

Whitehead group $\text{Wh}(\pi)$ is an abelian group which measures the extent to which an invertible matrix with entries in the group ring $\mathbb{Z}[\pi]$ can be diagonalized by generalized Gaussian elimination. Whitehead torsion has found wide application in the study of high-dimensional manifolds, such as in the s-cobordism theorem.

The computation of the Whitehead group $\text{Wh}(\pi)$ was initiated by G. Higman in 1940. He showed that $\text{Wh}(\pi)$ is trivial for some groups π , such as the trivial group $\{1\}$, and the infinite cyclic group \mathbb{Z} , and non-trivial for others, such as the finite cyclic group \mathbb{Z}_5 . This set the pattern for later developments, due to Bass, Milnor, Swan and others. By 1970 it had become clear that the computation of $\text{Wh}(\pi)$ for finite groups π could be attempted using the methods of algebraic number theory, such as localization and completion, and representation theory. These methods tend to fail for infinite groups π : instead the methods of combinatorial group theory and topology are used to prove that $\text{Wh}(\pi)$ is trivial for suitable classes of torsion-free groups.

In the last ten years Oliver has written a series of research papers on the Whitehead groups of finite groups, providing the most powerful computational technology available, at such a rate that it has been difficult for even experts in algebraic K-theory to keep up with the wealth of material. This book is a welcome opportunity to take stock, and survey the landscape. It is basically a pure algebra book, aimed at the reader who wants to learn the modern techniques, especially the p -adic logarithm introduced by Oliver. The author is quite high-minded, eschewing the simple device of a page or two tabulating the Whitehead groups for some particular finite groups, developing instead all the results needed to work out such a table. The only application considered here is to the congruence subgroup problem. However, the main applications have been to the computations of the Wall surgery obstruction groups of finite groups, over which the book draws a veil. Perhaps it is unreasonable to require a detailed account, but it is the important work of Wall and Bak in the early 70s which made the need for the new computational techniques apparent, and it is the L-theorists who are the largest category of users.

Overall, this is an excellent book for any K- or L-theorist who has to work with computations of the Whitehead groups of finite groups, and is already familiar with the classic text of Bass.

A. RANICKI

EVANS, D. E. and TAKESAKI, M. *Operator algebras and applications*, Volume 1: Structure theory; K-theory, geometry and topology; Volume 2: Mathematical physics and subfactors (London Mathematical Society Lecture Note Series 135, 136, Cambridge University Press, Cambridge 1988) Vol. 1, viii+244 pp, paper: 0 521 36843 X, £17.50; Vol. 2, viii+240 pp, paper: 0 521 36844 8, £17.50.

There is a tradition from the sixties that there are occasional, but irregular, conferences on operator algebras and some aspects of their applications. A number of these conferences have produced excellent proceedings afterwards: the most important was the Kingston Conference in 1980, and more recently there was the US-Japan Seminar, Kyoto, July 1983. This tradition continues. The two volumes under review contain research and expository articles from the participants in a UK-USA Joint Seminar on Operator Algebras held in July 1987 in Warwick and a few papers from those that had given talks at the symposium earlier in the year at the University of Warwick. The papers in these volumes provide good coverage of the material in the titles in a random way within each broad subject. There are expository articles alongside research calculations on technical problems; the standard of the papers is good. Unfortunately the volumes are a typographical muddle as they were produced from camera-ready copy supplied by the authors (the title on the cover of Volume 1 has an error in it); the advantage of the production is the informal style of some of the papers.

This review is not the place to provide a detailed assessment of the individual articles in these two volumes, a service which is fulfilled by the specialist reviewing journals. However the scope of these notes may be illustrated by mentioning a few papers and a brief view of what is in them. P. Baum and A. Connes in 'K-theory for discrete groups' discuss and motivate their conjecture that a certain natural homomorphism, defined by them, from a suitable geometrical K-theory group associated with a discrete group G into the corresponding C^* -algebra K-theory group of

the reduced C^* -algebra is an isomorphism. V. F. R. Jones in a paper 'Subfactors and related topics' surveys some of the known relationships between commuting squares of subfactors of a factor, vertex models, the Yang–Baxter equation, knot theory, quantum groups and field theory. With such broad cover this paper is a brief account of the current situation. This is followed by a long technical paper 'Quantized groups, string algebras, and Galois theory for algebras' by A. Ocneanu in which he introduces a Galois type invariant for the position of a subalgebra inside an algebra. S. Popa discusses amenability of type II_1 factors. B. Blackadar surveys various problems concerning comparison theories for simple C^* -algebras using the existing Murray-von Neumann comparison theory for projections in a von Neumann algebra as a guide. There are papers on operator algebras from the borders of mathematical physics to those of geometry, Lie groups and topology. These volumes are recommended to those who work in operator algebras and their closely related areas.

ALLAN M. SINCLAIR

KIRWAN, F. *An introduction to intersection homology theory* (Pitman Research Notes in Mathematics Series 187, Longman Scientific and Technical, 1988) 169 pp. 0 582 02879 5, £15.

A manifold is a topological space which is locally homeomorphic to a Euclidean space of a fixed dimension. The homology and cohomology of a manifold are related by the Poincaré duality isomorphisms. A singular space is a topological space which is a manifold except for some singularities. The Zeeman dihomology sequence related the singularities to the failure of Poincaré duality in a singular space. Sullivan and McCrory advocated this spectral sequence as a tool for the study of the topology of singular spaces. This bore fruit in the remarkable intersection homology theory developed over the last ten years by Goresky and MacPherson. The intersection homology groups of a singular space are defined using the singularities. In particular, singular spaces have Poincaré duality with respect to intersection homology. A singular space without singularities is a manifold, in which case the intersection homology groups coincide with the classically defined homology groups.

This book is the written record of what must have been an enjoyable graduate course introducing intersection homology to a non-specialist audience. It gives a clear account of the basic theory and the major applications. Chapter 1 gives the three main examples: the cohomology of complex projective varieties, de Rham and L^2 -cohomology, and Morse theory for singular spaces. In each case intersection homology reaches parts other homology theories cannot reach. Chapter 2 is a useful review of the relevant homological algebra, including sheaf cohomology. Chapter 3 contains the first definition of the intersection homology groups, using chain complexes. Chapter 4 deals with L^2 -cohomology, the analytic counterpart of the theory. Chapter 5 gives the sheaf-theoretic construction. Finally, Chapters 6, 7 and 8 deal with the applications of intersection homology to respectively the Weil conjectures relating algebraic number theory and algebraic geometry, the D-module theory relating differential equations and algebraic geometry, and the Kazhdan-Lustig conjecture in the representation theory of Lie algebras.

The level of exposition is more accessible than the articles in A. Borel et al., *Intersection cohomology* (Progress in Mathematics 50, Birkhäuser, Basel 1984), which are aimed at a specialist audience.

One can only wish that more graduate courses would lead to such stimulating and valuable lecture notes.

A. RANICKI

REID, M. *Undergraduate algebraic geometry* (London Mathematical Society Student Texts 12, Cambridge University Press, Cambridge 1988), viii + 129 pp, hard covers: 0 521 35559 1, £20; paper: 0 521 35662 8, £7.50.

A desire for honesty, via rigour, in our undergraduate courses is often attained at the cost of implementing (dishonest) axiomatic methods. Traditional undergraduate topics concerning, for