REALIZATION OF THE PRIMARY TERRESTRIAL REFERENCE FRAME

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ABSTRACT. The PTRF is based on 43 sites with 64 SSC collocation points with the optimum geographic distribution, which were selected from all stations of the ITRF89 according to the criterion of the minimum value of the errors of 7 parameters of transformation. The ITRF89 was computed by the IERS Terrestrial Frame Section in Institut Geographique National - IGN and contains 192 VLBI and SLR stations (points) with 119 collocation ones. The PTRF has been compared with the ITRF89. The errors of the 7 parameters of transformation between the PTRF and 18 individual SSC as well as the mean square errors of station coordinates are of the same order as those for the ITRF89. The transformation parameters between the ITRF89 and the PTRF are negligible and their errors are of the order of 3 mm.

1. Introduction

Geodetic and geodynamic investigations on the level of subcentimeter accuracy are the goals of the next decade. A well defined and stable terrestrial reference frame is needed for these purposes (Mueller et al., 1989).

The IERS Terrestrial Reference Frames ITRF88, ITRF89 (Boucher, 1988; Boucher et al. 1989; IERS, 1989-1990) are the best combine solutions developed from previous BIH Terrestrial Systems - BTS 1984 - BTS 1987 (BIH; 1986-1989; Boucher et al., 1984, 1985, 1986, 1988). The ITRF89 is defined by 192 VLBI, SLR and LLR stations which coordinates were computed as a combination of 18 Sets of Station Coordinates - SSC (IERS, 1990).

The accuracy of the transformation parameters between the individual SSC and the ITRF89 is now of the order of 1 to 10 cm, and depends on the number, the distribution and the accuracy of collocation stations in each SSC. The accuracy of the ITRF89, defined by the accuracy of 7 parameters of transformation depends mostly on distribution of the SSC collocation stations. The ITRF89 stations are located mostly in Western Europe and North America. The most number of such stations, which are not located homogeneously all over the Earth do not improve the accuracy of the ITRF89. A terrestrial reference frame can be defined with the same accuracy by smaller number of stations with accurate coordinates and optimum geographic distribution (Kosek et al., 1990). In the paper the choice of the optimum number and geographic distribution of stations taken from the ITRF89 set of stations was done and new terrestrial reference frame named the Primary Terrestrial Reference Frame - PTRF was determined. The PTRF consisted of 64 SSC collocation points with a good distribution and accuracy of their coordinates is presented as a combination of 18 Sets of Station Coordinates. Such system can be used in geodynamic investigations and for other global geodetic activities. The ITRF with larger number of stations with

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coordinates in the PTRF system can be used for all other purposes.

2. Method of analysis

The sites of the PTRF with the optimum geographic distribution were selected from the ITRF89 stations according to the criterion of the minimum value of the errors of 7 parameters of transformations obtained by the least-squares adjustment.

The observational equation for the least-squares adjustment written for the $i-th$ station is given by the following formula:

$$\begin{pmatrix} X_k \\ Y_k \\ Z_k \end{pmatrix} = \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} - \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} - \begin{pmatrix} D & -R3 & R2 \\ R3 & D & -R1 \\ -R2 & R1 & D \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} \tag{1}$$

where: X_k, Y_k, Z_k are disturbed by the white noise station coordinates X_i, Y_i, Z_i,
T_1, T_2, T_3 are the translation parameters,
D is the excess to 1 of the scale factor,
$R1$, $R2$, $R3$ are the small Euler rotation angles.

The equation (1) have been weighted by dividing it by the mean square error of a station computed from the mean square errors of station coordinates given in the ITRF89 solution:

$$m_i = \sqrt{(m_{x_i}^2 + m_{y_i}^2 + m_{z_i}^2)}, \tag{2}$$

The errors of 7 parameters of transformation are given by the following formula:

$$m_j = m_o\sqrt{(A^T A)_{jj}^{-1}} \quad ,\text{for } j = 1,2,\ldots,7 \tag{3}$$

A is the $t \times l$ matrix of the observational equation coefficients with $t = 7$ and $l = 3N$,
N is the number of stations.

In this equation the errors of the parameters of transformation depend only on the values of the diagonal elements of the matrix $(A^T A)^{-1}$ since the value of m_o is constant for the same number of stations. These diagonal elements depend on the stations distribution as well as on the m_i errors.

First the points of the highest accuracy of coordinates have been chosen in each site. Next the stations of not good distribution and accuracy have been eliminated sequentially according to the criterion of the minimum value of the errors of 7 parameters of transformation. These errors are minimum when the trace of the variance covariance matrix has a minimum value $(tr(A^T A)^{-1} = min)$. In order to choose the optimum distribution of $N-1$ stations from the set of N the $\binom{N}{N-1}$ combinations were analysed. From N number of stations a number of $N-1$ such stations were selected, for which the value of $tr(A^T A)^{-1} = min$. It enables elimination of one station and after that the number of $N-2$ stations from $N-1$ stations can be selected. All collocation stations of VLBI and SLR or LLR instruments have been included in the computation of the PTRF in order to get better tie of the VLBI to SLR systems, though for some of them like METSAHOVI, FLAGSTAFF, PASADENA, YUMA, PLATTEVILLE and SAN DIEGO the trace matrix analysis show that they ought to be eliminated.

Additionally points like: GOLDSTONE (S009, S014, S019), FORT DAVIS (M006), CANBERRA (S003) and MADRID (S001, S010) were also included to the computation in order to get the solution for the transformation parameters for each SSC for which at least 3 stations are necessary. All the SSC collocation stations of the Southern hemisphere are important for the computation of the PTRF, because of the small number of stations in this hemisphere. The improvement of the accuracy of their coordinates is very important for the better definition of the Terrestrial Reference Frame and the PTRF.

Finally, the PTRF consisting of 43 sites with 64 SSC collocation points was computed (Fig. 1). Using the least squares adjustment and the IGN program for ITRF computation (Boucher et al., 1988, 1989) the coordinates X_i, Y_i, Z_i, of the PTRF as well as the transformation parameters: T_1, T_2, T_3, D, $R1$, $R2$, $R3$, between this frame and the individual SSC were obtained (Tab. 1.) on the base of formula (1). The scale and the origin of the SSC-CSR89L02 have been adopted for definition of the ITRF88, ITRF89 and the PTRF systems and they were held fixed in the adjustment of the PTRF. The orientation of the PTRF axes is adopted to be the same as in the ITRF88 and ITRF89 systems and differs about fixed rotation angles from the SSC-CSR89L02 system. The transformation parameters from the PTRF to the individual SSC's are in a very good agreement with the transformation parameters from ITRF89 to the same individual SSC's (Tab. 1.,2.). The differences are smaller than their RMS errors except for such SSC like: GAOUA90L01 (T_2) and LPAC90L01 (T_2, D). The errors of these transformation parameters are of the same order as for the ITRF89. The errors of the coordinates of the PTRF stations (Tab. 3.) and the $\sigma_o = 1.35cm$ are of the same order as for the ITRF89 ($\sigma_o = 1.34cm$).

The transformation parameters from the PTRF to ITRF89 reference frames are negligible (Tab. 1.) and their RMS are of the order of 3 mm.

3. Conclusions

The PTRF consisting of 64 SSC collocation points in 43 sets carefully chosen defines the terrestrial system as well as the ITRF89 consisting of 119 SSC collocation points. About half a number of SSC collocation points due to not homogeneous distribution and lower coordinate accuracy have no influence on the accuracy of determination of the ITRF89. More number of the eliminated collocation station are located in Europe and North America. The PTRF defines well an ITRF for global geodynamic investigations. The ITRF with larger number of stations is useful for other geodetic activities.

4. Acknowledgements

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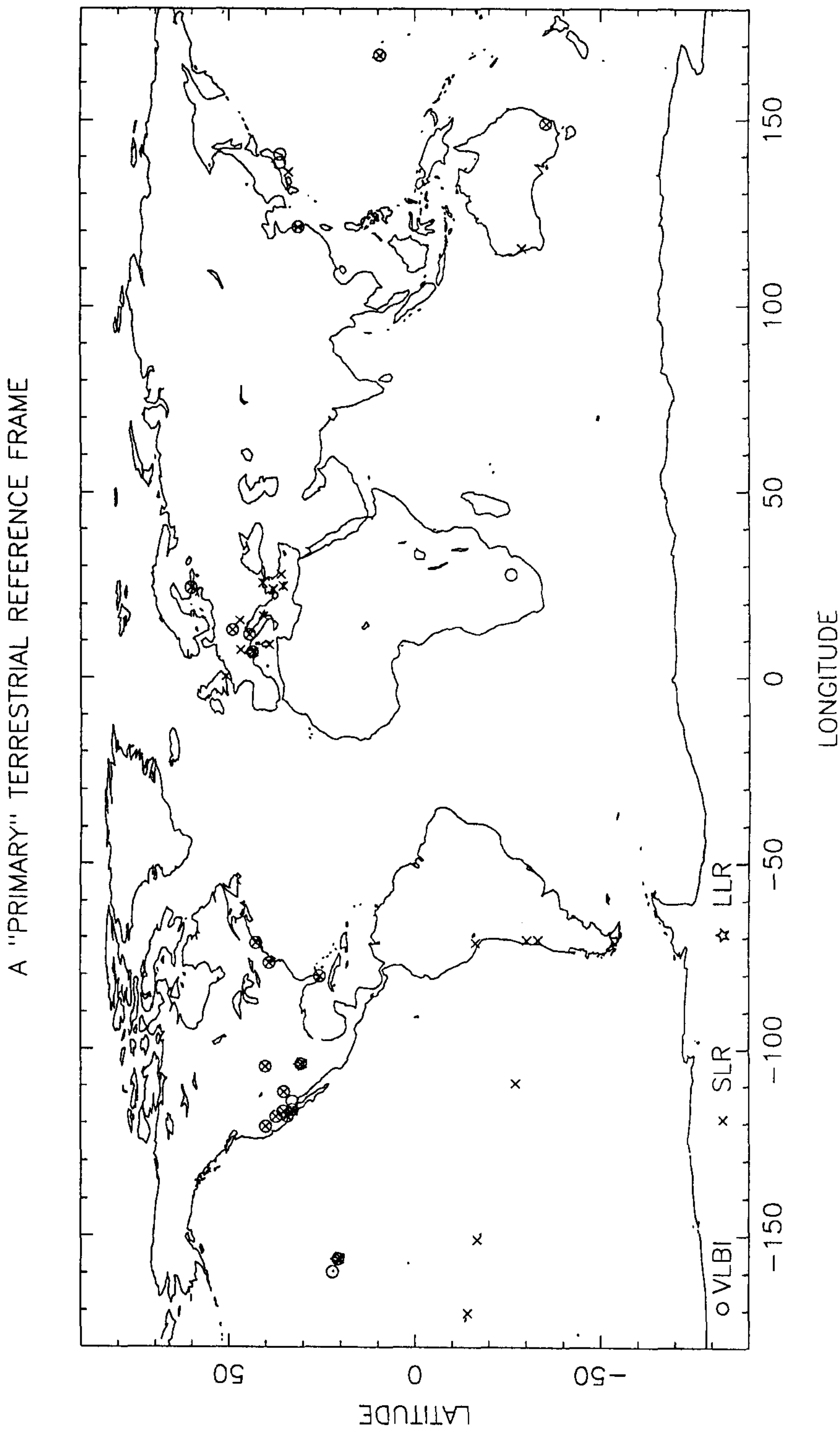
A "PRIMARY" TERRESTRIAL REFERENCE FRAME
o VLBI
× SLR
☆ LLR
LONGITUDE
LATITUDE
−150
−100
−50
0
50
100
150
50
0
−50

Table 1. Transformation parameters from the Primary Terrestrial Reference Frame to individual SSC systems and to the ITRF89 at epoch 1988.0.

SSC	T1	T2 [cm]	T3	D 10^{-8}	R1	R2 [mas]	R3	Coll. stations chosen	elimin.
GSFC 90R02	161.4	-86.0	52.0	-0.7	2.1	1.5	0.2	21	15
	1.1	1.2	1.1	0.2	0.5	0.4	0.4		
NGS 90R01	-0.2	-4.7	5.1	-0.3	-3.8	10.5	-0.2	14	12
	1.2	1.2	1.2	0.2	0.5	0.5	0.4		
USNO 90R01	-8.5	-16.5	3.5	-1.8	-0.4	2.2	0.5	4	4
	2.0	2.4	2.3	0.3	1.1	0.8	0.6		
SO 88R01	162.8	-107.3	36.9	-0.5	-7.4	7.0	-2.0	3	2
	7.8	4.7	6.4	0.5	2.5	3.4	1.4		
NAOMZ 89R01	-10.1	13.3	-4.2	1.3	-4.1	9.1	-1.4	3	3
	9.0	8.6	7.5	1.2	3.3	3.5	2.5		
JPL 90R02	-0.2	-4.6	9.1	-3.1	2.3	1.6	0.1	3	2
	2.7	2.9	2.8	0.4	1.4	1.2	1.5		
CSR 89L02	0.0	0.0	0.0	0.0	-3.4	4.3	-15.9	46	37
	-	-	-	-	-	-	-		
GSFC 89L01	-2.0	-0.7	1.7	0.1	-0.7	5.2	-7.5	40	35
	1.1	1.1	1.0	0.2	0.4	0.4	0.4		
DGFII 89L03	0.4	-0.3	4.6	-0.2	-286.1	-46.0	3.4	18	4
	1.5	1.5	1.4	0.2	0.6	0.6	0.5		
DUT 90LO1	-0.4	-1.2	-5.9	-1.1	0.1	1.7	-4.3	22	9
	1.3	1.3	1.3	0.2	0.5	0.5	0.5		
ZIPE 90L01	-1.8	0.5	-8.0	0.1	0.5	0.8	0.6	29	9
	1.3	1.3	1.2	0.2	0.5	0.5	0.4		
S0 90L01	-0.5	0.5	1.1	-0.6	-3.7	5.2	-14.3	30	12
	1.3	1.3	1.2	0.2	0.5	0.5	0.4		
GAOUA 90L01	-2.0	0.4	-0.6	-1.0	1.1	2.6	-0.8	26	14
	1.4	1.3	1.3	0.2	0.5	0.5	0.4		
LPAC 90L01	0.8	1.9	8.9	-0.6	2.2	-0.1	-4.2	20	7
	1.7	1.6	1.6	0.2	0.6	0.7	0.6		
UTXMO 90M01	-2.4	-0.7	34.3	-4.2	-0.8	0.2	-3.1	6	0
	5.9	7.7	5.4	1.0	2.4	1.7	3.3		
JPL 90M01	-9.8	-1.8	0.4	-2.0	1.5	3.2	-1.4	6	0
	4.5	5.0	4.7	0.6	2.3	1.7	1.3		
CERGA 89M01	-13.2	3.6	-4.7	-5.3	0.6	3.5	-11.5	5	0
	4.6	5.3	4.8	0.6	2.5	1.7	1.3		
S0 86M01	-17.2	21.8	6.8	-1.8	16.0	6.9	-6.1	4	0
	9.1	10.5	9.5	1.2	5.0	3.5	2.6		
ITRF89	-0.1	-0.6	-0.1	0.0	-0.1	0.0	0.0	64	55
	0.3	0.3	0.3	0.1	0.1	0.1	0.1		

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Table 2. Differences between transformation parameters from the PTRF to the individual SSC systems and from the ITRF89 to the individual SSC systems.

SSC	T1	T2 [cm]	T3	D 10^{-8}	R1	R2 [mas]	R3
GSFC 90R02	0.2	1.0	0.1	0.1	0.3	-0.2	0.0
NGS 90R01	0.0	0.6	0.3	0.0	0.1	-0.1	0.2
USNO 90R01	-0.6	0.5	-1.8	0.0	-0.4	0.3	0.4
SO 88R01	1.6	-0.5	-0.2	0.1	-0.2	-0.5	-0.4
NAOMZ 89R01	-2.3	1.5	1.2	0.1	-0.2	1.1	0.1
JPL 90R02	1.5	-0.8	2.2	0.5	0.1	-0.2	0.2
CSR 89L02	0.0	0.0	0.0	0.0	0.0	0.0	0.0
GSFC 89L01	-1.0	0.3	-0.7	0.1	0.3	-0.1	0.2
DGFII 89L03	0.4	0.7	0.2	0.0	0.2	0.0	0.0
DUT 90LO1	0.1	1.1	0.2	0.0	0.1	0.0	0.1
ZIPE 90L01	1.3	0.1	0.6	0.0	0.2	-0.2	-0.1
S0 90L01	0.1	1.2	-0.6	0.2	-0.2	0.2	-0.1
GAOUA 90L01	1.3	2.2	1.1	0.2	0.3	-0.6	0.0
LPAC 90L01	-0.7	2.8	0.0	-0.4	-0.1	0.6	0.5
UTXMO 90M01	0.2	1.3	0.1	0.1	0.1	-0.1	0.0
JPL 90M01	0.2	1.0	0.2	0.0	0.1	-0.1	0.0
CERGA 89M01	0.2	1.0	0.1	0.0	0.1	-0.1	0.0
S0 86M01	0.2	1.0	0.1	0.1	0.1	-0.1	0.1

Table 3. The station coordinates of the Primary Terrestrial Reference Frame.

STATION		X	m_x	Y [m]	m_y	Z	m_z
S010	CANBERRA	-4460935.076	.022	2682765.771	.021	-3674381.711	.021
S010	MADRID-R	4849336.797	.019	-360488.893	.019	4114748.621	.020
S014	GOLDSTON	-2351129.050	.015	-4655477.095	.015	3660956.884	.015
M102	WASHINGT	1130686.678	.011	-4831353.025	.011	3994110.875	.011
M002	MAUI I	-5465998.456	.012	-2404408.544	.012	2242228.403	.013
S001	JOHANNES	5085442.829	.024	2668263.391	.025	-2768697.273	.024
S002	WESTFORD	1492404.925	.012	-4457266.488	.012	4296881.692	.013
S003	FORT DAV	-1324210.866	.010	-5332023.144	.011	3232118.375	.011
S001	KASHIMA	-3997892.273	.020	3276581.312	.019	3724118.288	.022
S001	KAUAI	-5543845.987	.014	-2054564.165	.013	2387813.766	.013
S003	KWAJALEI	-6143536.502	.017	1363997.208	.017	1034707.353	.019
S001	BOLOGNE	4461370.198	.016	919596.751	.017	4449559.046	.018
M013	GOLDSTON	-2356494.019	.011	-4646607.672	.012	3668426.599	.012
S009	GOLDSTON	-2356170.906	.011	-4646755.885	.011	3668470.584	.011
M003	MONUMENT	-2386289.312	.011	-4802346.566	.012	3444883.962	.012
M004	OWENS VA	-2410421.134	.013	-4477800.428	.013	3838690.309	.013
M002	PLATTEVI	-1240708.017	.013	-4720454.337	.013	4094481.633	.013
S001	RICHMOND	961258.174	.013	-5674090.035	.013	2740533.732	.014
S009	SHANGHAI	-2831686.666	.018	4675733.885	.018	3275327.806	.020
S004	WETTZELL	4075540.093	.014	931735.168	.014	4801629.244	.014

Table 3. cont.

	STATION	X	m_x	Y [m]	m_y	Z	m_z
M001	YUMA	-2196777.811	.018	-4887337.065	.018	3448425.231	.018
S001	MADRID-R	4849092.729	.067	-360180.596	.068	4115108.964	.070
M003	SAN DIEG	-2428826.596	.012	-4799754.325	.015	3417267.044	.015
M006	FORT DAV	-1330020.936	.010	-5328401.851	.012	3236480.796	.011
M001	FORT DAV	-1330125.263	.010	-5328526.640	.011	3236150.244	.011
M001	YARRAGAD	-2389006.532	.020	5043329.288	.019	-3078525.383	.020
M001	WESTFORD	1492453.790	.012	-4457278.752	.012	4296815.886	.013
M001	KWAJALEI	-6143447.268	.018	1364700.185	.017	1034163.112	.019
M001	SAMOA	-6100045.855	.029	-996203.144	.029	-1568976.327	.029
M002	EASTER I	-1884984.374	.021	-5357608.169	.020	-2892853.406	.020
M105	WASHINGT	1130719.790	.011	-4831350.573	.011	3994106.477	.011
M002	QUINCY	-2517234.690	.013	-4198556.250	.013	4076569.654	.013
M001	MONUMENT	-2386277.917	.011	-4802354.367	.012	3444881.438	.012
M001	PLATTEVI	-1240678.122	.013	-4720463.373	.013	4094480.608	.013
M001	OWENS VA	-2410422.357	.013	-4477802.689	.013	3838686.692	.013
M002	GOLDSTON	-2350861.547	.015	-4655546.275	.015	3660997.827	.015
M003	HUAHINE	-5345865.399	.025	-2958246.717	.025	-1824623.889	.025
M001	MAUI I	-5466006.470	.012	-2404427.953	.012	2242187.475	.013
M002	RICHMOND	961319.050	.013	-5674090.966	.013	2740489.520	.014
M001	SANTIAGO	1769699.798	.029	-5044612.926	.030	-3468260.050	.029
M001	CERRO TO	1815517.158	.029	-5213464.875	.029	-3187999.408	.029
M001	ASKITES	4353444.929	.021	2082666.283	.021	4156506.648	.021
M001	KATAVIA	4573400.091	.022	2409322.200	.021	3723881.751	.022
M002	DIONYSOS	4595216.461	.020	2039435.345	.021	3912629.488	.021
M001	ROUMELLI	4728694.704	.018	2174373.372	.018	3674572.923	.018
M002	CAGLIARI	4893398.119	.019	772673.316	.019	4004140.939	.020
M002	BOLOGNE	4461399.729	.016	919566.848	.017	4449510.490	.018
S001	METSAHOV	2892595.614	.024	1311807.759	.024	5512610.719	.024
S001	ZIMMERWA	4331283.630	.017	567549.537	.017	4633139.941	.017
S002	WETTZELL	4075530.090	.013	931781.316	.013	4801618.172	.014
S001	CERGA GR	4581691.822	.015	556159.394	.015	4389359.383	.015
S001	SHANGHAI	-2831087.790	.019	4676203.485	.018	3275172.826	.021
S001	SIMOSATO	-3822388.354	.018	3699363.497	.017	3507573.095	.019
S002	LUSTBUEH	4194426.735	.017	1162693.874	.017	4647246.544	.017
S001	HERSTMON	4033463.849	.017	23662.354	.017	4924305.017	.017
S007	CANBERRA	-4446476.781	.018	2678127.080	.018	-3696251.774	.018
M002	FLAGSTAF	-1923976.627	.018	-4850871.713	.018	3658574.856	.018
M001	PASADENA	-2493211.862	.018	-4655229.531	.018	3565574.503	.018
S001	AREQUIPA	1942792.008	.018	-5804077.671	.018	-1796919.305	.018
S001	MATERA	4641965.108	.017	1393069.947	.017	4133262.218	.017
S003	CANBERRA	-4447548.616	.018	2677133.980	.018	-3694996.475	.019

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Table 3. cont.

	STATION	X	m_x	Y	m_y	Z	m_z
				[m]			
S002	FORT DAV	-1330781.240	.010	-5328755.585	.011	3235697.676	.011
S005	MAUI I	-5466006.954	.012	-2404428.165	.012	2242188.478	.013
S002	CERGA GR	4581692.400	.015	556195.867	.015	4389354.945	.016

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