STUDIES IN GLACIER PHYSICS ON THE PENNY ICE CAP, BAFFIN ISLAND, 1953

Part III: SEISMIC SOUNDING

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Abstract. Firn and ice thickness measurements were carried out by seismic refraction and reflection methods on a flat col of the highland snowfields of the Penny Ice Cap and on a medium-sized valley glacier (Highway Glacier). The longitudinal wave velocities were found to vary from some 1000 m./sec. (3280 ft./sec.) in firn to 3810 m./sec. (12,500 ft./sec.) in ice and approximately 6000 m./sec. (20,000 ft./sec.) in the bedrock (gneiss). The thickness of firn and ice at the firn col was found to be 254 m. (834 ft.). On Highway Glacier some 80 reflections were evaluated, giving position, dip and strike of the bedrock surface. A longitudinal profile of Highway Glacier from the junction of three main tributary glaciers to the tongue is given; the ice thickness slowly decreases. At the junction, the bedrock is 400 m. (1310 ft.) deep, there is no deep basin as might be expected from the surface features. The mean slope of the glacier surface is about 3° of arc and of the bed about 1°.

Zusammenfassung. Mit Hilfe der Refraktions- und Reflexionsmethode sind Eisdickenbestimmungen auf dem "Penny Ice-Cap" durchgeführt worden, die sich auf einen flachen Firnpass im Hochland und einen Talgletscher mittlerer Grösse (Highway Glacier) erstrecken. Die Ausbreitungsgeschwindigkeiten der longitudinalen Wellen nahmen von 1000 m/sec in oberflächennahem Firn bis auf 3810 m/sec in Eis zu und betrugen im Fels (Gneis) ungefähr 6000 m/sec. Die Dicke von Firn und Eis im Gebiete der Passhöhe betrug 254 m. Auf dem Talgletscher wurden in Bezug auf Position, Streichen und Fallen der Felsoberfläche ca. 80 Reflexionen ausgewertet. Vom Vereinigungspunkt der drei wichtigsten Gletscherarme bis zur Zunge kann ein Längsprofil mitgeteilt werden, das von 400 m Dicke an langsam abnehmende Gletschermächtigkeiten erkennen lässt. Beim Hauptvereinigungspunkt der Gletscher war keine wesentliche Übertiefung festzustellen, wie sie aus den örtlichen Verhältnissen eigentlich hätte erwartet werden können. Die mittlere Oberflächenneigung des Gletschers betrug ca. 3°, die der Sohle nur ca. 1°.

Fig. 1. Highway Glacier from summit of Mt. Battle, looking north

Photograph by W. H. Ward

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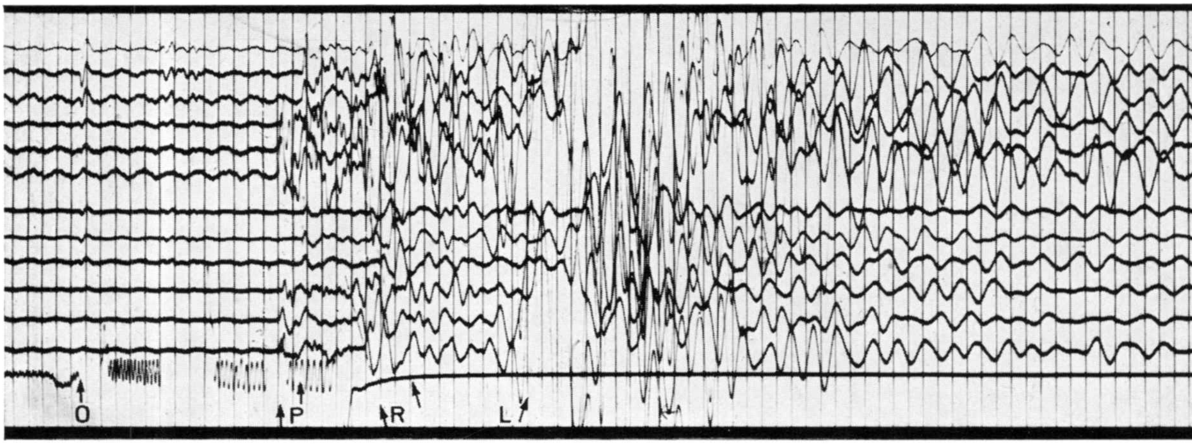

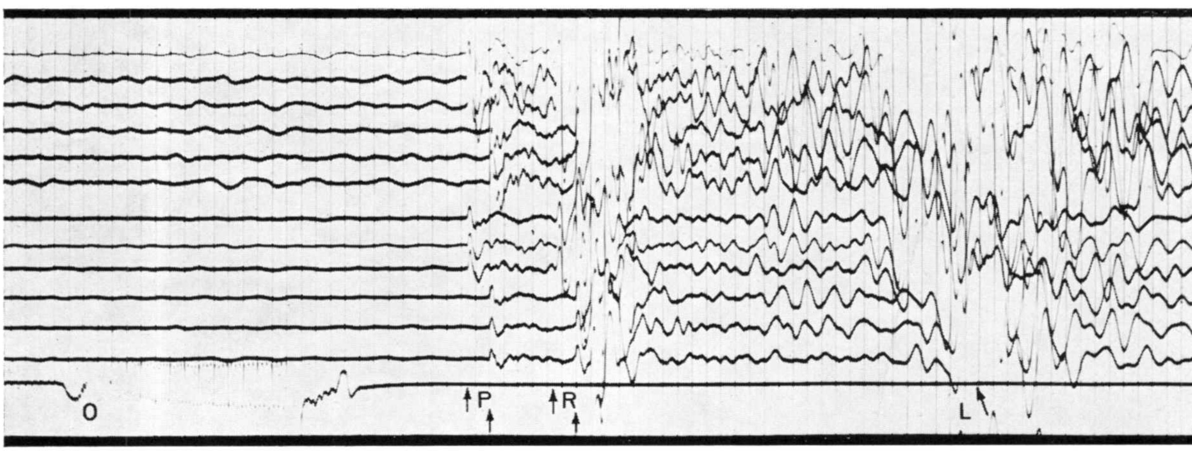

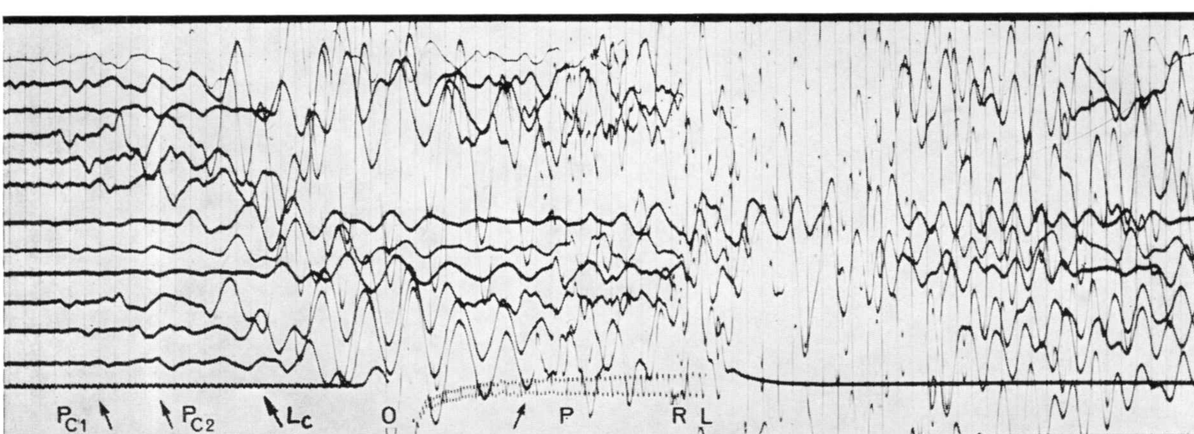

Fig. 4a (*top*). *Typical record of reflection shooting. Vertical timing lines 1/100 sec. apart. O = shot instant, P = direct longitudinal wave, R = reflected longitudinal wave, L = surface wave (see p. 544)*
Fig. 4b (*centre*). *Reflection record showing R with much higher intensity than P*
Fig. 9 (*bottom*). *Record thought to show the movement of a crevasse (see p. 546)*

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Scope of the Seismic Work

In some recent investigations of polar ice caps[1, 2, 3] seismic soundings and gravimetric surveys provided the necessary profiles for calculating the total mass of ice. A preliminary study of the air-photographs of the Penny Ice Cap (Fig. 4 of reference [4]) made it clear that in the present case thousands of soundings all over the ice cap would be necessary for even a very rough estimate of the mean thickness of the ice, an enterprise quite out of the question for a small group working without the aid of mechanical transport. Therefore soundings could be made only in a very limited region, which might have been chosen almost anywhere, as the ice thicknesses in any one part of the ice cap (the centre for instance) did not appear to be very different from another and were highly variable everywhere. Therefore it was decided to make soundings on a medium-sized glacier (Highway Glacier) between the base camp and the meteorological-glaciological Camp A1 (Fig. 1, p. 539; see also Fig. 3 of reference [4]) and on the adjacent firn fields at A2, a region that suited the general plans of the expedition. In this region a few special problems could be studied which resulted from the investigations of other expedition members or from the morphological features of the glacier. At A2, 1920 m. (6300 ft.), a refraction survey of velocities in the firn layers down to more or less dense ice was carried out in connection with the glaciological work of Ward and Baird[4] (Part I of this series of papers). This refraction work gave also an estimate of the ice thickness in the very flat firn pass where A2 was situated. The presence of a remarkably flat "square" on Highway Glacier at the junction of three major glaciers (called "Concordia Platz") raised the problem as to whether the rock bed would form a basin as at the original Concordia Platz on the Great Aletsch Glacier, or a more or less plain slope. The resolution of this limited problem took as much time, or even more, than the rest of the reflection soundings carried out on the lower parts of Highway Glacier. There special attention was given to a cross-section where Ward took measurements of the rate of movement of several surface points. All the soundings between "Concordia Platz" (A3) and the tongue of Highway Glacier enabled several cross-sections and a longitudinal section of some 12 km. (7·5 miles) of the glacier to be constructed and revealed geomorphological details in the neighbourhood of Pangnirtung Pass, which was studied by the geomorphologist Thompson.[5, 6]

General Organization and Equipment

Seismic work started at the most elevated position at A2, where men and equipment were flown by a Norseman aircraft. The soundings were carried out from 27 May until 9 August with a break during the melting season from 7 July until 29 July. Working in groups of two to four men, 140 man-days were spent on transportation, surveying and seismic shooting. The latter took half of the time, 28 per cent were taken by transportation, 12 per cent by surveying and 10 per cent to establish the equipment and for repairs, etc. These figures refer to the number of man-days, but it should be mentioned that the days of transportation were sometimes far longer and more strenuous than the others.

Transportation on snow and bare ice was done with different sorts of sledges, across the moraines from the tongue of Highway Glacier to base camp by back-packing, and along the shore of Glacier Lake by towing in a rubber boat. The equipment was rather heavy, originally most of it had been mounted on lorries and used for oilfield work and the total packed weight was some 320 kg. (700 lb.). For operation it had to be unloaded and erected inside a tent and this made it advisable to operate from as few stations as possible. The equipment consisted of 6 geophones, 6 amplifiers with high level and low level output, a 16-channel recording camera (12 of the galvanometers were connected to the amplifier outputs, one recorded the time break and 3 were kept in reserve), and the blasting equipment with telephones. It was supplied to the expedition by Magnolia Petroleum Company and worked excellently. The power was taken from four motor-cycle batteries, which were charged by a very light 400-watt generator (DKW Type GG 400). The surveying was done with a Wild theodolite, Type T O.

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REFRACTION METHOD

A number of longitudinal wave velocities were determined by refraction profiling, partly with a number of geophones in line, but mostly by moving the shot points along the profile. This gave interesting values at the higher elevations of A2, *i.e.* in firn. The time-distance curve is shown in Fig. 2, p. 543. From the velocity-distance relation, the graph of velocity versus depth given in Fig. 3, p. 543 has been calculated by means of the formula

$$z(v)=\frac{1}{\pi}\int_{0}^{\Delta(v)}\cosh^{-1}\frac{v}{c(\Delta)}d\Delta \qquad (1)$$

with z=depth, v=horizontal velocities (variable with depth), Δ=variable distance along the profile, $\Delta\,(v)$=distance where the velocity $\frac{d\Delta}{dt}=c\,(\Delta)$ takes the value v, t= travel time. The velocity curve from the centre of the Greenland Ice Cap, given by Holtzscherer[2, 7] is reproduced on the same figure.

The two curves agree very well in shape. At the shallower depths the higher velocities on the Penny Ice Cap can easily be understood because of the regular occurrence of surface melting and the faster compaction at higher subsurface temperatures (the temperature at the depth of zero seasonal change determined by Ward[9] at Camp A1 was $-13{\cdot}3^{\circ}$ C.). Further down, the Greenland velocities are higher due to the much lower temperature. Holtzscherer[2] has compared the French velocity measurements with those of Brockamp and has found good agreement. The value of 3810 m./sec. found at depth at A2 again agrees well with the Greenland values. The same good agreement was found on bare ice further down the glacier, namely 3720 m./sec. on the main part of Highway Glacier (temperature at depth of zero seasonal change about $-5{\cdot}8^{\circ}$ C.) and 3700 m./sec. on the tongue (temp. about $-5{\cdot}5^{\circ}$ C.). The bedrock at A2 is probably a granitic gneiss rich in feldspar, the most common rock in the vicinity, and the rather high velocity of 6000 ± 300 m./sec. was found.

The refraction method revealed a few values of depth. At A2 at the depth of 12·50 m. (41 ft.) an ice-layer of unknown thickness with the velocity of 3760 m./sec. (dense ice) could be found. It is not possible to correlate this ice layer with the data from the boring at A1[4], because the accumulation is much less there. It may represent a summer with exceptional melting. The bedrock showed up perfectly well, sloping northwards very slightly (about 2·8° of arc) at a place where the firn surface was sloping southwards from the flat col near the site of A2.* The bedrock surface has been calculated at 254 m.$\pm$12 m. below the snow surface. In spite of the curved trajectories assumed in the velocity survey, the calculation of the depth to bedrock was made with four different layers of firn with four different velocities. These velocities versus depth are given with the velocity curve in Fig. 3.

At the tongue the refraction method again proved to be of use for depth determination. The profile here existed over a sufficiently long distance only in one direction (the geophones were set up for reflection work) and consequently the two unknowns, velocity and slope, cannot be determined. But as the observed velocity 6100 m./sec. (terrain correction is applied already) lies sufficiently close to the value obtained for the rock at A2, it was assumed to be the true velocity in the rock and this implies a horizontal rockbed. The computed depth will be discussed later with the reflection results.

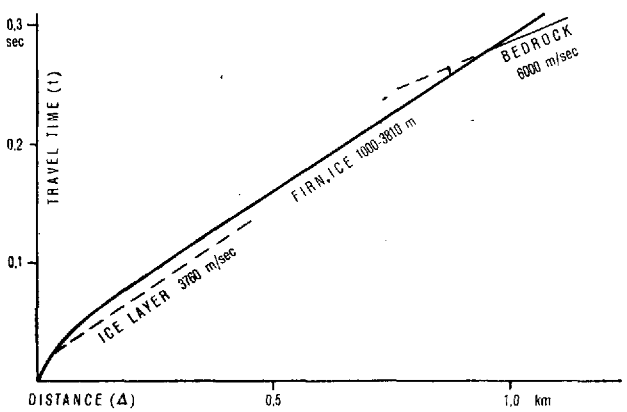
* Two profiles of shot points, extending northward and southward respectively, were laid out, but only the southern profile—up dip shooting—showed the segment in the travel time curve which corresponds to the bedrock. Nevertheless a fairly accurate calculation of the velocity in bedrock, and depth and dip of the ice-bedrock interface was possible. The spread of the 6 geophones in line was large enough to give an accurate down dip shooting segment when the most distant shots were fired. With a number of geophones at one end of the profile and a number of shots with similar spread at the other, both segments, up dip and down dip, can be obtained from the same stretch of bedrock.

Reflection Sounding

(a) *Shot-to-geophone disposition*

Reflection sounding on a valley glacier is a problem which is different in two important aspects to sounding on a thick ice cap, first the valley glacier is often shallow compared with big ice caps, and secondly the reflections cannot be expected to occur on a rock face which is parallel to the ice surface.[8]

On a shallow glacier, when the shot is fired in the proximity of the geophones, the surface waves will not have calmed down sufficiently by the time the reflected wave arrives, and the latter is not revealed on the record. The surface waves, however, have a wave velocity considerably smaller than the longitudinal wave, and if the shot is fired a sufficient distance from the geophones, the reflected wave arrives before the (much stronger) surface waves and can be detected easily. Many miles of cable have to be laid out when carrying out an extensive reflection shooting programme by this method and the amount of surface surveying of the shotpoint positions is correspondingly large.



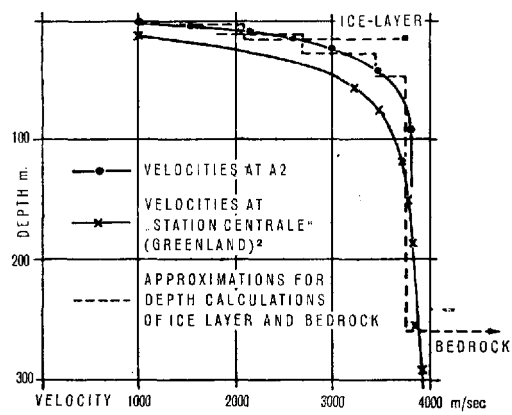


Fig. 2 (left). Travel time versus distance graph of refraction profile on firn at A2

Fig. 3 (right). Comparison between the variation of longitudinal wave velocity with depth at A2, Penny Ice Cap, altitude 1920 m., mean firn temperature about −13° C. and at the Greenland Ice Cap, altitude 2994 m., mean firn temperature −28° C. (after Holtzscherer)

Three geophones *not in line* are needed to determine the inclination of the reflecting rock face where it is not parallel to the ice surface, and if the geophones are placed at the corners of a right-angled triangle the evaluation is simpler. A numerical method for the evaluation of three-dimensional reflections is given in the appendix. The six geophones were set up near the recording tent at the corners of two equal squares with a common side, a pattern that provides ample possibilities for combinations of 3 geophones forming the apices of right-angled triangles. The side of the square was fixed at 60 m. (196 ft.). The shots were fired in line with either two or three of the geophones at distances from 450 to 1200 m. away, and this enabled an approximate evaluation of the ice depth to be made very quickly.

(b) *Shooting Technique*

When reflection sounding on Highway Glacier was started, two to three feet of snow with a density of about 0·3 grm./cm.³ covered the dense ice. Four different blasting methods were tried out: 1. Air-shooting, some three feet above the snow surface; 2. Surface-blasting on the snow; 3. Surface-blasting on the ice; 4. Blasting in shallow bore-holes in the ice. The first two methods gave remarkably strong reflections, but at frequencies too low for accurate measurements of the time of arrival of the reflection. The two latter methods gave comparable and very satisfactory records. Much more explosive was needed for surface-blasting on the ice, but as ample explosive was available it was preferable to use heavy charges than to spend time in drilling holes. Surface-blasting on the ice (with and without snow cover according to the season) was practised in most cases, and the charges ranged from ½ to 3 lb. (0·23 to 1·36 kg.). During the whole seismic work 200 lb. (90 kg.) of 60 per cent high velocity gelatin (Forcite) and 50 lb. (23 kg.) of 40 per cent Forcite were fired. A few seismocaps * were used for special investigations on wave velocities, but for most of the work ordinary short-period caps proved adequate.

(*c*) *Surveying*

At each set-up of a group of geophones two main shot-point lines were directed at right angles with the theodolite. The ice surface profiles along these lines were determined by tacheometry. This method, adequate on a more rugged glacier, was probably less accurate and slower than simple measurements of distance with a tape combined with a few vertical angles would have been. Long distances were not measured too accurately by stadia readings in every case and the error in the distance may exceed 1 per cent occasionally.† The altitudes may be erroneous by a little over 10 m., but the errors in the differences in height between neighbouring points are far smaller (30 cm. or less). The altitudes above sea level are determined from the elevations of a few points in the neighbourhood of Highway Glacier surveyed by Marmet.[9]

(*d*) *Records*

Fig. 4*a* (p. 540) is a copy of a record with very sharp reflections *R* on all six traces. The lower six give the same records as the upper set but with smaller amplification. Each group of three geophones in line which receive the direct longitudinal wave *P* at the same time do *not* mark the reflections simultaneously. This shows clearly that the reflections arrive from one side of the profile, the side where the geophones are that record the first and fourth traces. Excellent reflections were obtained from depths greater than 200 m. and in cases with focusing effects at the rock surface even as shallow as 150 m. Focusing was observed in many cases and gave reflections *R* with a much higher intensity than the direct longitudinal wave *P* (*e.g.* Fig. 4*b*). This effect can be explained partly by the sensitivity of the geophones which is greatest in the vertical direction.

(*e*) *Results*

The results of the soundings have been collected on a large scale map (Fig. 5 (p. 545) is a simplified version on a small scale) from which the block diagram in Fig. 6 (p. 547) containing the cross-sections I–V, the cross-sections in Fig. 7 (p. 545) and the longitudinal profile in Fig. 8 (p. 547) are constructed. The original map contains some 80 reflection points with strike and dip of the rock-ice interface. A satisfactory contour map of the bed of Highway Glacier however cannot be given, as there are still some large areas not covered by soundings. But parts of the cross-sections and the general line of the longitudinal section are given with high accuracy. Very often two determinations from different shot points of the same part of the reflecting rock face did not differ by more than one or two metres and very seldom exceeded 10 metres. A slight correction, less than 3 per cent, might be necessary for the whole of the results due to a slight drop in wave velocity towards the bottom of the glacier where the ice may be warmer.

The cross-sections of Fig. 7 do not call for much comment. They are of astonishingly regular U-shape, and, because of the high quality of the reflections, are believed to represent the surface of the bedrock itself. Any narrow gorges cut into the main profile would not be discovered by the reflection soundings. The longitudinal section (Fig. 8) is drawn through the deepest points of the cross-sections, which are found by interpolation and are less accurate than the measured points.

The greatest thickness of ice, which has been calculated, is 397 m. at a point close to the centre

* Caps giving the instant of explosion with higher accuracy than ordinary caps.

† It was evident from several checks that the velocity of the direct wave was sufficiently constant to calculate the distance just as accurately from the velocity times the travel time.

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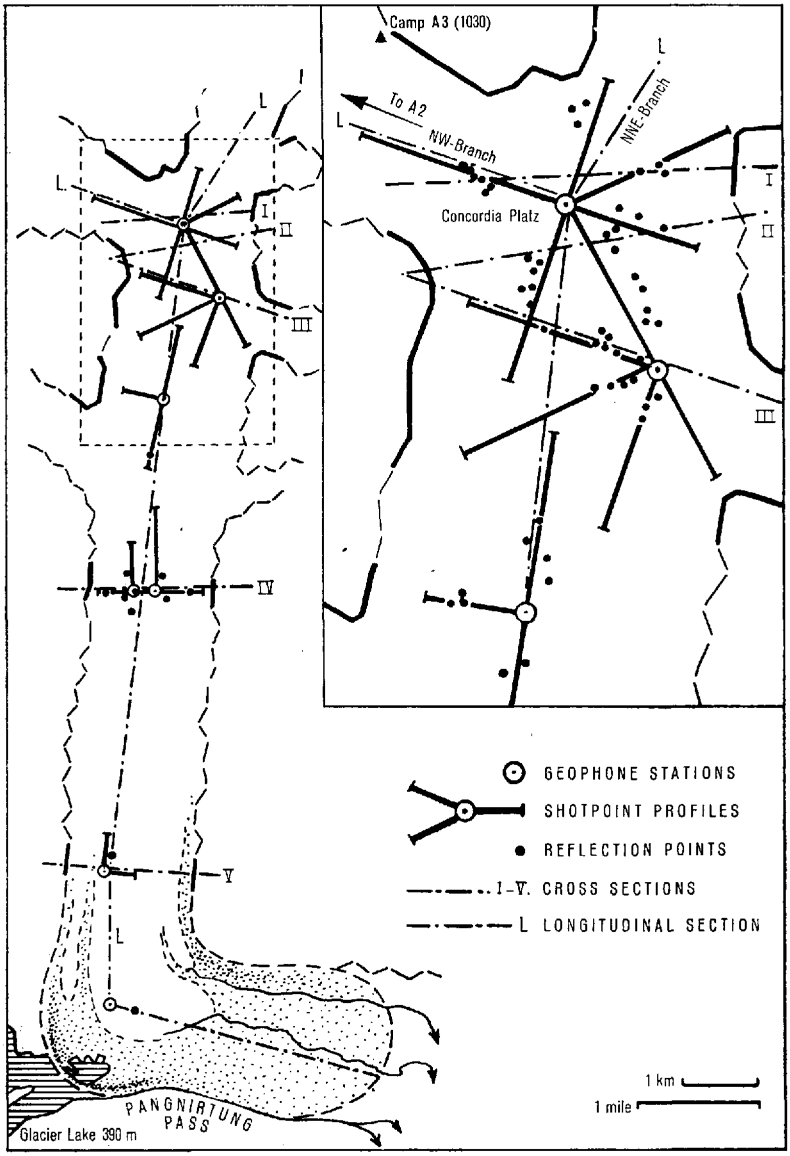

Fig. 5 (left). Sketch map of Highway Glacier with seismic profiles and some reflection points

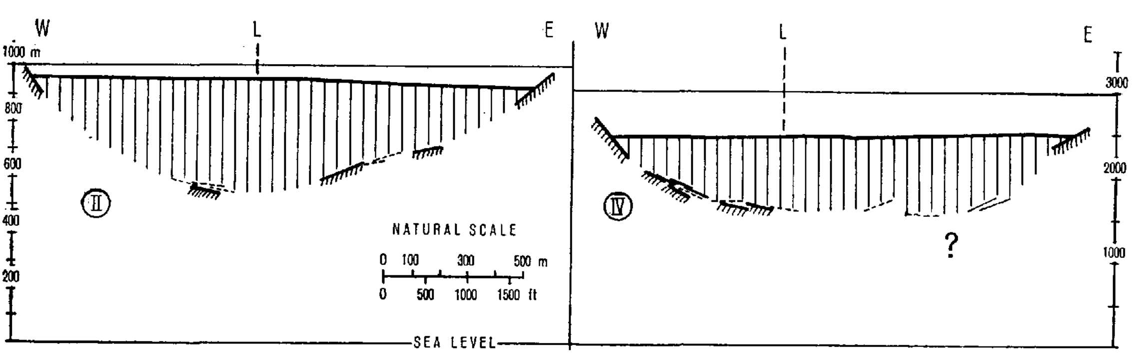

Fig. 7 (below). Cross-sections II and IV. Full lines are calculated points less than 50 m. north and south of section, dashed lines are points more than 50 m. away projected along the strike lines onto the section. The doubtful results on the east side of Section IV are due to poor records of "Concordia." This cannot be the maximum thickness however, as the bedrock slopes 9°–10° at this point. But from the generally flat character of the bedrock surface at "Concordia" the deepest point must lie nearby at a depth of less than 440 m.

It is difficult to state whether a slight basin is present or not at "Concordia". The longitudinal profile is drawn perfectly flat for more than one mile, but it could descend from south to north for probably some 30 m., to form a basin of that depth, 1 km. long and not more than 400 m. wide. This is a very shallow basin compared with the present and ancient basins formed by glaciers in the Alps. It is curious that no deep rock basin exists at the bottom of "Concordia Platz" as one might anticipate from surface features. There is no doubt that "Concordia Platz" lies at the junction of major structural trends where the underlying bedrock might be expected to be shattered and vulnerable to glacial erosion.

The slope of the glacier bed downstream of "Concordia" is fairly uniform and of the order of 1° of arc (=1·75 per cent) for some 8 km., compared with 3° at the glacier surface. The glacier steepens towards its outlet into the Pangnirtung Pass, but no reflection survey was made there. One single record with reflections difficult to detect was obtained in the region of the tongue, giving the ice-thickness of 150 m. This result however was checked nearby by the refraction method, which gave a depth of 177·5 m. The difference in depth might be explained by inaccuracy, a sloping rock surface, or a layer of loose deposits between the ice and the rock. The existence of this layer seems possible for several reasons. It is very likely that a fairly thick layer of loose material was spread out across Pangnirtung Pass when Highway Glacier pushed out into that valley. The seismic results suggest the same conclusion. The poor reflections and a definite loss of energy of the refracted wave could be accounted for by a boundary between ice and a loose deposit (probably frozen). In one case the first arrival due to the refracted wave was not discovered before its existence became evident from more distant shots! The existence of a layer of loose material (gravel, sand, moraine) would alter depth of the bedrock surface given above. The velocity in the "gravel" must be known in order to recalculate the refraction observations. Above the freezing point the velocity certainly would be less than the velocity in ice, and in this case, which is not likely, the total depth would be less. If the ice-gravel interface is determined by the reflection data at 150 m., then the thickness of the loose deposit would be less than 177·5−150·0=27·5 m. and the total depth could not be very different. It is almost certain however that the ground beneath the glacier is frozen, and the velocity in this case is likely to be greater than in ice.[2] If the velocity is assumed to be 4800 m./sec. as found by Holtzscherer in Greenland (for the present case a more or less arbitrary value), the depth of the bedrock surface is calculated now at 193 m. below the ice surface, and the thickness of the frozen "gravel" is 43 m. In this case the records of the medial layer would be lost amongst the traces of the first arrivals and it is impossible from the records to prove directly the presence of such a layer.

Local "Earthquakes" from Moving Crevasses

In two occasions the records showed movement of the ground some tenth of a second before the dynamite was blasted. It is believed that the opening of crevasses was the reason. Fig. 9 (p. 540) gives one of the records, showing two successive deflections with the arrival of the direct (P_{c1} and P_{c2}) and the surface waves (L_c). The deflections due to the blast (P, R, L) follow later.

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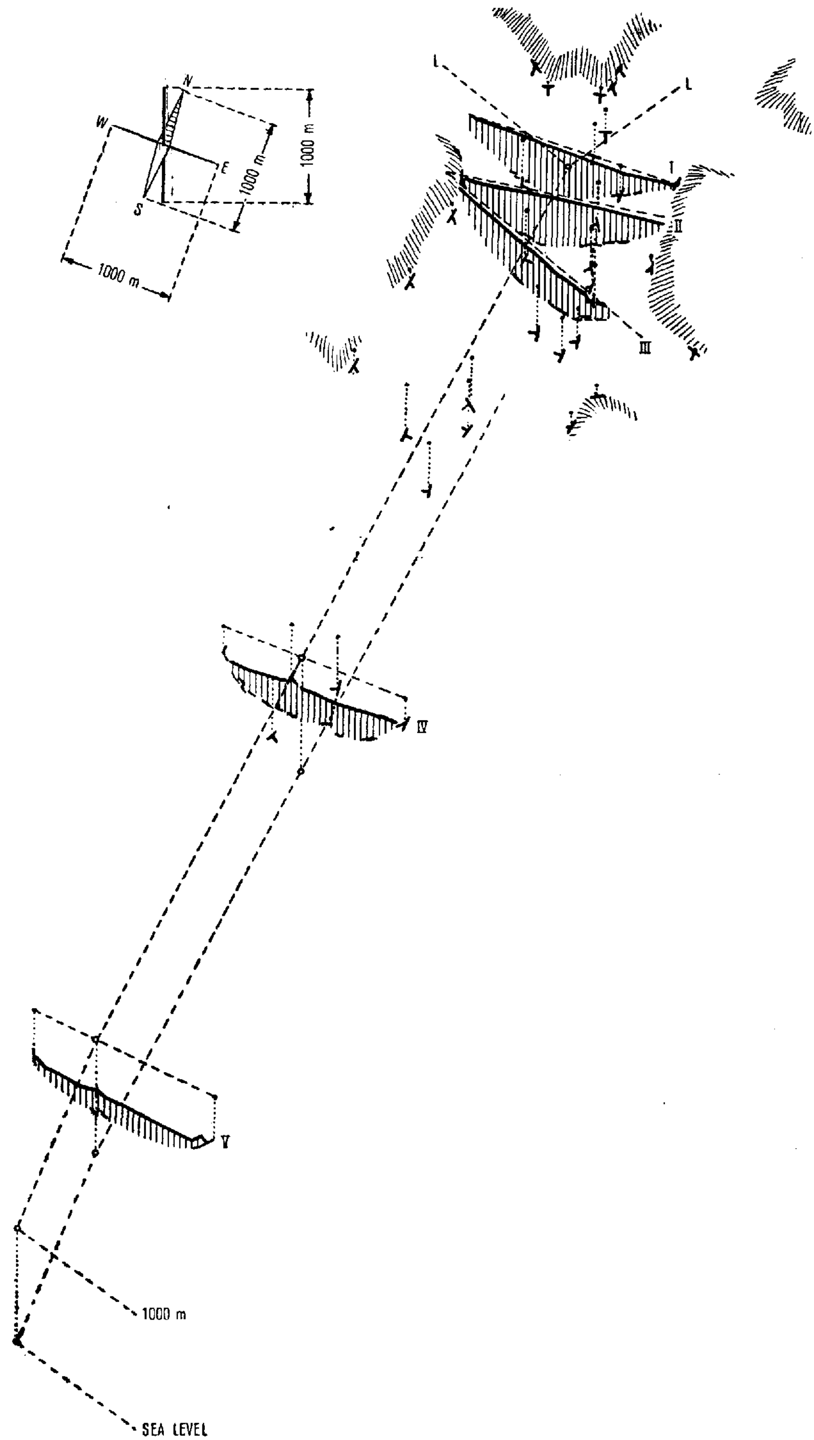

Fig. 6. Block diagram of Highway Glacier with cross-sections I–V and a few scattered reflection points, showing the depth from the 1000 m. level, the strike and the dip of the bedrock surface

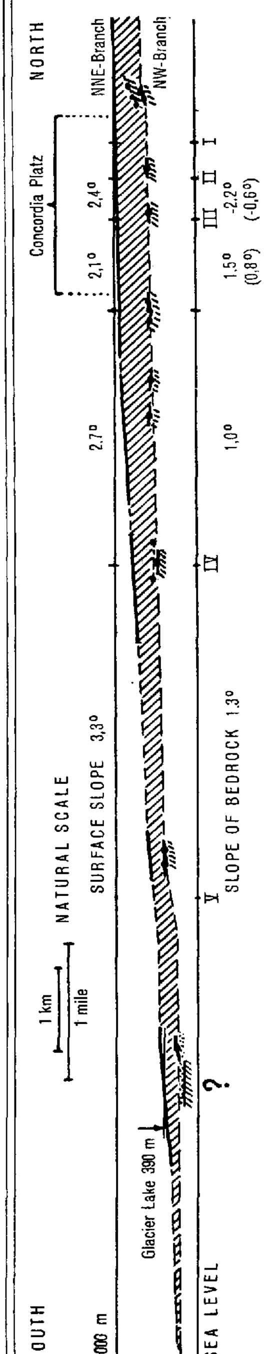

Fig. 8. Longitudinal section through Highway Glacier

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APPENDIX

METHOD OF COMPUTING DIP AND LOCATION OF THE REFLECTING ROCK SURFACE

In cases where the glacier surface and the rock bed are plane and parallel, only one geophone and one shot are necessary to determine the ice thickness. More geophones and shots can be used to improve the results. But on a valley glacier where the reflection may come from any direction, records on at least three geophones not in line are necessary to determine the position and orientation of the reflecting rock surface. The following method is based on the assumption that the reflections come from a plane rock surface. To meet this assumption as nearly as possible on an irregular bed the reflecting element should be small; that is to say the geophones must be kept close together. But some distance between the geophones is necessary to give measurable time differences between the reflections.

The ray paths of the reflected waves to all geophones intersect in an imaginary point which is the mirror image of the shot point in the reflecting plane. The rigorous method for calculating the co-ordinates of the image shot-point leads to three second-order equations, which may be solved analytically or graphically, but neither method is convenient under arctic camping conditions. An approximate solution is developed here, which requires only the use of a slide rule. The method is based on cartesian coordinates and deals with non-level surfaces. It differs from the method given by Rock[10] and by Lawlor[11], which is based on spherical coordinates and is developed for horizontal profiles only.

Position of the Image Shot-point

(a) Determination from records of four geophones

The four geophones G_1, G_2, G_3, G_4 (see Fig. 10 p. 551) lie in a plane not too far from level* and at the corners of a square. Rectangular coordinates (u, v, w) are chosen with the origin O at the centre of the square. G_1 and G_3 give the direction of the u-axis, G_2 and G_4 the v-axis. The w-axis is perpendicular to the plane of the geophones. (By setting out the geophones with a theodolite, the horizontal projections of the axes are rectangular and not the axes themselves, an error that can be neglected on a fairly level surface.)* The travel times r_1, r_2, r_3, r_4 of the reflected waves are found from the record for the respective geophones, and as the velocity c is known from the direct waves and the known distances between the shot-point S and any of the geophones, the distances between the image shot-point P and the geophones can be calculated. These distances are great compared with those between the geophones. The distance $OP=p$ is then approximately equal to the arithmetic mean of G_1P and G_3P, $p\cong\frac{r_1+r_3}{2}\,.\,c$, and the angle ϕ_u between O–P and the u-axis is given approximately by the equation

$$\cos\phi_u\cong-\frac{(r_3-r_1)\,.\,c}{u_3-u_1}\;\dagger \qquad (1)$$

The distance p is also approximately equal to the arithmetic mean of G_2P and G_4P, that is $p\cong\frac{r_2+r_4}{2}\,.\,c$, and the angle ϕ_v between O–P and the v-axis is given approximately by the equation

$$\cos\phi_v\cong-\frac{(r_4-r_2)\,.\,c}{v_4-v_2}\;\dagger \qquad (1a)$$

The difference between the two means for p is used to check whether the reflecting element

* For a surface steeper than a few degrees see equations (5a).

† u_1, u_3, v_2, v_4 are the coordinates of respective geophones; r_3-r_1 and r_4-r_2 are the ΔT-values in common use by geophysicists.

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approximates sufficiently to a plane rock surface. If the difference is large, the calculations are made with two triangular groups of three geophones, as described below. The direction from O to P is fully described by $\cos\phi_u$ and $\cos\phi_v$, as only the lower half-space is involved in the problem. The third direction cosine can be calculated however from the equation:

$$\cos^2\phi_u+\cos^2\phi_v+\cos^2\phi_w=1$$

The coordinates of P can be given now as:

$$u_P=p\,.\,\cos\phi_u \qquad w_P=p\,.\,\cos\phi_w$$
$$v_P=p\,.\,\cos\phi_v \qquad \ldots\ldots \quad (2)$$

Assuming p to be five times $G_1\,G_3=u_3-u_1=G_2\,G_4=v_4-v_2$, the error in p is not greater than 1/2 per cent and the error in the angles ϕ_u and ϕ_v not more than a few minutes of arc. These errors are far smaller than experimental errors.

The system of coordinates u, v, w with geophones in the u, v-plane is generally tilted by a small angle from horizontal,* generally also in a different direction for each set-up of geophones. To combine the results of a seismic survey, the positions of P have to be transformed to a system (or a number of parallel systems) of coordinates with a horizontal x, y-plane and a vertical z-axis. We make the x- and y-axes coincide with the horizontal projections of the u- and v-axes. Then, if the two systems of coordinates are rotated by only a small angle relative to each other, an arbitrary point Q_n with its coordinates u_n, v_n, w_n is transformed into the x-, y-, z- system by means of the equations (3):

$$\begin{aligned} x_n &\cong u_n+\kappa v_n-\tau w_n \\ y_n &\cong -\kappa u_n+v_n+\psi w_n \\ z_n &\cong \tau u_n-\psi v_n+w_n \end{aligned} \qquad \ldots\ldots \quad (3)$$

The values of κ, τ, ψ can be found from the known values of x_n, y_n, z_n at the points G_1, G_2, G_3, G_4 and are:

$$\begin{aligned} \kappa &\cong 0 \\ \tau &\cong \frac{z_3-z_1}{u_3-u_1}\cong\frac{z_3-z_1}{x_3-x_1} \\ \psi &\cong -\frac{z_4-z_2}{v_4-v_2}\cong-\frac{z_4-z_2}{y_4-y_2} \end{aligned} \qquad \ldots\ldots \quad (4)$$

The coordinates of P in the x-, y-, z- system are then calculated from:

$$\begin{aligned} x_P &= p\,.\,\cos\phi_x\cong p\left[\cos\phi_u-\frac{z_3-z_1}{x_3-x_1}\,.\,\cos\phi_w\right] \\ y_P &= p\,.\,\cos\phi_y\cong p\left[\cos\phi_v-\frac{z_4-z_2}{y_4-y_2}\,.\,\cos\phi_w\right] \\ z_P &= p\,.\,\cos\phi_z\cong p\left[\cos\phi_w+\frac{z_3-z_1}{x_3-x_1}\,.\,\cos\phi_u+\frac{z_4-z_2}{y_4-y_2}\cos\phi_v\right] \end{aligned} \qquad \ldots\ldots \quad (5)$$

with $$p=\frac{r_1+r_2+r_3+r_4}{4}\,c$$

$$\cos\phi_u\cong-\frac{(r_3-r_1)c}{x_3-x_1},\ \cos\phi_v\cong-\frac{(r_4-r_2)c}{y_4-y_2}$$

and $$\cos\phi_w=-\sqrt{1-\cos^2\phi_u-\cos^2\phi_v}$$

For axes u, v of any steepness with rectangular horizontal projections (directed in practice with

* For a surface steeper than a few degrees see equations (5*a*).

the theodolite), the angles ϕ_x, ϕ_y and ϕ_z are related accurately to ϕ_u and ϕ_v by the equations:

$$\cos\phi_u = \frac{x_3-x_1}{u_3-u_1}\cos\psi_x + \frac{z_3-z_1}{u_3-u_1}\cos\phi_z$$
$$\cos\phi_v = \frac{y_4-y_2}{v_4-v_2}\cos\psi_y + \frac{z_4-z_2}{v_4-v_2}\cos\phi_z \quad \text{(5a)}$$
$$1 = \cos^2\phi_x + \cos^2\phi_y + \cos^2\phi_z$$

leading to highly complicated formulas. If accuracies of about 1° are sufficient (the same error is inherent in the seismic readings), ϕ_x, ϕ_y and ϕ_z may be found easily enough by means of the stereographic projection. (With large stereographic nets $\frac{1}{4}$° or even higher accuracies seem possible). For the use of the "Wulff" stereographic net see reference[12]; different techniques, using dividers, can easily be developed however.

(*b*) *Determination from records of three geophones*

In cases where the four geophones do not lie in a plane, or where the two means of the reflection times calculated from pairs of geophones are not equal, or when one geophone is missing, the coordinates of the image shot-point can be calculated in a slightly different way. Only three geophones of the square are available, lying at the corners of a right-angled triangle (see Fig 11, p. 551). The two axes of rectangular coordinates on the ground are given by the two shorter sides of the triangle. They are rotated by 45° relative to the axes of the previous case and are designated by u' and v'. The geophone at the origin O′ is called G_0', the one on the u'-axis is G_x', and the one on the v'-axis is G_y'. The same indices are used for coordinates, travel times and distances of the respective geophones. The angles analogous to ϕ_u and ϕ_v, called ϕ_{ux}' and ϕ_{vy}', may now be found from:

$$\cos\phi_{ux}' \cong -\frac{(r_x'-r_0')c}{u_x'-u_0'} \text{ and } \cos\phi_{vy}' \cong -\frac{(r_y'-r_0')c}{v_y'-v_0'} \quad \text{(6)}$$

Unfortunately $\cos\phi_{ux}'$ and $\cos\phi_{vy}'$ do not refer to the same point and a correction has to be applied, *i.e.* $\cos\phi_u'$ and $\cos\phi_v'$ have to be calculated. If $G_x'\,O' = d'$, $MP = m$ and $O'\,P = p'$, then

$$m^2 = (p')^2 + \frac{(d')^2}{4} + p' \,.\, d' \,.\, \cos\phi_u'.$$

Also m^2 can be written in terms of d', p' and $\cos\phi_{ux}'$:

$$m^2 = \frac{(d')^2\cos^2\phi_{ux}'}{2} + (p')^2 - \frac{(d')^2}{4} \pm \frac{d'\cos\phi_{ux}'}{2}\sqrt{(d')^2\cos^2\phi_{ux}' + 4(p')^2 - (d')^2}$$

and from this,

$$\cos\phi_u' = \frac{1}{p'd'}\left[\frac{(d')^2\cos^2\phi_{ux}'}{2} - \frac{(d')^2}{2} \pm \frac{d'\cos\phi_{ux}'}{2}\sqrt{(d')^2\cos^2\phi_{ux}' + 4(p')^2 - (d')^2}\right].$$

This equation is simplified by putting $p'/d' = k$. Thus:

$$\cos\phi_u' = \frac{1}{2k}\left[\sin^2\phi_{ux}' \pm \cos\phi_{ux}'\sqrt{4k^2 - \sin^2\phi_{ux}'}\right] \quad \text{(7)}$$

which expanded becomes

$$\cos\phi_u' = \frac{\sin^2\phi_{ux}'}{2k} \pm \cos\phi_{ux}'\left[1 - \frac{\sin^2\phi_{ux}'}{8k^2} - \frac{\sin^4\phi_{ux}'}{128k^4} - \frac{1\,.\,3}{2\,.\,4\,.\,6}\left(\frac{\sin^2\phi_{ux}'}{4k^2}\right)^3 - \ldots\right] \quad \text{(7a)}$$

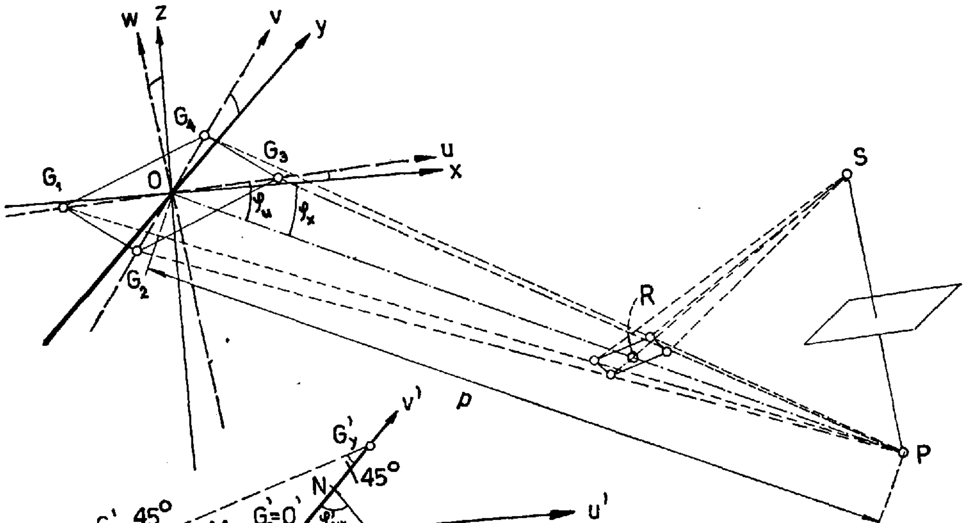
The first approximation for $\cos\phi_u'$ when $k >> 1$ is:

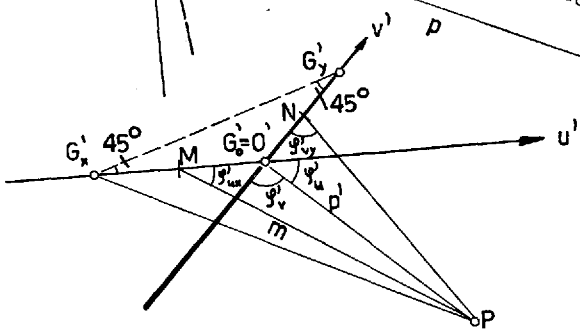
$$\cos\phi_u' \cong \cos\phi_{ux}' + \frac{\sin^2\phi_{ux}'}{2k} \quad \text{(8)}$$

Since k is found from $\frac{r_0'\,.\,c}{u_x'-u_0'}$, k can be positive or negative depending on the sign of $u_x'-u_0'$.

The values of $\frac{\sin^2 \phi_{ux}'}{2}$ may be tabulated for frequent use and a few values are given below:

$\cos \phi_{ux}'$:	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$\frac{\sin^2 \phi_{ux}'}{2}$:	0·50	0·49	0·48	0·45	0·42	0·37	0·32	0·25	0·18	0·09	0·00





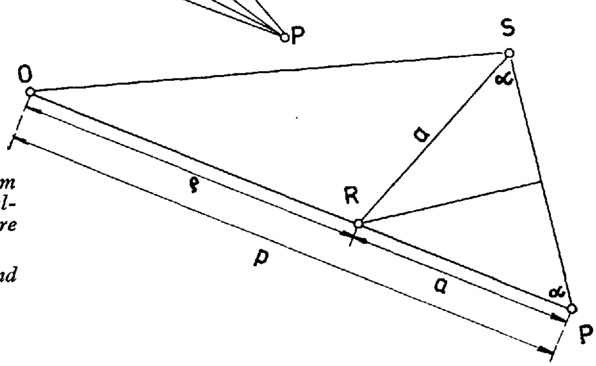


Fig. 10 (top). Space diagram and symbols for reflection calculations when 4 geophones are available

Fig. 11 (centre). Space diagram and symbols for reflection calculations when 3 geophones are available

Fig. 12 (bottom). Diagram and symbols for calculating R

In a similar way $\cos \phi_v'$ is found and the third direction cosine is given by the equation

$$\cos^2 \phi_u' + \cos^2 \phi_v' + \cos^2 \phi_w' = 1.$$

The transformation into the x'-, y'-, z'-system is made as above in equations (3) and (4), and this leads to the final equations for the coordinates of P in the x'-, y'-, z'-system.

$$x_P' = p' \cdot \cos \phi_x' \cong p' \left[\cos \phi_u' - \frac{z_x' - z_0'}{x_x' - x_0'} \cos \phi_w' \right]$$

$$y_P' = p' \cdot \cos \phi_y' \cong p' \left[\cos \phi_v' - \frac{z_y' - z_0'}{y_y' - y_0'} \cos \phi_w' \right] \quad . \quad . \quad . \quad . \quad .$$

$$z_P' = p' \cdot \cos \phi_z' \cong p' \left[\cos \phi_w' + \frac{z_x' - z_0'}{x_x' - x_0'} \cos \phi_u' + \frac{z_y' - z_0'}{y_y' - y_0'} \cos \phi_v' \right]$$

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with $p'=r_0' . c$;

$$\cos\phi_u'=\cos\phi_{ux}'+\frac{\sin^2\phi_{ux}'}{2}\,\frac{x_x'-x_0'}{p'};\quad \cos\phi_{ux}'=-\frac{(r_x'-r_0')c}{x_x'-x_0'};$$

$$\cos\phi_v'=\cos\phi_{vy}'+\frac{\sin^2\phi_{vy}'}{2}\,\frac{y_y'-y_0'}{p'};\quad \cos\phi_{vy}'=-\frac{(r_y'-r_0')c}{y_y'-y_0'};$$

$$\cos\phi_w'=-\sqrt{1-\cos^2\phi_u'-\cos^2\phi_v'}.$$

Equations similar to (5a) can be given for axes u' and v' of any steepness; here again the stereographic projection can be applied advantageously.

Determination of Position and Dip of the Reflecting Bedrock

The sides of the triangle forming the plane through O, S and P (Fig. 12, p. 551) can be determined as follows:

OP$=p$, known from the travel time of the reflected wave.

OS$=s$, known from the travel time of the direct wave or from surface surveying.

and $$\text{SP}=\sqrt{(x_S-x_P)^2+(y_S-y_P)^2+(z_S-z_P)^2}=b.$$

The reflection-point R is found by erecting the mean perpendicular on b. From $\cos\alpha=\frac{p^2+b^2-s^2}{2pb}$, SR=PR are found and then OR$=\rho$ is found from:

$$\rho=p\frac{p^2-s^2}{b^2+p^2-s^2} \qquad (10)$$

By replacing p by ρ in (5), the coordinates x_R, y_R, z_R of the reflection point R can be calculated. The dip δ of the reflecting surface at R and the direction ω of dip (angle between the x-axis and the horizontal projection of the steepest slope) are found from

$$\cos\delta=\frac{z_S-z_P}{b},\ \text{and}\ \tan\omega=\frac{y_P-y_S}{x_P-x_S} \qquad (11)$$

Similar equations are obtained in terms of the primed symbols.

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