The pupils should be asked to state in words, in various ways, what they have proved.

PETER COMRIE.

Similar Figures.—(1) Let A, B, C represent Aberdeen, Glasgow, Edinburgh respectively on a map of Scotland. Let D, E, F represent the same places in order on a map of Scotland on a different scale. The straight line AB represents the road from Aberdeen to Glasgow, the straight line AC represents the road from Aberdeen to Edinburgh. The angle A represents the angle between these roads. The angle D represents the same angle. Hence $\widehat{A}=\widehat{D}$. Similarly $\widehat{B}=\widehat{E}$; $\widehat{C}=\widehat{F}$. The $\triangle$s ABC, DEF are equiangular. Now suppose the scale of the first map is, say, 7″ to 1 mile, and the scale of the second, 3″ to 1 mile.

Then $$\frac{AB}{DE}=\frac{7}{3};\ \frac{BC}{EF}=\frac{7}{3};\ \frac{CA}{FD}=\frac{7}{3};\ \text{or, } \frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}.$$

Hence: *If two triangles are equiangular, the ratio of corresponding sides is the same for all; and conversely.*

(2) $$\frac{\triangle ABC}{\triangle DEF}=\frac{\textit{area of first map of Scotland}}{\textit{area of second map of Scotland}},$$

$$\text{or }=\frac{\textit{area of the first map of any county}}{\textit{area of the second map of the same county}},$$

$$\text{or }=\frac{\textit{any area on first map}}{\textit{corresponding area on second map}}.$$

Now describe on AB, DE the similarly situated squares ABPQ, DEXY. P and X represent the same place; Q and Y represent the same place: ABPQ, DEXY are corresponding areas. Hence

$$\frac{\triangle ABC}{\triangle DEF}=\frac{ABPQ}{DEXY}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{CA^2}{FD^2}.$$

The ratio of the areas of similar triangles is the square of the ratio of corresponding sides of the triangles.

(3) Let ABCDE, PQRST be corresponding polygonal "counties" on the two maps. Then

$$\frac{\triangle ABC}{\triangle PQR}=\frac{\triangle ACD}{\triangle PRS}=\frac{\triangle ADE}{\triangle PST}=\frac{\triangle ABCDE}{\triangle PQRST}=\frac{AB^2}{PQ^2}.$$

Or, *similar polygons may be divided into the same number of similar triangles, which bear to one another the same ratio as the polygons, and this ratio is the square of the ratio of corresponding sides.*

(4) Let $\triangle ABC$ have A a right angle; draw AD the perpendicular from A to BC. Then $\triangle s$ABC, ABD, ACD, being equiangular, may be regarded as three maps of the same three places in a country, on different scales. And $\triangle ABC = \triangle ABD + \triangle ACD$. Therefore any area of the map represented by ABC = sum of corresponding areas of the maps represented by ABD, ACD.

$$\therefore \quad BC^2 = AB^2 + AC^2.$$

The above are meant as illustrations of the corresponding propositions in geometry, or as the "proofs" necessary for a working knowledge of the propositions, in a preliminary course of geometry which includes similar figures.

P. PINKERTON.

Internal and External Bisectors, and an Example of Continuity.—I. To draw quickly a good figure of A, B, C, I, I_1, I_2, I_3, etc. Draw a circle, and a chord BC. Mark L the middle point of the arc below BC by the "engineer's method," viz., with B as centre and a radius as near BL as can be judged by the eye, make a mark on the arc, with C as centre and the same radius make another mark on the arc, judge by the eye the middle point of the arc between these marks; this is L. With centre L, radius LB or LC, describe a circle: I and I_1 lie on this circle. Mark M the middle point of the arc above BC. With centre M, radius MB or MC, describe a circle: I_2 and I_3 lie on this circle. Now I and I_1 lie on AL; I_2 and I_3 on AM. Mark A on the first circle, so that MA lies conveniently on the paper. The various collinearities and perpendicularities justify the figure to the eye; the properties of the mid-points of II_1, etc., I_2I_3, etc., and the loci of I, I_1, I_2, I_3 as A varies, are emphasised.