Easy Proof of Von Staudt's Theorem.

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The proof usually given of Von Staudt's Theorem is entirely analytical in character. The following proof is geometrical:—

If A, B, C, D be the vertices of a tetrahedron whose opposite faces are α, β, γ, δ, and if l be any line, to prove that $l[ABCD] \equiv l[\alpha\beta\gamma\delta]$ where lA denotes the plane through the line l and the point A and $l\alpha$ denotes the intersection of the line l with the plane α.

Consider the system of conicoids having $ABCD$ as a self-conjugate tetrahedron. A unique conicoid of the system can be made to pass through *3* given points, and if these points be chosen on l, we obtain a conicoid containing l, and self-conjugate to $ABCD$. Consider the configuration l, A, B, C, D. Reciprocating with regard to the above conicoid, l reciprocates into l (being a generator), A into α, B into β, C into γ, and D into δ (because the tetrahedron is self-conjugate).

Hence $l[ABCD] \equiv l[\alpha\beta\gamma\delta]$.