Therefore, we get:

\[
\begin{align*}
&\left( A^A \right) \left( A^A \right) \ldots \text{to } S_1 \text{ factors} \\
&\times \left( B^B \right) \left( B^B \right) \ldots \text{to } S_2 \text{ factors} \\
&\times \ldots \ldots \ldots \ldots \ldots \ldots \\
&\times \left( N^N \right) \left( N^N \right) \ldots \text{to } S_n \text{ factors}
\end{align*}
\]

\[
\left( \begin{array}{c}
A + A + \ldots + \text{to } S_1 \text{ terms} \\
+ \left( B + B + \ldots + \text{to } S_2 \text{ terms} \right) \\
+ \ldots \ldots \ldots \ldots \ldots \\
+ \left( N + N + \ldots + \text{to } S_n \text{ terms} \right)
\end{array} \right)
\]

\[
\left( A + A + \ldots + \text{to } S_1 \text{ terms} \right)
\]

\[
\left( + \left( B + B + \ldots + \text{to } S_2 \text{ terms} \right) \right)
\]

\[
\ldots
\]

\[
\left( + \left( N + N + \ldots + \text{to } S_n \text{ terms} \right) \right)
\]

\[
\left( \frac{S_1 A + S_2 B + \ldots + S_n N}{S_1 + S_2 + \ldots + S_n} \right)^{S_1 A + S_2 B + \ldots + S_n N}
\]

I am, Sir,

Yours faithfully,

STEUART E. MACNAGHTEN.

18, Lincoln's Inn Fields, W.C.

4 December 1905.

ON THE USE OF O\textsuperscript{OM} SELECT PREMIUMS FOR VALUATION PURPOSES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Mr. King remarks, in his recent paper on the Valuation in groups of Whole-life Policies by Select Mortality Tables (§ 4), that the reserves by the O\textsuperscript{OM} Select Tables are greater than by any table or combination of tables hitherto used. It is interesting to notice that the great stringency of the O\textsuperscript{OM} Tables is due mainly to the net premiums employed. This is made clear if the reserves by the Select and Aggregate Tables for Model Office, No. 1, at the end of 50 years, are analyzed by the formula which is given by Mr. King.

<table>
<thead>
<tr>
<th>Basis of Valuation</th>
<th>Value of Sums Assured</th>
<th>Value of Net Premiums</th>
<th>Actual Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>O\textsuperscript{OM}, 3 %</td>
<td>1,450,683</td>
<td>750,263</td>
<td>700,420</td>
</tr>
<tr>
<td>O\textsuperscript{M}, 3 %</td>
<td>1,449,006</td>
<td>760,007</td>
<td>688,999</td>
</tr>
<tr>
<td>Difference</td>
<td>1,677</td>
<td>9,744</td>
<td>11,421</td>
</tr>
</tbody>
</table>

Of the whole difference in the reserves, 85 per-cent is due to the difference in the value of the net premiums.
The result of substituting $O^{M}$ net premiums for net premiums by the $O^{M}$ Table in the $O^{M}$ 3 per-cent valuation can be very easily obtained by the aid of a formula given by Mr. King (J.I.A., xxxvii, p. 465). The result by the combined basis, which may be described as $O^{M}$ and $O^{M}$, 3 per-cent, is as follows:

<table>
<thead>
<tr>
<th>Basis of Valuation</th>
<th>Value of Sums Assured</th>
<th>Value of Net Premiums</th>
<th>Actual Reserves</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^{M}$ and $O^{M}$, 3 %</td>
<td>1,449,006</td>
<td>750,324</td>
<td>698,682</td>
</tr>
</tbody>
</table>

Here the value of the net premiums is practically the same as in the Select valuation. The difference in the value of the sums assured remains, and on the whole the result is a fairly close approximation to the Select valuation.

$O^{M}$ net premiums can be employed in other combinations, and various results for Model Office, No. 1, at the end of 50 years, are given in the following table in comparison with valuations involving $O^{M}$ net premiums. The results of using ultimate factors after five years, and throughout, have been obtained by the aid of Mr. Diver's table of the values of $\phi_{e^x+t}$ (Table I in his paper). The valuations are all on the basis of interest at 3 per-cent.

<table>
<thead>
<tr>
<th>Basis for Net Premiums</th>
<th>Basis for Valuation Factors</th>
<th>Actual Reserve</th>
<th>Comparative Reserve $O^{M}$, 3 % =10,000</th>
<th>Comparative Reserve $O^{M}$, 3 % =10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ . . . . . . . . . . . . . . .</td>
<td>688,989</td>
<td>10,000</td>
<td>9,837</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ first five years</td>
<td>693,884</td>
<td>10,071</td>
<td>9,907</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ after five years</td>
<td>698,214</td>
<td>10,134</td>
<td>9,969</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ throughout</td>
<td>698,682</td>
<td>10,141</td>
<td>9,975</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ . . . . . . . . . . . . . . .</td>
<td>700,420</td>
<td>10,166</td>
<td>10,000</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ first five years</td>
<td>700,815</td>
<td>10,171</td>
<td>10,006</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>Ultimate factors after five years</td>
<td>703,580</td>
<td>10,212</td>
<td>10,045</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ first five years</td>
<td>705,147</td>
<td>10,234</td>
<td>10,067</td>
</tr>
<tr>
<td>$O^{M}$</td>
<td>$O^{M}$ after five years</td>
<td>707,911</td>
<td>10,274</td>
<td>10,107</td>
</tr>
</tbody>
</table>

It may be remarked in passing that, if the valuation be made by Select tables, the difference made by employing ultimate factors instead of Select after five years is less than the error that may be introduced by using nearest ages at entry and at valuation in place of exact ages, and is in fact for practical purposes inappreciable.

The combination of $O^{M}$ net premiums with $O^{M(0)}$ valuation factors
Correspondence. [Jan. 1906.

throughout may appear to be excessively stringent; but it differs from the O\textsuperscript{O}\textsuperscript{M} Select valuation less than the latter from the H\textsuperscript{M}. and in the case of an office whose mortality has the same relation to the O\textsuperscript{O}\textsuperscript{M} Table that the latter has to the H\textsuperscript{M} Table the combination O\textsuperscript{O}\textsuperscript{M} and O\textsuperscript{O}\textsuperscript{M}(5) throughout would therefore not be so severe as a Select net premium valuation based upon its own experience.

The conclusion may be suggested that in the case of valuations which aim at the highest standard, net premiums based upon Aggregate tables should be abandoned in favour of Select net premiums, whether the valuations are based on Select tables in other respects or not.

I am, Sir,
Yours faithfully,
DUNCAN C. FRASER.

1, North John Street,
Liverpool.
5 December 1905.