Transport of Magnetic Helicity and Dynamics of Solar Active Regions

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Abstract. We introduce a method to calculate the magnetic helicity density in a given active-region vector magnetogram, and a lower limit of it, based on a linear force-free (Iff) approximation. Moreover, we provide a lower limit of the total magnetic helicity in the active region (AR). A time series of magnetograms can be used to calculate the rate of helicity transport. The results can be then correlated with manifestations of the dynamical activity in ARs, such as flares and filament eruptions.

1. Introduction and Method Description

Magnetic helicity in ARs is given by

\[ H_m = \int_V A \cdot B \, dV, \]

where \( B \) is the magnetic field vector, \( A \) is the vector potential and the integration refers to the volume \( V \) of the magnetic structure. Therefore, the integrand \( A \cdot B \) is a magnetic helicity density \( h_m \) at a given volume element. Following a Iff approximation (\( \alpha = \text{const} \)), \( h_m \) is thought to be given by (e.g. Pevtsov et al. 1995)

\[ h_m = \frac{B^2}{\alpha}; \quad \alpha = \text{const}. \]

Eq. (1) is incorrect. Indeed, for potential fields (\( \alpha = 0 \)) we find \( h_m \rightarrow \infty \), where we should find \( h_m = 0 \). In the Iff approximation \( h_m \) is, in fact, given by

\[ h_m = \frac{1}{\alpha} |B^2 - B \cdot B_p|; \quad \alpha = \text{const}, \]

where \( B_p \) is the potential field. From eq. (2) we now find \( h_m \rightarrow 0 \) for \( B \rightarrow B_p \). The overall \( \alpha \) can be calculated by a variety of methods (Leka 1999). From the resulting force-free field \( B_{ff} \), one may also calculate \( h'_{m} = (1/\alpha)|B_{ff}^2 - B_{ff} \cdot B_p| \). It turns out that \( |h'_{m}| \) is a lower limit of \( |h_m| \), since \( B_{ff} \) is the closest field to \( B_p \), for which \( h_m \rightarrow 0 \) (eq. (2)). Comparing \( B_{ff} \) and \( B_p \), one can now calculate \( h'_{m} \) anywhere in the AR. A helicity budget can then be found, i.e. \( H'_m = \int_V h'_m \, dV \). \( H'_m \) is a lower limit of the actual \( H_m \) in the AR. From a time series of vector magnetograms, one may find both \( H'_m(t) \) and \( (dH'_m/dt) \), i.e. the rate of helicity transport in the AR and compare it with dynamical activity in the AR.

Our method is applied to NOAA AR 9114 (Fig. 1). Notice the match between \( B \) (Fig. 1a) and \( B_{ff} \) (Fig. 1b). A change in the sign of \( \alpha \) forces \( h_m(t) \) to change sign (Fig. 1c). \( |h'_m(t)| \) is indeed a lower limit for \( |h_m(t)| \) (Fig. 1d). Notice the correspondence between the peaks of \( h_m(t) \) and \( H'_m(t) \) (Figs. 1a and 1e). This suggests that our method may be reasonable, although approximate.
Figure 1. Helicity calculation in NOAA AR 9114, observed by IVM. (a) The actual $B$ at a given time. (b) The best $B_{ff}$ at this time. Tic mark separation in a and b is 20". (c) Average $h_m(t)$. (d) Average $|h_m(t)|$ (squares, solid line) and $|h'_m(t)|$ (triangles, dashed line). (e) Minimum total magnetic helicity $H'_m(t)$.

2. Conclusion

Using only vector magnetograms, we estimate a lower limit of the total magnetic helicity budget and the rate of helicity transport in solar ARs for the first time. Helicity variations can be calculated only partially, with the formulation of Berger & Field (1984), for ARs close to disk center and by means of the highly uncertain velocity field (Chae 2001). Our method may prove quite useful, provided that the $B_{ff}$ approximation manages to capture the dynamical evolution in the active-region solar atmosphere.

References