)_o \sim 10^{-3} R_o$, the universe has been matter-dominated so that defining:

$$a_M \equiv \frac{8\pi G}{3}\varepsilon_M R^3 \approx 10^{-29} a_R^{3/2} m_{PL}^{1/2}\ , \tag{3}$$

$$\dot{R}^2 = \left(\frac{a_R}{R}\right)^2 + \frac{a_M}{R} - k\ . \tag{4}$$

Because the present scale $R_o > 10^{28}$ cm and temperature $T_o \sim 3K$, $RT > 10^{28}$, $S > 10^{87}$ and $a_R > 10^{56}\ m_{PL}^{-1}$, $a_M > 10^{57}\ m_{PL}^{-1}$. Defining $\Omega \equiv \rho/\rho_{CR} \equiv \rho(3H^2/8\pi G)$, Eq. (4) can be rewritten:

$$1 - \Omega^{-1} = \frac{3k}{8\pi G\rho R^2} = k\left[\left(\frac{a_R}{R}\right)^2 + \left(\frac{a_M}{R}\right)\right]^{-1}\ . \tag{5}$$

At present, $\Omega \sim 1$ and the closure of the universe is practically impossible to determine because the curvature term in Eq. (4) is and always has been so small relative to the radiation and matter energy density terms. This flatness or huge total entropy suggests huge dissipation in the early universe. Although this huge dissipation might be gravitational or hydrodynamic in origin, in the rapidly expanding early universe the GUTs' symmetry breaking leads naturally to extreme supercooling and entropy generation before the phase transition is completed. This happens because after supercooling the temperature rebounds almost up to $T_c$, the universe has meanwhile been expanding exponentially by a factor $> e^{100}$, so that the entropy increases by a factor $> 10^{87}$. Because the universe within the present horizon has expanded from a very small (presumably homogeneous) patch, the presently observable universe is homogeneous and practically devoid of magnetic monopoles or domain walls. Any huge dissipation would explain the flatness of the present universe. The exponential growth, brought about by the drawn-out first-order phase transition in the expanding universe, explains homogeneity of the present universe and the absence of magnetic monopoles.

The inflationary scenario predicts that $\Omega = 1$ to a very high accuracy ($10^{-9}$) and that galaxies are formed out of a scale-invariant spectrum of initial fluctuations. Both predictions are very hard to test. Dynamical observations show that, for mass clustered on the scale of superclusters, $\Omega \sim 0.2$, but neutrinos, photinos, gravitinos or other collisionless particles that do not cluster on this scale could make $\Omega$ larger. It is therefore important to push up the lower bound on $\Omega$, since if $\Omega$ exceeds unity significantly, the inflationary scenario would be disproven.

## THE UNIVERSE BEFORE THE SYMMETRY-BREAKING PHASE TRANSITION

Before the symmetry-breaking phase transition, the universe was dominated by the hugh vacuum energy density $\varepsilon_V(0)$ so that, in place of Eq. (4):

$$\dot{R}^2 = \frac{8\pi G}{3}(\rho_R + \rho_v)R^2 - k = \left(\frac{a'_R}{R}\right)^2 + \left(\frac{R}{b}\right)^2 - k \tag{6}$$

$$\ddot{R} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p)R = -\frac{a_R'^2}{R} + \frac{R}{b^2}, \tag{7}$$

$$a_R'^2 \equiv \frac{8\pi G}{3}\varepsilon_R R^4 = 2\pi(3/4\ a'_s)^{1/3} \cdot S'^{4/3}\ m_{PL}^{-2} \tag{8}$$

$$b^{-2} \equiv \frac{8\pi G}{3}\varepsilon_v = (7.0 \times 10^9 \text{ Gev})^2 = (3.5 \times 10^{23} \text{ cm}^{-1})^2$$

The primes refer to values before symmetry breaking so that we expect $S' \sim 1 \sim a'_R m_{PL}$. The vacuum energy density now defines a second length scale $b = 3 \times 10^{-24}$ cm. The ratio of these two scales in the symmetry-unbroken universe, $\alpha \equiv (2\ a'_R/b)^2$, is a measure of the radiation energy or total entropy before the GUTs' phase transition.

## AVOIDING THE INITIAL COSMOLOGICAL SINGULARITY

From Eq. (7) there is an inflection in R(t) at $R = (a'_R b)^{1/2}$, when $\varepsilon_R = \varepsilon_V$. From Eq. (6), $\dot{R}$ can vanish only if both $k = +1$ (closed universe) and $\alpha \leq 1$ (low entropy initial state). Thus, there are two possibilities for homogeneous isotropic radiation-dominated universes with positive vacuum energy:

1) If $k = -1$ (open universe) and/or $\alpha > 1$ (relatively high entropy initial state), there can be no extremum in R. We have an inflectional universe with a singular origin (Hot Big Bang) at which the radiation density $\varepsilon_r >> \varepsilon_v$. This is followed by a short coasting phase, followed by vacuum-dominated exponential growth during which the universe cools down, finally undergoing the phase transition to the present broken-symmetry universe. This inflectional model, in which the curvature term was insignificant in the early universe, is the one advocates of the inflationary scenario usually have in mind.

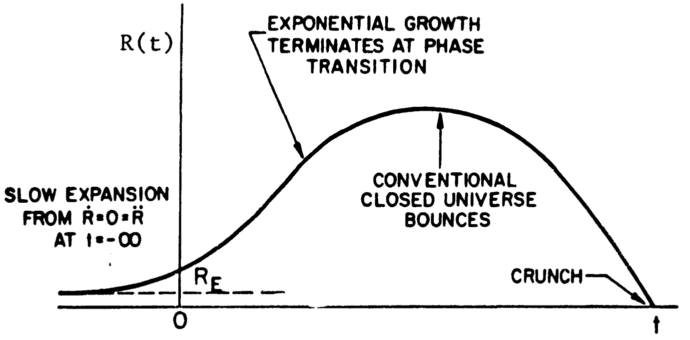
2) If $k = +1$ (closed universe) and $\alpha \equiv \sin^2\phi \lesssim 1$ (low entropy initial state), then we have a bouncing universe with an extremum in R(t)

$$R^2_{max,min} = \frac{b^2}{2} (1 \mp \cos \phi), \; R_{max} R_{min} = \frac{b^2}{2} \alpha$$

where the curvature and energy terms cancel. Such a low entropy is reasonable before the GUTs' phase transition. If there were a singular origin, R(t) would stay less than $R_{max} = b \sin \phi/2$ and the universe would have remained very small and very hot, never becoming our present universe; we must reject this possibility. There is, however, another reversing solution, $R(t) \gtrsim R_{min} = b \cos \phi/2$ in which $T < T_{max} \sim m_{PL} \sin \phi/2$. There is no initial singularity because the radiation energy never dominated the vacuum energy. Summarizing: If the universe is open and began radiation-dominated, there had to be an initial singularity. If it is closed and began not too hot, there was no initial singularity. We call this latter solution the Tepid Little Bang in contrast to the Hot Big Bang. If the inflationary scenario is correct, $\Omega = 1 \pm (10^{-9})$, so that it will be practically impossible to distinguish observationally between an open and closed universe. For those who prefer to maintain Einstein's gravitational theory, however, the Tepid Little Bang enjoys the theoretical advantage of avoiding the initial singularity which would otherwise signal the incompleteness of Einstein's theory.

Especially interesting is the critical case $\alpha = 1$ for which $R_{max} = R_{min} = b/\sqrt{2} \equiv R_E$. Then, at $R = R_E = (3/2\pi B)^{1/2} m_{PL}/\sigma{-}2$, the radiation and vacuum energies balance precisely so that $\dot{R} = R = 0$. (This fine-tuning may be a consequence of some supergravity theory or new cosmological principle.) This is the original Einstein static universe that is known to be gravitationally unstable. If, at $t = -\infty$, it begins to expand from $R = R_E$, it slowly goes into exponential expansion until the phase transition takes place and we enter into the conventional closed universe, with the usual baryosynthesis, nucleosynthesis and galaxy synthesis scenarios. If the universe is closed and began with the critical low entropy, this Tepid Little Bang avoids the initial singularity which would otherwise signal the breakdown of conventional gravitational theory.

Fig. 1. The Tepid Universe, which is gravitationally unstable, expands exponentially from finite size at t = -∞ and undergoes phase transition to the present universe.




Discussion

*Shklovskii:* The Anthropic principle (Dicke et al.) is very important.

*Peacock:* Do you think there is any prospect that GUTs will eventually explain why the cosmological constant is presently zero (or very small)? If a value $\Lambda = 0$ has to be assumed ad hoc, the theory of the phase transition is seriously incomplete.

*Bludman:* Hopefully, some future theory will explain the fact that, at present, $\Lambda = 0$. Accepting this empirical fact, symmetry-unbreaking requires an original cosmological constant of the sign and magnitude needed to avoid an initial singularity.

*Lukash:* The model you suggested begins with the Einstein unstable model, which, for this reason, cannot be continued to $t = -\infty$. Is that not the case?

*Bludman:* Because the Einstein universe is unstable, if it begins to grow, it enters exponential growth slowly. In contrast with the Hot Big Bang, the time before the phase transition is infinitely long.