METHOD OF SURFACE OF SECTION APPLIED TO A POSSIBLE CAPTURE ORIGIN OF JUPITER'S SATELLITES

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1. INTRODUCTION

The capture of satellites is considered to be a slow process, in which particles approaching the planet lose their energy gradually and fall down into the stable orbits around it. Therefore, in order to investigate the capture problem, it seems to be necessary to clarify the behaviors of orbits near the planet. Recently Hayashi et al. (1977) and Heppenheimer and Porco (1977) found that in the restricted three body problem, particles incident through Lagrange points L_1 and L_2 revolve always directly around Jupiter as long as the potential windows at L_1 and L_2 are small. This suggests the impossibility of the capture of retrograde satellites when the potential windows are small.

In the present paper, we investigate the behaviors of the ensemble of retrograde orbits for which the potential windows are large.

2. THE EQUATIONS OF MOTION

We adopt the following formulation of the restricted three body problem. The frame of reference is the rotating rectangular coordinates (x, y) with the angular velocity of the Sun around Jupiter. Both the Sun and Jupiter are at rest on the x-axis, Jupiter being at the origin. The direction of the x-axis is from the Sun to Jupiter. The system of units is such that the distance between the Sun and Jupiter, the angular velocity of their revolution, and the sum of their masses are all unities, respectively. Denoting the mass of Jupiter by μ, we have the equations of motion

$$\ddot{x} = 2\dot{y} + x + 1 - \mu - (1-\mu)\frac{x+1}{r_1^3} - \mu\frac{x}{r_2^3} \quad ,$$

$$\ddot{y} = -2\dot{x} + y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \quad ,$$

where

R. L. Duncombe (ed.), Dynamics of the Solar System, 181–184.

$$r_1 = \sqrt{(x+1)^2 + y^2} \;, \qquad r_2 = \sqrt{x^2 + y^2} \;.$$

The equations of motion admit the Jacobi integral

$$C = (x+1-\mu)^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - \dot{x}^2 - \dot{y}^2 \;.$$

3. THE BEHAVIORS OF THE RETROGRADE ORBITS IN THE SURFACE OF SECTION

The solution of the equations of motion can be represented as a trajectory in the 4-dimensional space (x,y,v,θ), where $v=\sqrt{\dot{x}^2+\dot{y}^2}$ and $\theta=\arctan(\dot{y}/\dot{x})$. If we fix the value of Jacobi constant C and put $y=0$, the trajectory can be represented as points in the surface of section (x,θ), where the corresponding orbits cross the x-axis (see Figure 1).

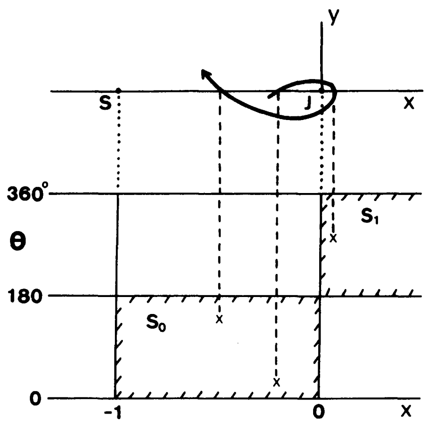

Figure 1. An orbit and the surface of section (x, θ)

Let Σ_0 and Σ_1 stand for the upper and lower halves of the surface of section respectively. In general, an orbit which crosses Σ_0 at a point P_0 crosses successively Σ_1 at P_1 and again Σ_0 at P_2. The transitions from P_0 to P_1 and from P_1 to P_2 are considered to be induced by mappings. Denoting the mapping by T, we have $P_1=TP_0$, $P_2=TP_1$, hence $P_2=T^2P_0$.

Let S_0 and S_1 be the region defined by $-1\leqq x\leqq 0$, $0°\leqq\theta\leqq 180°$ in Σ_0 and that defined by $x\geqq 0$, $180°\leqq\theta\leqq 360°$ in Σ_1, respectively. As long as the orbit around Jupiter is retrograde, the corresponding points in the surface of section are always in S_0 and S_1.

Let the point set W_1 in S_0 be such that $TP\in S_1$ and $T^2P\in S_0$ for any $P\in W_1$ and either $TP'\notin S_1$ or $T^2P'\notin S_0$ for $P'\notin W_1$. We assume that TW_1 is compact and that the points $x=0$, $\theta=180°$ and $x=0$, $\theta=360°$ are not included in TW_1. Then W_1 and T^2W_1 are also compact. Consequently T is topological on

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W_1 and TW_1. Besides the assumptions above, W_1 is assumed to include the points invariant under T^2. Then it can be shown that the boundaries of W_1 and T^2W_1 intersect each other at least at two points.

Define W_n for n=2,3,... by $W_n=T^{-2(n-1)}(W_1\cap T^{2(n-1)}W_{n-1})$. Evidently, the relations $W_1\supset W_2\supset\ldots\supset W_n\supset\ldots$ hold, and W_n shrinks to the invariant set W. If we define U_n by $U_n=W_n-W_{n+1}$ (n=1,2,...), it is obvious that U_n tends to a null set as $n\to\infty$. The orbits having the initial conditions in U_n will revolve retrogradely just n times around Jupiter in the future.

Because of the invariance of the equations of motion under the transformation $x\to x$, $y\to -y$, and $t\to -t$, the past behaviors of the orbit corresponding to the point (x,θ) in S_0 is simulated by the future behaviors of the orbit corresponding to the point $(x, 180°-\theta)$ in S_0. The invariant set W is symmetric with respect to the line θ=90°. Therefore, if P is not included in W, its mirror image with respect to the line θ=90°, is not included in W. This means the impossibility of the capture of the retrograde satellites in the restricted problem.

We denote the mirror image of U_n with respect to the line θ=90° by U'_n (n=1,2,...), and define the sets U_0 and U'_0 as $U_0=S_0-W_1$ and $U'_0=S_0-T^2W_1$. Then, the point set in S_0 corresponding to the ensemble of the orbits which revolve retrogradely just n times is expressed as

$$V_n = \bigcup_{\substack{k+l=n \\ k,l\geq 0}} (U_k\cap U'_l)$$

4. NUMERICAL RESULTS

The equations of motion with μ=0.001 have been integrated numerically for various values of C. It is shown that the sets W_1, TW_1, and T^2W_1 are compact at least when $C\gtrsim 2.5$. The remarkable result is that for $C\gtrsim 2.95$, the boundaries of the above sets, consequently those of W_n(n=2,3,...)too, are constituted only of the points corresponding to collision orbits. Figure 2 shows the structure of the ensemble of retrograde orbits in the region S_0 for C=3.0. In the figure, W_1 is the region inside the bold solid curve. T^2W_1 is inside the broken curve. The regions filled with circles, shaded horizontally and shaded vertically are U_1, U_2 and U_3, respectively. W_2, W_3 and W_4 are the regions obtained by subtracting U_1, U_2 and U_3 from W_1, W_2 and W_3 respectively. W_1 and T^2W_1 are symmetric with each other with respect to the line θ=90°. The approximate invariant region around the invariant point f corresponding to the class f periodic orbit (see e.g. Hénon 1965) is also shown by the dotted curve. Three crosses are the invariant points under T^6.

It has been shown numerically that most of the orbits corresponding to the points in U_1 escape from Jupiter for C=3.0. Therefore in this case V_n defined in the previous section corresponds to the orbits which approach Jupiter and escape after revolving retrogradely just n times around it. A preliminary estimate shows that the amount of such orbits decreases rather rapidly with the increase of n. Therefore the occurrence of the capture seems to be rare for the value of C examined.

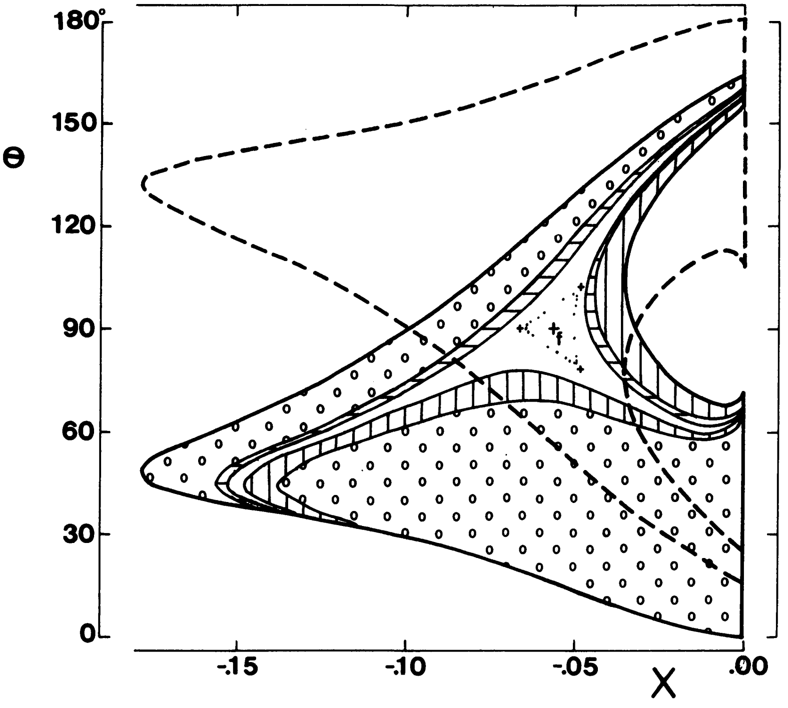

Figure 2. The region S_0 in the Surface of Section for μ=0.001, C=3.0

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DISCUSSION

Dvorak: How can you explain in the simple model of the cirular problem the capture of Jupiter's satellites? In this case the Hill's curves are closed!

Tanikawa: In this case, Hill's curves are not closed around Jupiter; and, for our result, shows that the capture is impossible in the restricted problem. The other forces (like dissipation) are needed.