Binary Stars in Young Clusters –
a Theoretical Perspective

Pavel Kroupa
Institut für Theoretische Physik und Astrophysik der Universität Kiel
Olshausenstraße 40, D-24118 Kiel, Germany

Abstract. The preponderance of binary systems in all known stellar populations makes them exciting dynamical agents for research on topics as varied as star formation, star-cluster dynamics and the interiors of young and old stars. Today we know that the Galactic-field binary population is probably a dynamically evolved version of the Taurus–Auriga pre-main sequence population, and that the initial distributions of binary-star orbital elements are probably universal. Furthermore, N-body calculations tentatively suggest that OB stars form in energetic binaries near cluster cores, and that binaries with 'forbidden' orbital elements that are produced in stellar encounters, may turn out to be very useful windows into stellar interiors, potentially allowing tests of pre-main sequence evolution theory as well as of models of main-sequence stars.

1. Introduction

Most 'stars' in the sky are multiple systems. Observations demonstrate that the binary proportion of 'isolated' pre-main sequence stars is typically much higher than in the Galactic-field (GF) (Ghez et al. 1997; Köhler & Leinert 1998; Duchene 1999). Differences such as these contain clues about the origins of various populations. The disruption of binary systems in their stellar birth aggregates may naturally reduce the binary proportion to the GF level. This is an exciting notion, opening the possibility of learning something about the configuration of the birth aggregate typical for present-day star formation (sf).

The study of the evolution of a primordial binary population in a young cluster thus becomes important for understanding the origin of the distribution of binary-star orbital elements. Cluster dynamics with a substantial primordial binary population differs from that of single-star clusters because the number of interacting systems changes with time owing to binary disruption affecting the relaxation process, and because binary systems contain additional degrees of freedom that single stars do not. These internal degrees of freedom affect the energy exchanges that occur during close encounters, changing the energy spectrum of escaping stars and the energy budget of the whole cluster, ultimately being able to arrest core collapse in massive star clusters (e.g. Giannone & Molteni 1985; Giersz & Spurzem 2000).

This text addresses related questions, with the caveat that many important and interesting problems remain to be studied. Complementary discussions are
available in Kroupa (2000a, KI; Kroupa 2000b, KII), this contribution (KIII) concentrating primarily on the implications of stellar-dynamical interactions on the binary-star orbital characteristics and high-velocity stars.

In the following, the mass, \( m \), period, \( P \) (always in days), eccentricity, \( e \), and mass-ratio, \( q = m_2/m_1; m_2 \leq m_1 \), of a stellar system (a binary, or a single with no \( P \), \( e \), \( q \)) are collectively referred to as its dynamical properties. The primordial, or birth, distribution functions of these quantities are the initial mass function (IMF), and the initial period, eccentricity and mass-ratio distributions (IPF, IEF, IMRF, respectively). The distribution functions are written as, \( f_x \), where \( f_x \, dx \) is, for example, the number of orbits with orbital parameter \( x \) between \( x \) and \( x + dx \). The total (all periods and all primaries) binary proportion is \( \int f_{tot} = N_{bin}/(N_{bin} + N_{sing}) \), with \( N_{bin} \) being the number of binaries and \( N_{sing} \) the number of single stars in the sample.

There are two main mechanisms that evolve the dynamical properties: eigenevolution through system-internal processes that redistribute angular momentum and energy (e.g. tidal circularisation, disk–companion-star interactions, outflows), and stimulated evolution through encounters with neighbours. Eigenevolution affects short-period systems \( (P \sim 10^3 \text{ days}) \), whereas stimulated evolution affects long-period systems.

One key goal of the work reported here is to find out if the IPF, IEF and IMRF are universal, or if systematic variations with sf conditions are evident in the available data (Durisen & Sterzik 1994).

2. Origin of the Galactic Field Population

The difference between the period distribution of T Tauri (TT) binaries, \( f_p^{TT} \), and of GF late-type binaries, \( f_p^{GF} \), \( (f_p^{TT} \approx 2f_p^{GF} \text{ for } P \gtrsim 10^4 \text{ days}) \) as well as between the respective mass-ratio distributions, \( f_q^{TT} \) and \( f_q^{GF} \), \( (f_q^{TT} > f_q^{GF} \text{ for } q \lesssim 0.35, \text{ Leinert et al. 1993}) \) can readily be understood if most GF stars stem from clusters that are dynamically equivalent to a cluster consisting of \( N_{bin} = 200 \) binaries with a half-mass radius \( R_{0.5} = 0.8 \text{ pc} \) (the dominant mode cluster). This result is obtained by performing \( N \)-body calculations of a library of clusters with different properties (in this instance different \( R_{0.5} \) but the same \( N_{bin} = 200 \)) (Kroupa 1995a, K1). The approach is called inverse dynamical population synthesis (IDPS) because the typical sf structure is inferred from the dynamical properties of GF stars; dynamical population synthesis (DPS) being the construction of a synthetic GF population from a distribution of stellar birth aggregates (a future goal).

How IDPS works is demonstrated in figs. 4 and 5 of K1. The resulting dominant-mode cluster, in which the pre-main sequence period and mass-ratio distributions evolve to the observed GF distributions, is remarkably similar to the most-common embedded cluster observed in the Orion molecular cloud (Lada & Lada 1991), which is also held to be the dominant mode of sf.

This work shows that the discordant TT and GF populations can be unified; the latter being a dynamically evolved version of the former. With IDPS the typical sf structure can thus be identified if the properties of primordial binary systems are assumed to be universally similar to the Taurus–Auriga population. Conversely, adopting (retrospectively) the embedded clusters described by Lada
Figure 1. The overall model GF mass-ratio distribution \(0.1 \leq m_{1,2} \leq 1.1 \, M_\odot\) is the solid histogram, whereas the IMRF (random pairing from the GF IMF) is shown as the dashed histogram (from fig.12 in K2). The peak at \(q = 1\) results from adjustment of orbital parameters during pre-main sequence eigenevolution. Observational data from Reid & Gizis (1997) are shown as solid dots, after removing WD companions and scaling to the model. This 8 pc sample is not complete and may be biased towards \(q = 1\) systems (Henry et al. 1997). Nevertheless, the agreement between model and data is striking.

& Lada (1991) as the typical sf events, it follows that the IPF, IEF and IMRF are remarkably universal. This is supported by the period distribution of binaries in the ONC and the Pleiades; both can be viewed as being evolved versions of a Taurus–Auriga-like distribution (Kroupa, Petr & McCaughrean 1999; Kroupa, Aarseth & Hurley 2000).

A more realistic primordial binary population by Kroupa (1995b, K2), including the entire range of orbital periods and a model for eigenevolution together with many more \(N\)-body computations, shows that the GF population can be reproduced beautifully, if most stars form in the above mentioned dominant-mode cluster (e.g. Fig. 1). An elaborate study of this model GF population confirms that essentially all triple and quadruple systems in the GF must be primordial, that the dependence of the model binary proportion on spectral type reproduces the observations, and shows that the observed specific angular momentum distribution, \(f_J\), of molecular cloud cores is a smooth extension to large values of the primordial binary-system \(f_J\).

3. The Binary Star – Star Cluster Connection: Fundamentals

Mass segregation and massive sub-system: Massive systems sink towards the centre through energy equipartition, thereby gaining potential energy. This heats the cluster which expands. Once the massive stars assemble near the cluster core they interact, exchanging companions for more massive contemporaries,
ejecting the divorced partners, and being ejected themselves when one of the involved systems hardens sufficiently (for more details see KII).

**Hard and soft binaries:** Further energy exchanges involve *soft* and *hard* binaries. A soft binary has a binding energy, \(|e_b| \ll e_k\), where \(e_k\) is the average kinetic energy of systems in the cluster. For a hard binary, \(|e_b| \gg e_k\). The average energy transfer can be understood with the equipartition argument: Let the orbital velocity of the reduced particle, which conveniently describes the binary, be \(v = v_{\text{orb}}\), and \(\sigma\) be the velocity dispersion in the cluster. An encounter of a soft binary \((v_{\text{orb}} \ll \sigma)\) with another cluster system (in this gedanken experiment with the same mass as the reduced particle) leads to the reduced particle gaining kinetic energy, at the expense of the field star. This increases \(v\) and thus reduces \(|e_b|\), further increasing the cross section for additional encounters. Since the binding energy of soft binaries is usually feeble, the extra energy input through the encounter typically leaves the soft binary unbound, and the corresponding cooling of the field is negligible. Similarly, for a hard binary, \(v_{\text{orb}} \gg \sigma\), and an encounter reduces the kinetic energy of the reduced particle, thus increasing \(|e_b|\) of the binary. The field star leaves with an increased kinetic energy. Heggie (1975) and Hills (1975) provide, respectively, detailed mathematical and numerical analysis of the cross sections for these processes and arrive at the *Heggie–Hills law for stimulated evolution*, namely that in a cluster, soft binaries get softer, whereas hard binaries harden (see also Hut 1983).

Thus, disruption of soft binaries can cool the cluster, and the hardening of hard binaries heats the cluster. The details depend sensitively on the number ratio of hard to soft binaries, and, assuming universal initial dynamical properties, ultimately on the binding energy of the cluster. Cooling of a cluster was first noted in the Trapezium-Cluster computations of KPM, and it turns out that the most active cooling sources are those with \(e_k \lesssim |e_b| \lesssim 100 e_k\) (KII).

**Early quasi-equilibrium:** During the earliest phase (typically shorter than a few initial crossing times, \(t_{\text{cr}}\)), the cluster expands as a result of mass segregation aided by the immediate onset of some binary hardening (for the relative importance of both processes see KII), which causes a decay of the velocity dispersion in the expanding cluster. At the same time, the softest binaries are disrupted until the cutoff, \(e_{\text{bcut}}\), in the binary-star binding-energy distribution satisfies \(|e_{\text{bcut}}| > e_k\). At this critical ('thermal') time \(t_t\), the cluster’s evolution changes by a reduced expansion rate. More details are available in KI.

The cutoff orbital period in a cluster thus indicates the maximum concentration the cluster has ever had.

**Binary-disruption time-scale:** The time-scale for the disruption of binaries in a young cluster is \(t_{\text{cr}}\), because it takes a few global crossing times for most systems to have crossed at least once through the central, dense region. This is demonstrated in fig. 2 of KI, where \(f_{\text{bc}}(t)\) is plotted for clusters with \(N = 800, 3000, 10000\) stars but the same \(t_{\text{cr}}\). At the same time, \(f_P\) and \(f_q\) evolve through disruption of the systems with smallest binding energy (i.e. large \(P\) and small \(q\)). An important empirical example is the ONC which has an \(f_P\) that is depleted at \(P \gtrsim 10^7\) days (Scally, Clarke & McCaughrean 1999), which may be a result of binary disruption. This was predicted to be the case by Kroupa (1995c, K3), who also suggests that \(f_q\) should be significantly depleted at small mass ratios in the Trapezium Cluster. BD–BD and BD–star systems...
are disrupted efficiently because of their small $|e_b|$, so that typically $f_{BD} \approx 0.2$ after a few $t_{cr}$ (fig. 6 in Kroupa 2000c).

Should an observer catch a cluster before it is many $t_{cr}$ old, a *radially increasing binary proportion* would be evident, with a decrease in the outer cluster regions as a result of the transient halo of predominantly single stars forming as a result of the encounters near the restless core. Such a radial signature of youth persists until the cluster is well mixed, but the expanding binary-depleted halo remains (KPM). It would be a dramatic discovery if such a radial dependency of $f_{tot}(r)$ would be found in the ONC.

**Ejection of stars:** A cluster looses stars as a result of many weak two-body encounters that elevate them above the escape energy (e.g. Lee & Goodman 1995). Rare, very close fly-bys of single stars in a cluster potential can produce relatively large kinetic energy changes and thus ejection events, leaving the other star more bound to the cluster. Analytical estimates for idealised clusters (single, equal-mass stars) show that the ejection events are about 4 times less likely than losses through evaporation (Binney & Tremaine 1987).

However, in realistic young clusters that have many primordial binaries, three-body encounters (binary–‘single’, or binary–very-hard binary) are not rare, because the cross section is comparable to the semi-major axes of the binaries, which span a large range, typically up to the ’thermal cutoff’ given by the velocity dispersion in the relaxed cluster. Such events can lead to the binary hardening and the ’single’-star receding with the kinetic energy surplus gained from the hardened binary. The ejected star may be the initial companion of the binary, if the companion is less massive than the incoming system (e.g. fig. 6 in Hills 1975; Hills 1977). Conservation of linear momentum implies that the binary recoils with a corresponding velocity in the other direction to that of the ’single’ star, and if the gain in binding energy of the binary was sufficient, this binary can also leave the cluster. An observer finds two systems on opposite sides of a young cluster. Examples of such an event are probably the two early B stars lying on opposite sides of the Monoceros R2 cluster (Carpenter et al. 1997), as well as the two early B stars straddling the embedded cluster S 255-IR (Zinnecker, McCaughrean & Wilking 1993). In both cases, at least one of the B stars must be a binary.

Binary–binary interactions are also not rare in realistic clusters, and, if the encountering binaries have similar $e_b$, can lead to complex behaviour, with temporarily bound but unstable four- or three-body systems usually decaying into one binary and two single stars. After a time-lag, which is typically a few times the dynamical time of the chaotic small-N subsystem, the result can be ejections with velocities up to a few 10 km/s (Sterzik & Durisen 1998). The four-body systems usually decay in two steps, with one single star being ejected first, followed later by the other single star. Since the first ejection event is likely to be less energetic than the second, which includes an already hardened system, and is not likely to be in the same direction as the first ejection event because the system is chaotic, the observer may find three systems receding away from the cluster with different velocities, with at least one being a binary. Since the second decay is likely to release more energy, the alignment would be such that the two more massive (i.e. brighter) ’stars’ (one being the hardened binary) lie nearly co-linearly on opposite sides of the cluster, and a third, less massive

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system (from the first ejection event) typically lying closer to the cluster but at some angle relative to the other two. An example of this may be the possible multiple ejection event that lead to the run-away stars AE Aurigae, µ Columbae and the eccentric binary ι Orionis (Hoogerwerf, de Bruijne & de Zeeuw 2000). An example of an extreme runaway 5\(M_\odot\) star is HIP 60350, which can also be traced back to a cluster (Tenjes et al. 2000).

Run-aways can, however, also be produced as sling-shot events in supernova explosions (Portegies Zwart 2000a), so that it is important to build-up statistics from \(N\)-body calculations to seek possible specific characteristics of run-aways produced by both mechanisms (see also Leonard 1995).

The kinematical and dynamical properties of run-aways can be used to constrain the binary populations and morphological properties of the parent clusters (Clarke & Pringle 1992). Thus, binary-rich clusters produce significantly more high kinetic-energy stars than single-star clusters (fig. 5 in K3). Initially more concentrated clusters also eject more high-energy stars. The highest velocities achieved in these clusters \((N_{\text{bin}} = 200\) or \(N_{\text{sing}} = 400\)) are 40 km/s, but rare, star-grazing encounters can expel stars at a few 100 km/s (Leonard 1991). There are well-defined correlations between binary proportion, \(P\) and \(m_{\text{sys}}\) with ejection velocity. This demonstrates that the dynamical properties retain a memory of the dynamical events, which will be useful when interpreting the distribution of young stars around sf regions on a statistical basis (Leonard & Duncan 1990; Sterzik & Durisen 1998; Kroupa 1998). Such data will become available with the upcoming astrometry satellites DIVA (Röser 1999) and GAIA (Gilmore et al. 1998).

Massive stars are also ejected, mostly through violent interactions in the cluster core. The velocities of pre-supernova dynamically ejected stars from the cluster models of KI (central density as in the ONC, no initial mass segregation and random pairing from the IMF) are displayed in Figs. 2 and 3. These figures demonstrate that the binary proportion among the ejected stars is very low, as expected, and that stars with \(m \gtrsim 5M_\odot\) do not achieve \(v \gtrsim 40\) km/s in any of the models by 3.5 Myr. Also evident is that the \(N = 10000\) cluster ejects no massive stars within 3.5 Myr, and a small number of low-mass stars. This is a result of the larger number of massive stars in this model, the massive-star sub-population forming the core thus having a longer relaxation time (i.e. being 'less collisional'). That the primordial binary population has a significant effect on the ejection rate is also apparent, the \(f_{\text{tot}} = 1\) clusters having a K2-period-distribution, whereas the \(f_{\text{tot}} = 0.6\) models have a log-normal period distribution, and thus a larger binding energy per binary. Barely any ejections occur if \(f_{\text{tot}} = 0\).

One immediate but still preliminary interpretation of these results is that OB stars probably must form in energetic binaries (small \(P\) and large \(q \lesssim 1\), and probably already near the centre of their parent cluster, to explain the observed high-velocity OB run-aways, thus tentatively supporting the formation scenario of Bonnell, Bate & Zinnecker (1998).

At later times (5–50 Myr), fig. 6 in KI shows that between 10 and 50 per cent of all surviving OB stars lie at distances larger than 2\(R_{\text{tid}}\) from their parent cluster, which has a tidal radius \(R_{\text{tid}}\) (this includes companions flung out when their primary explodes).
Figure 2. Velocities of ejected massive stars at \( t = 3.5 \) Myr for the clusters discussed in KI (\( N = 800, 3000 \) and \( 10000 \), from top to bottom, with \( f_{\text{tot}} = 1, 0.6 \) and 0, from left to right; note that all clusters here have the same IMF). Triangles indicate binaries. The top three panels contain data from \( N_{\text{run}} = 10 \) \( N \)-body calculations, for the middle three panels \( N_{\text{run}} = 5 \), and for the bottom panel \( N_{\text{run}} = 2 \). Thus, the number of points in the bottom panel should be divided by two to mentally normalise the 7 data sets. The vertical dotted lines indicate the escape velocity, \( v_{\text{esc}} \), at \( t = 0 \) from the cluster centre. The first supernova explodes at \( t \approx 4.5 \) Myr.

Figure 3. As Fig. 2, but for low-mass stars. The horizontal dotted lines indicate the brown dwarfs.
A binary involved in an energetic interaction, in which a companion may be exchanged, usually ends up hardened and having a large $e$, and possibly a fast space motion. If $e$ is so large that the peri-astron separation, $R_{\text{peri}}$, is smaller than the sum of the stellar radii, $R$, then the stars merge to form a rejuvenated, and if massive, possibly an exotic star that may have been ejected from its cluster (Leonard 1995; Portegies Zwart 2000b). If the eccentricity is not large enough for immediate physical collision, but $R_{\text{peri}}$ is a few times $R$ but smaller than some critical value $R_{\text{coll}}$, then the binary orbit will tidally circularise rapidly, the stars merging by the time the system is observed. If $R > R_{\text{coll}}$ then the binary will circularise without merging within the age of the system. If the system hardens or softens while circularising depends on the angular momentum transfer between the stellar spins and orbit.

Such eccentric systems can appear in the $e - \log_{10} P$ diagram in a region usually avoided by binaries as a result of eigenevolution (short $P$ and large $e$), and are thus referred to as forbidden orbits (K2, fig. 5). A few candidate systems may exist (see K2), and one particularly interesting example is G1 586A, which has $P = 890$ days and $e = 0.975$. It is discussed at length by Goldman & Mazeh (1994) as a prime example of how to test various tidal circularisation theories. The rate of production of such orbits depends on the number of stars in the cluster and on its concentration, and is therefore greatest during the first few crossing times. Fig. 3 in KI shows the average number of forbidden orbits per cluster as a function of time, and Fig. 4 plots the corresponding $e - \log_{10} P$ diagrams. Note that not all forbidden orbits have large centre-of-mass velocities.

Tidal circularisation depends on the coupling between the velocity gradient (shear) in the stellar envelope due to the tide, and the turbulent viscosity, and thus on the internal constitution of the star. It is very efficient for fully convective stars, because the depth of the convection zone is essentially the handle that connects the outer regions of the star with its interior, but the theories for tidal circularisation are very uncertain.

Zahn & Bouchet (1989) show that most of the circularisation of bloated fully convective pre-main sequence binaries occurs within the Hayashi phase ($10^5 - 10^6$ yr). A longer evolution time-scale obtains for older systems, and the longest period which is circular may be useful as a clock to infer the age (or inversely the circularisation time-scale if the age is known) of the system (see various contributions in Duquennoy & Mayor 1992). Goldman & Mazeh (1994) study the circularisation of binaries with initially $e \approx 1$, and find that the time-scales for changes in semi-major axis, $a$, and $e$, are $\tau_a \equiv a/a \propto P^{16/3} (1 - e)^{15/2}$ and $\tau_e \equiv e/\dot{e} \propto P^{16/3} (1 - e)^{13/2}$. For $e = 0.5, 0.8$, $(1 - e)^{15/2} = 0.0055, 5.7 \times 10^{-6}$ and $(1 - e)^{13/2} = 0.011, 2.9 \times 10^{-5}$, respectively, demonstrating that the time-scales are likely to be very short for 'forbidden systems' with large $e$.

Since forbidden systems have a short period ($P \lesssim 10^3$ days), changes in $e$ and $P$ may be directly observable over a few to many periods, in which case tidal circularisation theory may be testable if the odd forbidden binary can be found that experienced an encounter recently. It may be worth re-checking the orbital parameters of, for example, EK Cep and P2486 (table A2 in Mathieu 1994), and of the high-proper-motion system G253-44, which has $P = 19.38$ days, $e = 0.52$
Figure 4. Eccentricity–period diagram at $t = 3.5$ Myr for the clusters with $f_{\text{tot}} = 1.0$ and 0.6 shown in Figs. 2 and 3 (no binaries form in the displayed region in the clusters with $f_{\text{tot}} = 0$). Open circles have a system mass $m \geq 8 M_\odot$, filled circles have $m < 8 M_\odot$; crosses show systems with velocities $v > v_{\text{esc}} = 4.3, 6.6, 9.8$ km/sec (top to bottom panel). The long-dashed line delineates forbidden orbits (on its left) from 'normal' binaries (K2), and is taken from Duquennoy & Mayor (1991). Forbidden orbits are not eigenevolved once they are produced (cf. fig. 5 in K2).

(Mazeh, Mayor & Latham 1997). Concerning the ages of forbidden systems such as above mentioned Gl 586A, it is worth keeping in mind that the time the system had for circularisation is not the nuclear age of the stars, since the encounter that produced the large $e$ may have happened long after the birth of the stars or system.

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