*From square of b take $4ac$;
 Square root extract, and b subtract ;
 Divide by $2a$; you've x always.*

Another mnemonic for the same values, due to Mr N. D. Beatson Bell, is

*When you have written $-b$,
 The double sign put down ;
 Then $b^2 - 4ac$
 With square-root mark you crown ;
 Beneath it all a line you trace,
 Beneath which line $2a$ you place.*

The value of the co-efficient of refraction of light in two important cases is got from the following :—

*When rays d , pass from air to glass,
 The value of μ is three by two ;
 But when they pass from air to water,
 The value of μ is one by three-quarter(s) !*

Eighth Meeting, June 12th, 1885.

THOMAS MUIR, Esq., LL.D., F.R.S.E., in the Chair.

Summation of certain Series.

By Professor TAIT.

[*Abstract.**]

The attempt to enumerate the possible distinct forms of knots of any order, though unsuccessful as yet, has led me to a number of curious results, some of which may perhaps be new. The general character of the methods employed will be obvious from an inspection of a few simple cases, and any one who has some practice in algebra may extend the results indefinitely.

* This Abstract is part of the paper read in June, entitled "On the detection of amphicheiral knots, with special reference to the mathematical processes involved." I have unfortunately mislaid the MS.—P.G.T.

Take, for instance, the series

$$r^m - n(r+s)^m + \frac{n \cdot n-1}{1 \cdot 2} (r+2s)^m - \&c.$$

where the coefficients are the terms of $(1-1)^n$, and the other factors are the m^{th} powers of the terms of an arithmetical series :— m being a positive integer. The well-known properties of exponential series give us an easy method of summing all expressions of this form. For we have

$$(\epsilon^{px} - \epsilon^{qx})^n = \epsilon^{npx} - n\epsilon^{(n-1)p+q}x + \frac{n \cdot n-1}{1 \cdot 2} \epsilon^{(n-2)p+2q}x^2 - \&c.$$

which may be written in the form

$$\begin{aligned} & \left((p-q)x + \frac{p^2-q^2}{2!}x^2 + \frac{p^3-q^3}{3!}x^3 + \&c. \right)^n \\ &= \sum \frac{1}{m!} \left(np^m - n(np+q-p)^m + \frac{n \cdot n-1}{1 \cdot 2} (np+2q-p)^m - \&c. \right) \end{aligned}$$

Make $np = r$, $q-p = s$; and p and q are known.

The required sum is then the coefficient of x^m in the expansion of

$$m! \left((p-q)x + \frac{p^2-q^2}{2!}x^2 + \dots \right)^n.$$

It vanishes therefore, so long as $m < n$; and for $m = n$ its value is

$$m! (p-q)^n = (-)^n m! s^m.$$

When the coefficients in the given series are the *alternate* terms of $(1-1)^n$, we have only to treat, as above, the expression

$$(\epsilon^{px} + \epsilon^{qx})^n \pm (\epsilon^{px} - \epsilon^{qx})^n.$$

Such results may be varied *ad libitum*, by introducing two or more quantities in place of x , and comparing coefficients of like terms :—e.g., as in finding, by the two methods of expansion, the term in $x^r y^s$ of the quantity

$$(\epsilon^{px} - \epsilon^{qy})^n.$$

But it suffices to have called attention to processes which can give endless varieties of results, some of which may have useful applications.