Can. J. Math., Vol. XXXV, No. 6, 1983 pp. 1001-1009

SPACES WITH A UNIQUE UNIFORMITY

RICHARD H. WARREN

1. Introduction. The major results in this paper are nine characterizations of completely regular spaces with a unique compatible uniformity. All prior results of this type assumed that the space is Tychonoff (i.e., completely regular and Hausdorff) until the appearance of a companion paper [**9**] which began this study. The more important characterizations use quasi-uniqueness of R_1-compactifications which relate to uniqueness of T_2-compactifications. The features of the other characterizations are: (i) compact subsets linked to Cauchy filters, (ii) C- and C^*-embeddings, and (iii) lifting continuous maps to uniformly continuous maps.

Section 2 contains information on T_0-identification spaces which we will use later in the paper. In Section 3 several properties of uniform identification spaces are developed so that they can be used later. The nine characterizations are established in Section 4. Also it is shown that a space with a unique compatible uniformity is normal if and only if each of its closed subspaces has a unique compatible uniformity.

There are several reasons for this study of completely regular spaces which may not be Hausdorff. First, it completes the work begun in [**9**]. Two new tools have been developed and used: uniform identification spaces and R_1-compactifications. It is shown that, except for the Stone-Čech compactification, all the characterizations in [**2**, **4**, **5**, **8**] are valid without a Hausdorff assumption. This study complements [**3**, **6**, **7**] which are in the framework of completely regular spaces which are not necessarily Hausdorff.

Recall that a topological space is called R_1 if and only if whenever $\{\bar{x}\} \neq \{\bar{y}\}$ there are disjoint open sets U and V such that $\{\bar{x}\} \subset U$ and $\{\bar{y}\} \subset V$ [**1**]. An R_1-compactification is a compact R_1-space in which the original space is densely embedded.

2. T_0-identification spaces. Background and extensive information about these spaces is contained in [**9**]. We are interested in some additional properties which are developed here for use in Section 4.

Received January 8, 1982 and in revised form December 1, 1982.

Remember that points x, y in a topological space $(X, \mathcal{T})$ are identified if and only if $\{\bar{x}\} = \{\bar{y}\}$ and the resulting quotient space $(Y, \mathcal{V})$ is a T_0-space, called the *T_0-identification space* of $(X, \mathcal{T})$. For $x \in X$, let D_x be the member of Y containing x. Then $f:X \to Y$ by $f(x) = D_x$ is a continuous, open, closed map onto Y and is called the *T_0-identification map*.

Throughout this section $(Y, \mathcal{V})$ will be the T_0-identification space of $(X, \mathcal{T})$, and D_x and f will be as designated in the two preceding sentences.

A topological space $(X, \mathcal{T})$ is said to be *C^*-embedded* in the space $(Z, \mathcal{U})$ if every bounded, real-valued, continuous function on X has a continuous extension to Z, possibly through a homeomorphism of $(X, \mathcal{T})$ onto a subspace of $(Z, \mathcal{U})$. Such a homeomorphism is called an *embedding* of $(X, \mathcal{T})$ into $(Z, \mathcal{U})$. Also, $(X, \mathcal{T})$ is said to be *densely embedded* in $(Z, \mathcal{U})$ if there is an embedding h of $(X, \mathcal{T})$ into $(Z, \mathcal{U})$ such that $h(X)$ is dense in Z. By $C^*(X)$ we denote the set of bounded, real-valued, continuous functions on X.

THEOREM 2.1. *Let $(X, \mathcal{T})$ be a subspace of $(S, \mathcal{S})$ whose T_0-identification space is $(T, \mathcal{U})$. If $(Y, \mathcal{V})$ is C^*-embedded in $(T, \mathcal{U})$, then $(X, \mathcal{T})$ is C^*-embedded in $(S, \mathcal{S})$.*

Proof. Let g be in $C^*(X)$. As a result of Lemma 2.1 in **[9]**, we may define a real-valued function h on Y by $h(D_x) = g(x)$ for each $D_x \in Y$. Hence $g = h \circ f$ and h is continuous. By assumption h has an extension k in $C^*(T)$. Let $e:S \to T$ be the quotient map $e(s) = [s]$ where $[s]$ is the equivalence class containing s. Then $k \circ e$ is in $C^*(S)$ and is an extension of g.

THEOREM 2.2. *Let $(Y, \mathcal{V})$ be a dense subspace of the T_0-space $(T, \mathcal{U})$. Then there is a topological space $(S, \mathcal{S})$ such that $(T, \mathcal{U})$ is the T_0-identification of $(S, \mathcal{S})$ and $(X, \mathcal{T})$ is densely embedded in $(S, \mathcal{S})$. Furthermore, if $(X, \mathcal{T})$ is C^*-embedded in $(S, \mathcal{S})$, then $(Y, \mathcal{V})$ is C^*-embedded in $(T, \mathcal{U})$.*

Proof. Let $S = X \cup (T \setminus Y)$, so without loss of generality we may assume $T \cap X = \emptyset$. For each open subset A of T form

$$A' = \cup\{D_x:D_x \in A \cap Y\} \cup A\setminus Y.$$

Then $\{A':A \in \mathcal{U}\}$ is a topology on S. As usual, define $x \approx y$ for $x, y \in S$ if and only if $\{\bar{x}\}^s = \{\bar{y}\}^s$. Note that when x and y are distinct points in S, then $x \approx y$ if and only if $x, y \in X$ and $x \sim y$ in X. Thus $\approx$ determines the members of T, with the identification of $\{t\}$ with t whenever $t \in T \setminus Y$. It is easy to show that the quotient topology on T agrees with $\mathcal{U}$ and that $(X, \mathcal{T})$ is a dense subspace of $(S, \mathcal{S})$.

Let h be in $C^*(Y)$. Then $h \circ f$ is in $C^*(X)$ and has an extension j in $C^*(S)$. As a result of Lemma 2.1 in [**9**], we may define a real-valued function k on T by $k(D_x) = j(x)$ for $D_x \in Y$ and $k(t) = j(t)$ for $t \in T \setminus Y$. Let $e:S \to T$ be the quotient map. Then $j = k \circ e$, k is in $C^*(T)$ and $k|_Y = h$.

THEOREM 2.3. *Let* X^* *be an* R_1*-compactification of* $(X, \mathscr{T})$ *and* Y^* *the* T_0*-identification space of* X^*. *Then* Y^* *is a* T_2*-compactification of* $(Y, \mathscr{V})$.

Proof. Let k be a dense embedding of X into X^* and let g be the T_0-identification map of X^* onto Y^*. By Lemma 2.1 in [**9**], we may define a map $h:Y \to Y^*$ by $h(D_x) = g(k(x))$. Then $h(Y)$ is homeomorphic to Y and $h(Y)$ is dense in Y^*. It is known that a compact R_1-space is completely regular. Thus Y^* is Tychonoff.

THEOREM 2.4. $(X, \mathscr{T})$ *is locally compact if and only if* $(Y, \mathscr{V})$ *is locally compact.*

Proof. Since f is an open, onto mapping, $(Y, \mathscr{V})$ is locally compact when $(X, \mathscr{T})$ is. On the other hand, since for each $G \in \mathscr{T}$, $f(G) \in \mathscr{V}$ and $f^{-1}(f(G)) = G$, it follows that if $(Y, \mathscr{V})$ is locally compact, then $(X, \mathscr{T})$ is.

3. Uniform identification spaces. These spaces originated in [**9**] where they became the major tool for extending uniform results from Tychonoff spaces to completely regular spaces.

Let $(X, \mathscr{H})$ be a uniform space. For $x, y \in X$, define $x \sim y$ if and only if $y \in H(x)$ for each $H \in \mathscr{H}$. Then $\sim$ is an equivalence relation on X. Throughout this section Y is the set of equivalence classes, D_x is the member of Y containing x, $f:X \to Y$ by $f(x) = D_x$ and $\mathscr{K}$ is the quotient uniformity on Y induced by f. Then $(Y, \mathscr{K})$ is called the *uniform identification space* of $(X, \mathscr{H})$. The map f is uniformly continuous and is the T_0-identification map with respect to the induced topologies.

For our purposes in Section 4, the most useful feature about identifying uniform spaces is the following theorem which is proved in [**9**].

THEOREM 3.1. *Let* $(X, \mathscr{T})$ *be a topological space and let* $(Y, \mathscr{V})$ *be its* T_0*-identification space. Let* Θ *be the family of all uniformities on* X *compatible with* $\mathscr{T}$ *and let* Ω *be the family of all uniformities on* Y *compatible with* $\mathscr{V}$. *Then* Θ *and* Ω *are order isomorphic.*

THEOREM 3.2. $(X, \mathscr{H})$ *has property* (*) *if and only if* $(Y, \mathscr{K})$ *does.*

https://doi.org/10.4153/CJM-1983-055-9 Published online by Cambridge University Press

(*) Each entourage for the uniformity contains a set of the form $A \times A$ where the complement of A is compact.

Proof. ($\Rightarrow$) Let $K \in \mathcal{K}$. Then $f^{-1}(K) \in \mathcal{H}$ by Theorem 3.3 in [**9**] and there is $A \times A \subset f^{-1}(K)$ such that $X \setminus A$ is compact. Then $f(X \setminus A)$ is compact and

$$Y \setminus f(X \setminus A) \subset f(X \setminus (X \setminus A)) = f(A).$$

Thus

$$Y \setminus f(X \setminus A) \times Y \setminus f(X \setminus A) \subset f(A) \times f(A) \subset K.$$

($\Leftarrow$) Let $H \in \mathcal{H}$. Since $\mathcal{K}$ is a quotient uniformity, $f(H) \in \mathcal{K}$ and there is $B \times B \subset f(H)$ such that $Y \setminus B$ is compact. Then $f^{-1}(Y \setminus B)$ is compact since for each open subset G of X, $f(G)$ is open and $f^{-1}(f(G)) = G$. Also

$$X \setminus f^{-1}(Y \setminus B) = f^{-1}(Y \setminus (Y \setminus B)) = f^{-1}(B).$$

Therefore

$$X \setminus f^{-1}(Y \setminus B) \times X \setminus f^{-1}(Y \setminus B) \subset H.$$

COROLLARY 3.1. *$(X, \mathcal{H})$ has property $(\substack{*\\*})$ if and only if $(Y, \mathcal{K})$ does.*

$(\substack{*\\*})$ The space is compact or the filter with base consisting of complements of compact subsets is a Cauchy filter.

Proof. A filter $\mathcal{F}$ in a uniform space is a Cauchy filter if and only if for each entourage H there is $F \in \mathcal{F}$ such that $F \times F \subset H$.

THEOREM 3.3. *$(X, \mathcal{H})$ is totally bounded if and only if $(Y, \mathcal{K})$ is totally bounded.*

Proof. Since f is uniformly continuous and onto, $(Y, \mathcal{K})$ is totally bounded when $(X, \mathcal{H})$ is. On the other hand, given $H \in \mathcal{H}$, by Theorem 3.3 in [**9**] there is $K \in \mathcal{K}$ such that $f^{-1}(K) \subset H$. Assuming $(Y, \mathcal{K})$ is totally bounded, there is a finite set $\{D_{x_1}, \dots, D_{x_n}\}$ such that

$$\cup \{K(D_{x_k}) : k = 1, \dots, n\} = Y.$$

So $Y = \{D_y : (D_{x_k}, D_y) \in K \text{ for some } k\}$. Hence

$$X = \cup \{D_y : (x_k, y) \in f^{-1}(K) \text{ for some } k\}$$
$$= \cup \{H(x_k) : k = 1, \dots, n\}.$$

4. Unique compatible uniformity. The first characterization of Tychonoff spaces with exactly one compatible uniformity was published by Doss

[**4**] and was extended to completely regular spaces in [**9**]. Additional characterizations of these Tychonoff spaces were implied by Dickinson [**2**] and given by Newns [**8**]. Dickinson's work is extended to completely regular spaces in Theorem 4.1 (b) and Newns' work in Theorem 4.1 (c), (d) and (d′).

THEOREM 4.1. *Let* $(X, \mathscr{T})$ *be a completely regular topological space. Then the following are equivalent.*

(a) *There is a unique uniformity on* X *compatible with* $\mathscr{T}$.

(b) $(X, \mathscr{T})$ *is compact or locally compact with a unique* (*up to* T_0*-identification*) R_1*-compactification.*

(c) $(X, \mathscr{T})$ *is compact or locally compact with the* 1*-point compactification of* $(X, \mathscr{T})$ *densely embedded in each* R_1*-compactification of* $(X, \mathscr{T})$.

(d) *Each entourage for each compatible uniformity on* X *contains a set of the form* $A \times A$ *where* $X \setminus A$ *is compact.*

(d′) $(X, \mathscr{T})$ *is compact or the filter with base consisting of complements of compact subsets of* X *is a Cauchy filter for every compatible uniformity on* X.

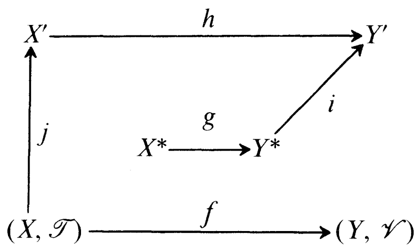
Proof. Let $(Y, \mathscr{V})$ be the T_0-identification space of $(X, \mathscr{T})$.

(a) ⇒ (b). Assume $(X, \mathscr{T})$ is not compact. Then $(Y, \mathscr{V})$ is not compact and has a unique compatible uniformity by Theorem 3.1. From [**2**] $(Y, \mathscr{V})$ is locally compact and by Theorem 2.4 $(X, \mathscr{T})$ is locally compact. In [**2**] it is shown that $(Y, \mathscr{V})$ has a unique compactification which is Tychonoff. Let X^* be the 1-point compactification of $(X, \mathscr{T})$ and Y^* its T_0-identification space. By Theorem 2.3 Y^* is a T_2-compactification of Y. If X' is any R_1-compactification of X and Y' its T_0-identification space, then by Theorem 2.3 Y' is a T_2-compactification of $(Y, \mathscr{V})$. Thus Y' is homeomorphic to Y^*.

(b) ⇒ (a). It is well-known that a compact, completely regular space admits exactly one compatible uniformity. If $(X, \mathscr{T})$ is not compact, then its 1-point compactification X^* is an R_1-space and hence completely regular. By Theorem 2.3 the T_0-identification space of X^* is a T_2-compactification of $(Y, \mathscr{V})$. If $(T, \mathscr{U})$ is any T_2-compactification of $(Y, \mathscr{V})$, then the space $(S, \mathscr{S})$ constructed in Theorem 2.2 is an R_1-compactification of $(X, \mathscr{T})$. By assumption $(T, \mathscr{U})$ is homeomorphic to the T_0-identification space of X^*. Hence $(Y, \mathscr{V})$ has a unique compactification which implies by [**8**] that $(Y, \mathscr{V})$ has a unique compatible uniformity. From Theorem 3.1 $(X, \mathscr{T})$ has a unique compatible uniformity.

(b) ⇒ (c). If $(X, \mathscr{T})$ is not compact, then let X^* be its 1-point compactification and Y^* the T_0-identification space of X^*. Let X' be any R_1-compactification of $(X, \mathscr{T})$ and Y' the T_0-identification space of X'.

There is an embedding map $j:X \to X'$ which is one to one, open, continuous and satisfies $\overline{j(X)} = X'$. There is a homeomorphism $i:Y^* \to Y'$. If f, g, h are the respective T_0-identification maps, then we have:



We may assume $X^* = X \cup \{\omega\}$ and $h \circ j = i \circ g$ on X. Since Y' is homeomorphic to the 1-point compactification of $(Y, \mathscr{V})$, we can choose $\alpha' \in Y'$ which is a point adjoined to Y. Then there is $\omega' \in X'$ satisfying $h(\omega') = \alpha'$. Define $k:X^* \to X'$ by $k(x) = j(x)$ for each $x \in X$ and $k(\omega) = \omega'$. It is easy to verify that k is one to one, open, continuous and that $k(X^*)$ is dense in X'.

(c) $\Rightarrow$ (b). Using the same notation as in the diagram for the previous proof, we now assume that $k:X^* \to X'$ is a dense embedding. Denote equivalence classes in X^* by D_x for $x \in X^*$ and in X' by $[x']$ for $x' \in X'$. Since

$$[k(x)] = hkg^{-1}(D_x) \quad \text{for each } x \in X^*,$$

we may define $i:Y^* \to Y'$ by $i(D_x) = [k(x)]$. Then it is easy to verify that i is a homeomorphism.

(a) $\Leftrightarrow$ (d). Coupling Theorems 3.1 and 3.2 with [**8**] shows that (a) is equivalent to $(Y, \mathscr{V})$ having a unique compatible uniformity, which is equivalent to each entourage for each compatible uniformity on Y containing a set of the form $B \times B$ where $Y \setminus B$ is compact, which is equivalent to (d).

(d) $\Leftrightarrow$ (d′). Use Corollary 3.1.

The next theorem takes two pieces of work [**5**, p. 95] from Tychonoff spaces to completely regular spaces. Other characterizations on p. 95 were generalized in [**9**].

THEOREM 4.2. *For a completely regular space* $(X, \mathscr{T})$ *the following are equivalent*:

(a) *There is a unique uniformity on* X *compatible with* $\mathscr{T}$.

(b) $(X, \mathscr{T})$ *is* C^*-*embedded in every completely regular space containing* $(X, \mathscr{T})$ *as a dense subspace.*

https://doi.org/10.4153/CJM-1983-055-9 Published online by Cambridge University Press

(c) *Every continuous image* $(Z, \mathscr{Z})$ *of* $(X, \mathscr{T})$ *is C-embedded in each completely regular space containing* $(Z, \mathscr{Z})$ *as a dense subspace.*

Proof. Let $(Y, \mathscr{V})$ be the T_0-identification space of $(X, \mathscr{T})$.

(a) $\Leftrightarrow$ (b). Let X be a dense subspace of the completely regular space S whose T_0-identification space is T. Denote the equivalence classes of S by $[s]$ where $s \in S$. If $s \in X$, then we may identify the equivalence class D_s of X with $[s]$ of S. Thus Y is densely embedded in T. It follows from Theorem 3.1 that (a) is equivalent to Y having a unique compatible uniformity, which by [**5**, p. 95] implies that Y is C^*-embedded in T, and by Theorem 2.1 X is C^*-embedded in S. On the other hand, (b) implies by Theorem 2.2 that Y is C^*-embedded in every Tychonoff space containing Y as a dense subspace, which by [**5**, p. 95] is equivalent to Y having a unique compatible uniformity.

(a) $\Rightarrow$ (c). If we assume $(Z, \mathscr{Z})$ is a dense subset of a completely regular space, then $(Z, \mathscr{Z})$ is completely regular. Let $(U, \mathscr{U})$ be the T_0-identification space of $(Z, \mathscr{Z})$. Since $(Y, \mathscr{V})$ has a unique compatible uniformity by Theorem 3.1 and since $(U, \mathscr{U})$ is the continuous image of $(Y, \mathscr{V})$, by [**5**, p. 95] $(U, \mathscr{U})$ is C-embedded in every Tychonoff space containing $(U, \mathscr{U})$ as a dense subspace. By Theorems 2.1 and 2.4 in [**9**], $(Z, \mathscr{Z})$ satisfies (c).

(c) $\Rightarrow$ (a). Since the identity map on X combined with (c) implies that Theorem 4.1 (e) in [**9**] holds, it follows that $(X, \mathscr{T})$ satisfies (a).

The next theorem generalizes the characterizations found in [**5**, p. 238].

THEOREM 4.3. *For a completely regular space* $(X, \mathscr{T})$ *the following are equivalent*:

(a) *There is a unique uniformity on X compatible with* $\mathscr{T}$.

(b) *Every continuous map from* $(X, \mathscr{T})$ *into a completely regular space is uniformly continuous for each compatible uniformity on X.*

(c) *Every function in* $C(X)$ *is uniformly continuous for each compatible uniformity on X.*

(d) *Every function in* $C^*(X)$ *is uniformly continuous for each compatible uniformity on X.*

(e) *Every function in* $C^*(X)$ *is uniformly continuous for each compatible, totally bounded uniformity on X.*

Proof. Let $(Y, \mathscr{V})$ be the T_0-identification space of $(X, \mathscr{T})$ and $f:X \to Y$ the T_0-identification map.

(a) $\Rightarrow$ (b). Let $h:(X, \mathscr{T}) \to (Z, \mathscr{Z})$ be continuous where $(Z, \mathscr{Z})$ is a completely regular space. Let $(U, \mathscr{U})$ be the T_0-identification space of $(Z,$

$\mathscr{Z}$) and $g:Z \to U$ the T_0-identification map. By assumption there is a unique uniformity $\mathscr{H}$ on X which induces $\mathscr{T}$ and by Theorem 3.1 a unique uniformity $\mathscr{K}$ on Y which induces $\mathscr{V}$. Let $\mathscr{L}$ be a uniformity on Z which induces $\mathscr{Z}$ and let $(U, \mathscr{M})$ be the uniform identification space of $(Z, \mathscr{L})$. Then by Corollary 3.2 in **[9]** the map f is uniformly continuous with respect to $\mathscr{H}$ and $\mathscr{K}$. By Theorem 3.3 in **[9]** for each $L \in \mathscr{L}$, $g(L) \in \mathscr{M}$. Define $k:Y \to U$ by $k(D_x) = gh(x)$. Then k is continuous and by **[5**, p. 238**]** k is uniformly continuous from $(Y, \mathscr{K})$ to $(U, \mathscr{M})$. Therefore h is uniformly continuous.

(b) $\Rightarrow$ (c). The real line is a completely regular space.

(c) $\Rightarrow$ (d). $C^*(X) \subset C(X)$.

(d) $\Rightarrow$ (e). Each totally bounded uniformity is a uniformity.

(e) $\Rightarrow$ (a). Let $g \in C^*(Y)$. Let $\mathscr{K}$ be a compatible, totally bounded, separated uniformity on Y. By Theorem 3.4 in **[9]** there is a uniformity $\mathscr{H}$ on X such that $(Y, \mathscr{K})$ is the uniform identification space of $(X, \mathscr{H})$. By Theorem 3.3 $(X, \mathscr{H})$ is totally bounded. Define h on X by $h(x) = gf(x)$. Thus $h \in C^*(X)$ and by assumption h is uniformly continuous. It follows that g is uniformly continuous and by **[5**, p. 238**]** $(Y, \mathscr{V})$ has a unique compatible uniformity. From Theorem 3.1 $(X, \mathscr{T})$ has a unique compatible uniformity.

Next we examine those completely regular spaces whose subspaces inherit a unique uniformity.

THEOREM 4.4. *A closed subspace of a normal space with a unique compatible uniformity has a unique compatible uniformity.*

Proof. Let S be a closed subspace of the normal space X and $f \in C(S)$. Then

$$A = \{x \in S:f(x) \geqq 1\} \quad \text{and} \quad B = \{x \in S:f(x) \leqq 0\}$$

are disjoint closed subsets of S. By Tietze's extension theorem there is an extension F of f to all of X. Since X has a unique uniformity, by Theorem 4.1 in **[9]**, $\{x \in X:F(x) \geqq 1\}$ or $\{x \in X:F(x) \leqq 0\}$ is compact. Hence A or B is compact, and by Theorem 4.1 in **[9]** S has a unique uniformity.

THEOREM 4.5. *If every closed subspace of a space $(X, \mathscr{T})$ has a unique compatible uniformity, then $(X, \mathscr{T})$ is normal.*

Proof. Let f be the T_0-identification map of $(X, \mathscr{T})$ onto $(Y, \mathscr{V})$ and let A be a closed subset of Y. Then it is assumed that $f^{-1}(A)$ has a unique uniformity. Also $f|_{f^{-1}(A)}$ is the T_0-identification map of $f^{-1}(A)$ onto A. By Theorem 3.1 A has a unique uniformity and so $(Y, \mathscr{V})$ is normal by

Theorem 4 in **[8]**. Then $(X, \mathscr{T})$ is normal because f is closed and continuous.

REFERENCES

1. A. S. Davis, *Indexed systems of neighborhoods for general topological spaces*, Amer. Math. Monthly *68* (1961), 886-893.

2. A. Dickinson, *Compactness conditions and uniform structures*, Amer. J. Math. *75* (1953), 224-228.

3. R. F. Dickman, Jr., J. R. Porter and L. R. Rubin, *Completely regular absolutes and projective objects*, Pacific J. Math. *94* (1981), 277-295.

4. R. Doss, *On uniform spaces with a unique structure*, Amer. J. Math. *71* (1949), 19-23.

5. L. Gillman and M. Jerison, *Rings of continuous functions* (Van Nostrand, Princeton, 1960).

6. N. C. Heldermann, *Developability and some new regularity axioms*, Can. J. Math. *33* (1981), 641-663.

7. D. C. Kent and G. D. Richardson, *Completely regular and ω-regular spaces*, Proc. AMS *82* (1981), 649-652.

8. W. F. Newns, *Uniform spaces with unique structure*, Amer. J. Math. *79* (1957), 48-52.

9. R. H. Warren, *Identification spaces and unique uniformity*, Pacific J. Math. *95* (1981), 483-492.

Military Programs Department,
General Electric Company,
King of Prussia, Pennsylvania