ON THE PARAMETRIC RESONANCE IN THIN DISK GALACTIC DYNAMO

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Abstract. The absence of parametric resonance by generation of the large-scale bisymmetric magnetic field in the framework of the simplest thin disk galactic dynamo model is shown.

Key words: galactic dynamo – bisymmetric mode – parametric resonance

The growth rate of the axisymmetric field generating by a turbulent dynamo is greater than the growth rate of the bisymmetric one. However, bisymmetric magnetic fields can be observed in some galaxies. It is not clear, why the slowly growing but nevertheless dominant bisymmetric configuration appears instead of the axisymmetric configuration. Chiba and Tosa 1990 recently proposed an idea to solve this problem. Real galaxies often have a spiral arm which rotates with an angular velocity Ω. Naturally, the galactic dynamo parameters are supposed to oscillate in time with an angular frequency $\tilde{\Omega} = \Omega - v(r)$, where $v(r)$ is the velocity of differential rotation of the galaxy. The bisymmetric magnetic field rotates with angular frequency ω. Based on the parametric resonance theory for Mathieu equation (Landau and Lifschitz 1969), one supposes the following condition of parametric resonance:

$$\tilde{\Omega} = 2\omega \tag{1}$$

which can provide an additional increase of the growth rate of the magnetic field in bisymmetric configuration. However, the equations of generation of the magnetic field are much more complicated than the Mathieu equation. Attempts to reduce the problem to the Mathieu equation (Hanasz et al. 1991, Schmitt and Rüdiger 1992) encounter some difficulties and the resonance condition is more complicated than (1).

Following Ruzmaikin et al. 1988 in the framework of the simplest thin disk galactic dynamo model we derive the equation for the large-scale magnetic field Q generating by a turbulent dynamo mechanism in thin galactic dynamo disk :

$$\frac{\partial Q}{\partial t} + (\boldsymbol{v} \cdot \nabla_\perp Q) = \gamma Q + \lambda^2 \Delta_\perp Q\,, \tag{2}$$

where $\quad \nabla_\perp = \{\, \frac{\partial}{r\partial r}(r\cdot),\ \frac{\partial}{r\partial \varphi},\ 0\},\quad \Delta_\perp = \nabla_\perp^2,$

γ is the local growth rate of magnetic field, $\boldsymbol{v}$ is the linear velocity of differential rotation of the galaxy toward the azimuth direction, and $\lambda = h/R$ is the ratio of a half-thickness of the disk h to its radius R. Here we do not take into account any more thin properties of the galactic media, e.g. meridional circulation. Using some evident assumptions regarding the structure of the large-scale magnetic field and replacing the diffusion term $\lambda^2 \Delta_\perp Q$ as $-\lambda^2 k^2 Q$ we obtain from (2) an ordinary differential equation

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$$\dot{Q} = (\gamma_0 \pm i\,\frac{v_0}{r_0} - \lambda^2 k^2\,)\,Q\,, \tag{3}$$

where a point means a derivation with respect to time. Equation (3) indicates that the growth rate of Q is $\Gamma_1 = \gamma_0 - \lambda^2 k^2$ and the frequency of its oscillations is $\Gamma_2 = \pm\, i\, v_0/r_0$. Separating the real and imaginary parts of equation (3) and taking $Q = Q_1 + i\,Q_2$ can yield a pair of first order equations, which can be expressed in matrix form as

$$\frac{d}{dt}\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{pmatrix} \Gamma_1 & -\Gamma_2 \\ \Gamma_2 & \Gamma_1 \end{pmatrix}\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = A(t)\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}. \tag{4}$$

The Mathieu equation $y'' + \sigma(t)y = 0$ can be also expressed in a similar way. However, its matrix has another structure:

$$A(t) = \begin{pmatrix} 0 & 1 \\ -\sigma(t) & 0 \end{pmatrix}. \tag{5}$$

Assume Γ_1 and Γ_2 are periodical functions of time. Then the equation (3) can be solved explicitly:

$$Q(t) = exp\left\{\int_0^t (\,\Gamma_1(\tau) + i\,\Gamma_2(\tau)\,)d\tau\right\} Q(t=0). \tag{6}$$

Here we take into account that the matrix $A(t)$ commutates by several values of t due to its special structure. The solution of the Mathieu equation cannot be written explicitly due to the non-commutativity of its matrix $A(t)$.

Introducing matrix $A(t)$ to be weakly periodically dependent on time

$$A(t) = A^{(0)} + A^{(1)} \sin \tilde{\Omega} t,$$

where matrices $A^{(0)}$ and $A^{(1)}$ are constants we obtain $Q(t) = F(t)\,e^{\kappa\, t}$, where κ is some certain constant completely determined by matrix $A^{(0)}$, and $F(t)$ is a complex bounded periodic function of time. The growth of Q from (10) evidently depends only on properties of matrices $A^{(0)}$ and $A^{(1)}$. Thus, there is no parametric resonance by any rate of frequencies $\tilde{\Omega}$ and ω.

Note, that the obtained result does not mean the parametric resonance in a galactic dynamo is impossible. However, such resonance can be reached only by taking into account more complicated phenomena which are not considered in the framework of the simplest model of the galactic dynamo.

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