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A family of non-invertible prime links: Corrigendum

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Wilbur Whitten has pointed out to me that there exist invertible links amongst my family of allegedly non-invertible links [3], and a letter from Neville Smythe has confirmed that the error lies in the third paragraph from the bottom of page 106. Professor Smythe also suggested that the use of F-isotopy was possibly a red herring and, accordingly, I have devised a simpler proof of the existence of infinitely many non-invertible prime links in S^3 (although Wilbur Whitten's paper [4] makes such a proof redundant).

Let k_1 be a non-invertible pretzel knot, and let V be a tame closed regular neighbourhood of k_1. There exists a simple closed curve k_2 on BdV such that k_1 and k_2 together bound a nonsingular annulus in V ([2], p. 26). The link $k_1 \cup k_2$ is therefore of genus zero, and so is prime if it is not geometrically splittable ([1], Theorem 3). $k_1 \cup k_2$ is certainly non-invertible.

Suppose that $k_1 \cup k_2$ was splittable, and let $\mathcal{C}$ be the class of all 3-cells in $S^3 - k_1$ which contain k_2 in their interiors. There exists a 3-cell $C \in \mathcal{C}$ such that

(i) Bd$C \cap$ BdV consists of finitely many simple closed curves which are not tangent curves, and

(ii) the number of curves of Bd$C \cap$ BdV is minimal for all 3-cells in $\mathcal{C}$.

Suppose some intersection curve bounds a disc D^* on BdV. Then D^* contains a subdisc D which is bounded by an intersection curve α, and

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which contains no intersection curves in its interior. Because k_2 lies in $\text{Int}C$, $k_2 \cap \text{Bd}D = \emptyset$; because k_2 lies on $\text{Bd}V$ but is not contained in any disc on $\text{Bd}V$ $(H_1(V)$ is generated by $k_2)$, it follows that $k_2 \subset \text{Bd}V - D$. Then if $D \subset C$, one component, C' say, of

$$C - \{\text{an open regular neighbourhood of } D\}$$

contains k_2 in its interior and lies in $S^3 - k_1$; $C' \in \mathsf{C}$. If $D \subset \text{Cl}(S^3-C)$, let $S \subset S^3 - k_2$ be the component of the 3-cell

$$\text{Cl}(S^3-C) - \{\text{an open regular neighbourhood of } D\}$$

which contains k_1 in its interior; S is a 3-cell, so $C' = \text{Cl}(S^3-S)$ is also a 3-cell, and $C' \in \mathsf{C}$. For both choices of C' , we have

$$\text{Bd}C' \cap \text{Bd}V \subset \text{Bd}C \cap \text{Bd}V - \{\alpha\} .$$

But this contradicts the minimality assumption involved in our choice of the 3-cell C in C , so no such curve α can exist; that is, no curve of $\text{Bd}C \cap \text{Bd}V$ can be nullhomologous on $\text{Bd}V$.

If $\text{Bd}C \cap \text{Bd}V$ is not empty, therefore, there exists an intersection curve α' which is not nullhomologous on $\text{Bd}V$, yet bounds a disc D' on $\text{Bd}C$ which lies in $\text{Cl}(S^3-V)$ and contains no intersection curves in its interior. This is impossible because $\text{Bd}V$ is incompressible in $S^3 - k_1$, and no such α' can exist; we conclude that $\text{Bd}C \cap \text{Bd}V = \emptyset$.

Then either $\text{Bd}C \subset \text{Int}V$, or $\text{Bd}C \subset (S^3-V)$. In the first case, $\text{Cl}(S^3-C)$ is a 3-cell (by Alexander's Theorem) which contains k_1 in its interior and itself lies in the interior of V - this is impossible because k_1 has order one in V . In the second case, both k_1 and k_2 lie in the component of $S^3 - \text{Bd}C$ which contains V , so $\text{Bd}C$ cannot split k_2 from k_1 , as assumed.

It follows that $k_1 \cup k_2$ is not geometrically splittable, and so is a prime link. Since there are infinitely many non-invertible pretzel knots, we may obtain infinitely many non-invertible prime links in this way.

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[3] James M. McPherson, "A family of non-invertible prime links", *Bull. Austral. Math. Soc.* 4 (1971), 105-108.

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