DYNAMICS of ARTIFICIAL SATELLITES and SPACE DEBRIS

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MEAN ORBITAL MOTION OF GEODETIC SATELLITES AND ITS APPLICATIONS

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Abstract. Averaging methods are convenient tools for studying long-periodic variations of the motion of artificial satellites. The main lines of a semi-analytical theory of the mean motion are given. We show how, when coupled with a careful reduction of the tracking data, this theory allows to determine parameters related to the temporal variations of the Earth gravity field (e.g. the amplitude of 18.6 years tide and the secular variation of even zonal harmonics). The theory is also very useful for other applications such as mission analysis.

1. Introduction

The most elegant, and so far most accurate, method for studying the global Earth gravity field in time and space (combined ocean-atmosphere-Earth) is to analyse the induced perturbations in the orbits of artificial satellites. Several "geodetic" satellites have now been routinely tracked by lasers for two decades. Low satellites (e.g. Starlette at 900 km altitude) are more sensitive to the atmosphere and to the Earth gravity field at mean wavelengths, whereas higher satellites (e.g. Lageos at 6000 km) are suitable for the study of long wavelengths, including their temporal variations, and the Earth rotation. The development of highly precise Earth gravity field models in view of precise orbit determination (see Schwintzer *et al.*, 1996; Tapley *et al.*, 1994) allows the determination of the temporal variations of mass distribution in the oceans, atmosphere and solid Earth. In order to provide enough information on these geodynamical phenomena from a satellite perturbation analysis, it is necessary to extract the secular and long periodic effects appearing in satellite orbits. A natural strategy is (1)

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to use dynamical arcs as long as possible, that is several tens of thousands of orbital revolutions (typically 10 to 20 years), and (2) to remove short periodic effects. This is exactly the spirit of averaging methods.

The method of averaging is a well-known tool in celestial mechanics (Moons, 1993). With such a technique, the first goal is to improve the accuracy of the solution by removing short periodic terms which can be corrupted by unknown local phenomena in time and space. Due to the limited accuracies of the available initial conditions for the artificial bodies and the averaging methods used for the computation, investigations developed in the 70s revealed large approximations (e.g. Wagner, 1973). Following these ideas, we have defined the concept of mean orbital motion more comprehensively and more precisly. This has led to the development of a semi-analytical theory of the mean orbital motion (Métris, 1991; Métris and Exertier, 1995). The theory is based on the concept of filtered elements permitting exact separation between short-period and long-period variations of the orbital motion. The theory initially developed for gravitational effects has been extended to dissipative ones (e.g. drag, radiation pressure). The purpose of this paper is to show which ingredients, theoretical and practical, have permitted to precisely describe the long term changes of satellite orbital elements. To provide an unprecedented information on geodynamical processes, investigations over long periods of time of the motion of Lageos and Starlette form the basis of our applications.

The organization of the paper is the following: after recalling the principle of the averaging method in Section 2, the analytical theory used to compute the mean orbital motion is summarized in Section 3. Section 4 is dedicated to examples of applications realized with the method.

2. Principle of the Averaging Method

Satellite motion can be described by the temporal evolution of six independent parameters. We will consider that the study of the orbital motion can be treated as a study of the temporal evolution of six signals. These signals result either from observations or from a dynamical model; the goal of any perturbation analysis is to compare both signals, observed and theoretical, permitting the fit of model parameters and initial conditions of the motion.

There is a considerable freedom in the selection of the solution technique with properly chosen variables. The averaging technique consists in transforming (filtering) the initial system of motion equations and processing the integration of the transformed equations to provide the theoretical signal. Thus, in order to set up an averaging method, it is necessary: (1) to properly define the transformation (filtering) to be applied both to observed and theoretical signals, and the variables used during this operation, (2) to develop the tools allowing to perform the filtering that leads to the theory of the mean orbital motion, and (3) to compare the solution with the observations properly reduced, and to analyse the residuals.

Given a set of variables $X_i(t)$ describing the satellite motion, the filtered variables $\overline{X}_i(t)$ over the period T result from the removal of, and only of, the short-period variations from $X_i(t)$, i.e. the periodic variations with a period smaller than T. The concept of filtered elements is completely defined by the specification of the set of elements X_i, and the cut-off period T.

The variables chosen for the processing of the transformation are of great importance[1]. From a practical point of view, Delaunay variables (Brouwer, 1959) have been chosen because they are canonical action-angle variables, they highlight the different frequencies, and they are close to the Keplerian elements. In absence of resonances, the cut-off period must be slightly larger than one day (e.g. 1.2 days), to permit the removal of all periodic terms related to the orbital and Earth sidereal periods. In the case of weak resonance between these two periods, the cut-off period is chosen so as to remove these resonant terms. Orbits in deep resonance have not been considered, because most current geodetic space missions avoid this kind of dynamical configuration.

3. Theory and Observations

3.1. THEORETICAL FILTERED DELAUNAY ELEMENTS

In classical methods, the temporal evolution of osculating elements is governed by a differential system (e.g. Lagrange or Hamilton or Gauss equations). Here, we shall examine if, in a similar way, filtered elements can be the result of the integration of a modified (averaged) Hamiltonian system. Only the main lines will be explained here; readers interested in more details can refer to (Métris, 1991; Métris and Exertier, 1995).

The differential equations of motion expressed in Delaunay canonical variables (v_i, V_i; $i = 1,3$) are derived from the Hamiltonian $\mathcal{H}$ of the disturbed system:

$$\dot{v}_i = \frac{\partial \mathcal{H}}{\partial V_i} \quad (i = 1,3), \qquad \dot{V}_i = -\frac{\partial \mathcal{H}}{\partial v_i} \quad (i = 1,3). \tag{1}$$

In absence of external time dependent parameters, the short period removal is accomplished by elimination of the mean anomaly l from the differential system. In this aim, a canonical change of variables has been performed:

$$\begin{array}{ccc} (l,g,h,L,G,H) & \longrightarrow & (l',g',h',L',G',H') \\ \mathcal{H}(l,g,h,L,G,H;\varepsilon) & \longrightarrow & \mathcal{H}'(-,g',h',L',G',H';\varepsilon) \end{array} \tag{2}$$

[1]In particular we have shown (Métris and Exertier, 1995) that the mean motion is not unique but depends on the choice of the set of variables X_i.

The transformation is constructed by means of Lie transforms according to the Deprit algorithm (Deprit, 1969). It is completely determined by its generating function W, which is developed in powers of ε:

$$W(l,g,h,L,G,H;\varepsilon)=\sum_{k\geq 0}\frac{\varepsilon^k}{k!}W_{k+1}=\frac{\partial}{\partial\varepsilon}\left[\sum_{k\geq 1}\frac{\varepsilon^k}{k!}W_k\right] \tag{3}$$

Following the standard process (see Deprit and Rom, 1970), at each order k, W_k is determined up to a function independent of l. Applying a Lie transform to a function of the old variables (v_i, V_i) permits to express it in function of the new variables (v_i', V_i'). In particular:

$$\left\{\begin{array}{ll} v_i = v_i' + & \left[\varepsilon\frac{\partial W_1}{\partial V_i}+\frac{\varepsilon^2}{2}\left(\frac{\partial W_2}{\partial V_i}+\left\{\frac{\partial W_1}{\partial V_i};W_1\right\}\right)\right](v_j',V_j')+\mathcal{O}(\varepsilon^3) \\ V_i = V_i' - & \left[\varepsilon\frac{\partial W_1}{\partial v_i}+\frac{\varepsilon^2}{2}\left(\frac{\partial W_2}{\partial v_i}+\left\{\frac{\partial W_1}{\partial v_i};W_1\right\}\right)\right](v_j',V_j')+\mathcal{O}(\varepsilon^3) \end{array}\right. \tag{4}$$

where $\{\phi;\psi\}$ stands for the Poisson bracket of ϕ and ψ. But in fact, we are looking for a set of variables $(\overline{v}_i', \overline{V}_i')$ such as:

$$\left\{\begin{array}{lll} \overline{v}_i' & =< v_i >_{l'} & =< v_i' >_{l'} + < v_i - v_i' >_{l'} \\ \overline{V}_i' & =< V_i >_{l'} & =< V_i' >_{l'} + < V_i - V_i' >_{l'} \end{array}\right. \tag{5}$$

As a result of the averaging, $< v_i' >= v_i'$ and $< V_i' >= V_i'$. Expression (4) allows the computation of $< v_i - v_i' >_{l'}$ and $< V_i - V_i' >_{l'}$ knowing the generating function W. If W_1 is chosen in such a way that $< W_1 >_l= 0$ (thanks to the integration constant) the mean values (5) are of second order.

It would be simpler to obtain the total equivalence between the two sets of variables (v_i', V_i') and $(\overline{v}_i', \overline{V}_i')$. But, it can be proved that this is not possible beyond the first order. This is the reason why $(\overline{v}_i', \overline{V}_i')$ can not be directly obtained by the integration (numerical, for example) of the canonical averaged system. An important step in our solution is precisely to add a change of variables (Eq. 5) which is not canonical.

The above algorithm states the bases of the computation of the theoretical filtered elements but it is not always usable in this form; in particular, for non-gravitational forces no Hamiltonian exists. Fortunately, we can show that, provided that the perturbation fulfills some conditions of periodicity (more or less satisfied in reality), the filtering can be performed by means of numerical quadratures, at least up to the first order of these small perturbations. This was applied for the drag and radiation pressure perturbations.

3.2. OBSERVED FILTERED DELAUNAY ELEMENTS

If we could directly observe the variables of the motion, it would be sufficient to apply a digital filter to get observed filtered variables. This is of course not the case: observations are more or less complicated functions of the elements. In classical methods, one simply computes the value of this function using theoretical elements and compares it to the observation. Unfortunately, in our case, the use of theoretical filtered elements to compute the function does not produce the searched theoretical filtered observation because the function is not linear with respect to the variables. The problem has been solved by Exertier (1988, 1990). The first step consists in converting the observations into variables of the motion via orbit fits (using short arcs) to get "observed osculating elements". Then, in a second step, observed osculating elements are filtered to produce "observed filtered elements". We have checked the robustness of this data reduction scheme and have shown that a level of 10^{-9} between a simulated observed signal and the corresponding theoretical signal can be achieved by this procedure.

At this stage a question may arise: what is the gain, if a classical method must be used anyway to compute observed filtered elements? Several advantages exist: (1) one can get very good results with only, let us say, one filtered element per month computed with typically one week of observations, (2) even with a larger density of filtered elements, the factor of compression of the useful information ranges from thousands to one million, and (3) being independent of the dynamical model used to compute them, these filtered elements are computed once for all.

4. Applications

If, as explained above, building a precise averaging method is a difficult task, it is rewarded by the following benefits: (1) we are free of short periodic variations always difficult to modelize, (2) the CPU time is reduced by more than a factor 10, and the same parameters are always recovered with very different methods.

4.1. GEODYNAMICS

The temporal variations of the external gravitational field represent a dynamic aspect of the mass redistribution within the ocean-atmosphere-solid Earth system. Satellite solutions place bounds on the aggregate mass redistribution ongoing within this system. In particular, Satellite Laser Ranging (SLR) observations acquired on Lageos for twenty years account for the drastic improvement seen in the long wavelength static, time-dependent, and tidal geopotential fields (Marshall *et al.*, 1995).

Among the temporal variations of the gravity field, tides are the best understood for they have the largest effects and occur at well known astronomical frequencies. For near-Earth orbit determination, the tide modeling problem represents a challenge: to improve the long wavelength tidal terms, which give rise to long period perturbations. This is particularly true for the 18.6 year ocean and solid-Earth tides. The analysis of 15 years of SLR data on Lageos reduced in terms of mean orbital elements, has permitted to extract without ambiguities a 18.6 year periodic signal in the mean ascending node of the orbit related to these phenomena. The amplitude of the signal, when expressed in terms of oceanic tide, is of 2 cm $\pm$ 0.1 cm with zero phase, the residuals on the node being at the level of 85 mas (Exertier *et al.*, 1995).

To date, direct assessment of non-tidal changes in the geopotential has been restricted to variations in a few zonal harmonics. The coefficients (secular, annual and semi-annual) for the J_2 harmonic have been determined from our study on the mean orbital motion of Lageos (*ibid.*). As an example, we found a secular variation of J_2 of $(2.4 \pm 0.3) \times 10^{-11}$/yr, which is in good agreement with other results (e.g. Gegout and Cazenave, 1993; Eanes 1995).

The most challenging aspect in the study of temporal gravitational variations will be to attempt to separate the contributions from individual geophysical processes, given the estimate of their overall effect from satellite determinations. In this field, the semi-analytical theory of the mean orbital motion will certainly play an increasing role.

4.2. NON-CONSERVATIVE FORCES

Of particular importance in the development of contemporary gravity models are the laser geodetic satellites. These satellites are passive targets constructed as solid, dense spheres. Their simple form reduces both the magnitude and complexity of their surface forces. Since separation and modeling of conservative and non-conservative forces acting on these satellites is easier to achieve than with complex satellite forms, they have provided the most important data for geopotential recovery.

The semi-analytical theory of the mean orbital motion has been extended to non-conservative forces. In particular, we have treated the problem of averaging solar radiation pressure including its associated shadowing effects, and drag. Atmospheric drag is a significant non-conservative force modeling problem for new missions orbiting at low-Earth altitudes (between 350 km and 500 km) (Bruinsma *et al.*, 1996), and satellites with complex shapes. The analysis of 12 years of Starlette SLR data reduced in terms of mean orbital elements, has permitted to extract properly the decrease of the mean semi-major axis due to drag. This has permitted to evaluate the performance over long periods of recent atmospheric density models: DTM

(Barlier *et al.*, 1978; Berger *et al.*, 1996) and MSIS86 (Hedin *et al.*, 1987). The two main conclusions are: (1) DTM94 and MSIS86 are comparable in quality, and (2) they produce much better results for long term evolution of filtered elements than for short periodic variations of osculating elements.

The averaging method can be very useful in other ways: since the residuals are clean of fast variations, it is easier to study new long period phenomena. Moreover, with this process, we can also work on residuals issued from other sources. For example, we used so called residual excitations (Eanes, 1995) produced at UT/CSR to improve the model of thermal forces acting on Lageos (Métris *et al.*, 1995). This kind of work could hardly be performed by classical methods (using osculating motion) because the physical model is at the same time complicated and poorly known. Thus, one needs many tests, the interpretation of which must be very visual.

4.3. MISSION ANALYSIS

Planning and designing a satellite mission requires powerful computational tools, which are used to determine the orbital parameters satisfying the mission's objectives with minimum cost. Lifetime estimations are an important part of mission analysis, and the concept of mean motion in this context is of particular interest for two reasons: the averaged orbital elements reflect only the long period perturbations, which enhances their interpretability, and secondly, the computational speed as compared to the classical exceeds a factor 15. The latter reason allows the fast computation of several lifetime scenarios, with varying initial conditions and a predicted solar activity. The solar activity predictions are the weak point in satellite lifetime predictions, since they are not very accurate; errors of the order of 20% (Brown, 1992) must be reckoned with when the prediction is given several years before solar maximum. Thus, the lifetime of a particular (low-Earth) satellite in a given configuration is a function of the predicted solar activity, with large error bars. Any other model error will be negligible compared to it.

Lifetimes for the German satellite CHAMP, to be launched into a polar, circular orbit at an altitude of 500 km in 1999, were estimated. The lifetime estimates varied between 4 and 12 years under stronger or weaker solar regimes, and, depending on the solar activity level, orbit corrections during the mission will be necessary (Bruinsma *et al.*, 1996).

5. Conclusion

The concept of filtered elements applied to dedicated satellites appears to be a powerful tool for long term analysis. It is efficient for monitoring geophysical changes; good results have been obtained concerning the determination of $\dot{J}_2$ and of the 18.6 year tide. The theory of the mean motion has been also used to study non-gravitational forces. It is used intensively by CNES for mission analysis purposes.

References

Barlier, F., *et al.*: 1978, "A thermospheric model based on satellite drag data", *Aeronomica Acta* **A 185**.

Berger, Ch., Biancale, R., Ill, M., and Barlier, F.: 1996, "Improvement of the empirical thermospheric model DTM: DTM94. Comparative review on various temporal variations and prospects in space geodesy applications", *J. Geod.*, submitted.

Brouwer, D.: 1959, "Solution of the problem of artificial satellite theory without drag", *Astron. J.* **64**, 378–397.

Brown, G. M.: 1992, "The peak of solar cycle 22: Prediction in retrospect", *Ann. Geophys.* **10**, 453–461.

Bruinsma, S.L., Exertier, P., and Barlier, F.: 1996, "Long arc computation for low-orbiting satellites", *European Geophysical Society XXIe General Assembly – Symposium G13*, The Hague, The Netherlands.

Deprit, A.: 1969, "Canonical transformations depending on a small parameter", *Celest. Mech.* **1**, 12–30.

Deprit, A. and Rom, A.: 1970, "The main problem of artificial satellite theory for small and moderate eccentricities", *Celest. Mech.* **2**, 166–206.

Eanes, R. J.: 1995, "A study of temporal variations in Earth's gravitational field using Lageos-1 laser range observations", *CSR-95-7*, Center for Space Research, University of Texas, Austin.

Exertier, P.: 1988, *Orbitographie des Satellites Artificiels sur de Grandes Périodes de Temps: Possibillités d'Applications*, Thesis (in French).

Exertier, P.: 1990, "Precise determination of mean orbital elements from osculating elements by semi-analytical filtering", *Manuscr. Geod.* **15**, 115–123.

Exertier, P., Métris, G., Boudon, Y., and Barlier, F.: 1995, "Simultaneous determination of the 18.6 year ocean tide and $\dot{J}2$ from the mean orbital motion of Lageos", *XXIe General Assembly I.U.G.G. – Symposium G3*, Boulder, Colorado.

Gegout, P. and Cazenave, A.: 1993, "Temporal variations of the Earth's gravity field for 1985-1989 derived from Lageos", *Geophys. J. Int.* **114**, 347–359.

Hedin, A.E.: 1987, "MSIS-86 thermospheric model", *J. Geophys. Res.* **92(A5)**, 4649–4662.

Marshall, J.A., Klosko, S.M., and Ries, J.C.: 1995, "Dynamics of SLR tracked satellites", *Rev. Geophys., Suppl.* 353-360.

Métris, G.: 1991, *Théorie du Mouvement du Satellite Artificiel: Développement des Equations du Mouvement Moyen, Application à l'Etude des Longues Périodes*, Thesis (in French).

Métris, G. and Exertier, P.: 1995, "Semi-analytical theory of the mean orbital motion", *Astron. Astrophys.* **294**, 278–286.

Métris, G., Vokrouhlický, D., Ries, J.C., and Eanes, R.J.: 1996, "Non-gravitational effects and the Lageos eccentricity excitations", *J. Geophys. Res.*, in press.

Moons, M.: 1993, "Averaging approaches", in: *Proceedings of Artificial Satellite Theory Workshop*, U.S.N.O Washington D.C. (USA).

Schwintzer, P., Reigber, Ch., Bode, A., Kang, Z., Zhu, S.Y., Massmann, F.-H., Raimondo, J.C., Biancale, R., Balmino, G., Lemoine, J.M., Moynot, B., Marty, J. C., Barlier, F., and Boudon, Y.: 1996, "Long-wawelength global gravity field models: GRIM4-S4, GRIM4-C4", *J. Geod.*, in press.

Tapley, B.D., et al.: 1994, "The JGM3 gravity model", *Annales Geophysicae* **12**, Suppl. 1, C192.

Wagner, C.A.: 1973, "Zonal gravity harmonics from satellite long arcs by seminumerical method", *J. Geophys. Res.* **78**, 3271–3280.

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