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On the equation of state in Jovian seismology

J. PROVOST

Département Cassini, OCA, BP 229, 06304 NICE Cédex 4

B. MOSSER

DESPA, Observatoire de Paris-Meudon, 5, place Jules Janssen,92195 MEUDON Principal Cédex

G. CHABRIER

Ecole Normale Supérieure de Lyon, 46 Allée d'Italie, 69364 LYON Cédex 07

Abstract

We have investigated the effect on the seismological properties of the giant planet Jupiter of different descriptions of the equation of state (EOS) of the fluid hydrogen helium envelop. Recent Jovian models by Chabrier et al. 1992 use the equation of state of Saumon et al. (1992) which predicts the PPT (plasma phase transition). We show that different hypothesis at the level of the PPT induce large differences in the internal structure of the corresponding models, specially the sound speed. This gives rise to substantial differences of the oscillation frequencies up to 100 μHz, for modes of degrees ℓ up to 20. These results show the great capability of Jovian seismology to test the physics involved in the interior of the giant planets.

39.1 Description of the Jovian models

Models of Chabrier et al. (1992) are based on the following assumptions: hydrostatic equilibrium of the rotating planet, adiabatic equation of state for each region (rocks and ices in the core; mainly hydrogen and helium in the envelope, with small addition of denser elements). All these models are constrained by the values of the gravitational moments J_2 and J_4 measured by the Voyager mission. The models use the equation of state of Saumon et al. (1992) which predicts the plasma phase transition (PPT) of molecular to metallic hydrogen near the 1.2 Mbar level. The models considered mainly

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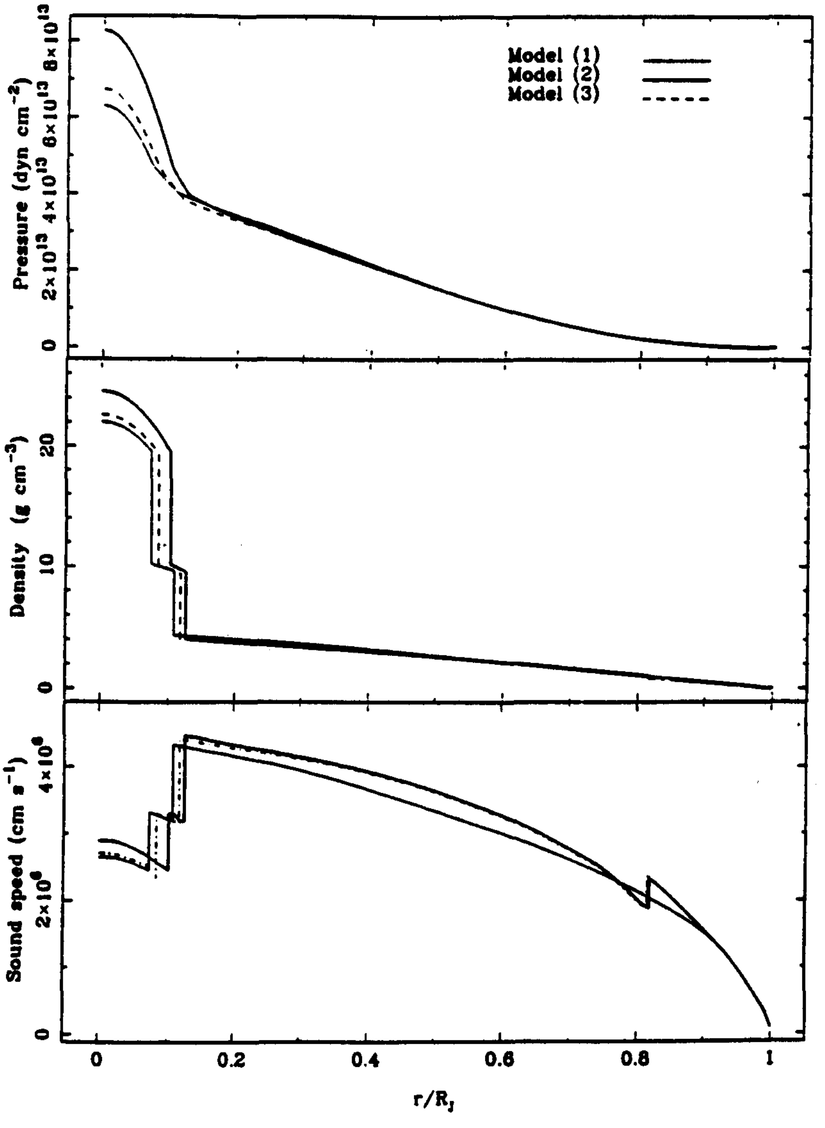

Fig. 39.1 Variation of the pressure, density and sound speed along the radius normalized to the Jovian radius, for the models of Chabrier et al. 1992.

differ by the description of the EOS. Model (1) has a simply interpolated EOS and does not include the treatment of the PPT. It corresponds therefore to a Jovian interior model without PPT. Models (2) and (3) include the PPT, with a slightly different treatment (there is a small jump in the Helium content at the level of the PPT for model (3)).

These models have a double core, constituted of a small rocky core surrounded by a thin ice layer. This induces two strong discontinuities, clearly

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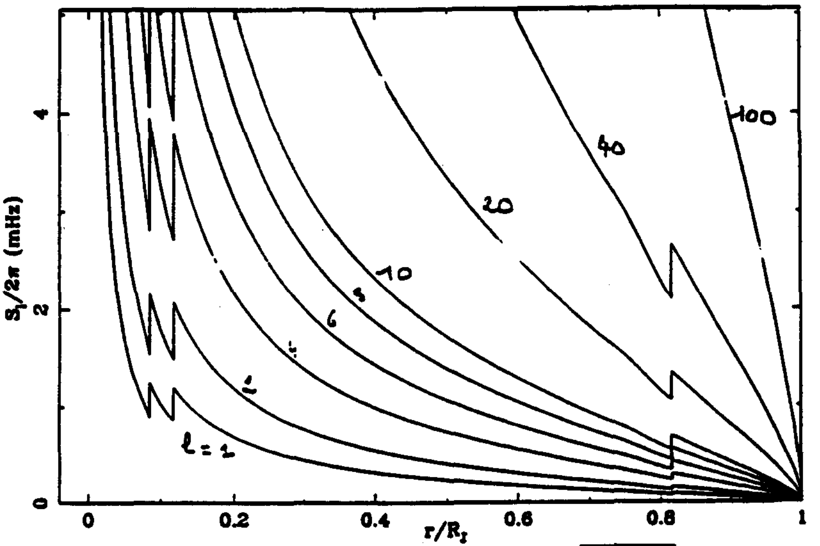

Fig. 39.2 Variation of the Lamb frequency $S_\ell = \sqrt{\ell(\ell+1)}c/r$ as a function of the radius for different degrees ℓ= 1, 2, 4, 6, 8, 10, 20, 40, 100, for the model (1). The acoustic cavity extends from the surface to the point where $S_\ell/2\pi = \nu$.

visible on the pressure p, density ρ and sound velocity c profiles, represented in Figure 1 (a, b, c). There are large differences of pressure, density and sound velocity between the different models (1), (2) and (3), roughly localized in the core of the planets for p and ρ. The differences of sound speed are much larger and occur in the whole planet: this will differently affect the frequencies of the oscillations according to their degrees. Note that the slightly different treatments of the external layers of the models (2) and (3) result in a rather large difference of the size of their core. As a result, the treatment of the equation of state in the external layers has an incidence on the deeper internal structure of the planet and specially on the variation of the sound speed.

39.2 Jovian oscillations

Jovian seismology will be a powerful tool to obtain detailed informations about the different layers of the planet as soon as oscillations of different degrees will be observed . The acoustic oscillations are trapped in a cavity with an upper boundary located in the surface layers. The lower boundary of an oscillation of frequency ν and degree ℓ is located at the level where the Lamb frequency, equal to $S_\ell = \sqrt{\ell(\ell+1)}c/r$ divided by 2π is equal to the frequency ν . Thus it depends on the degree of the mode and goes up from the center to the surface when the degree increases, as it can be seen in Figure 2. Since the frequency of an acoustic oscillation is mainly

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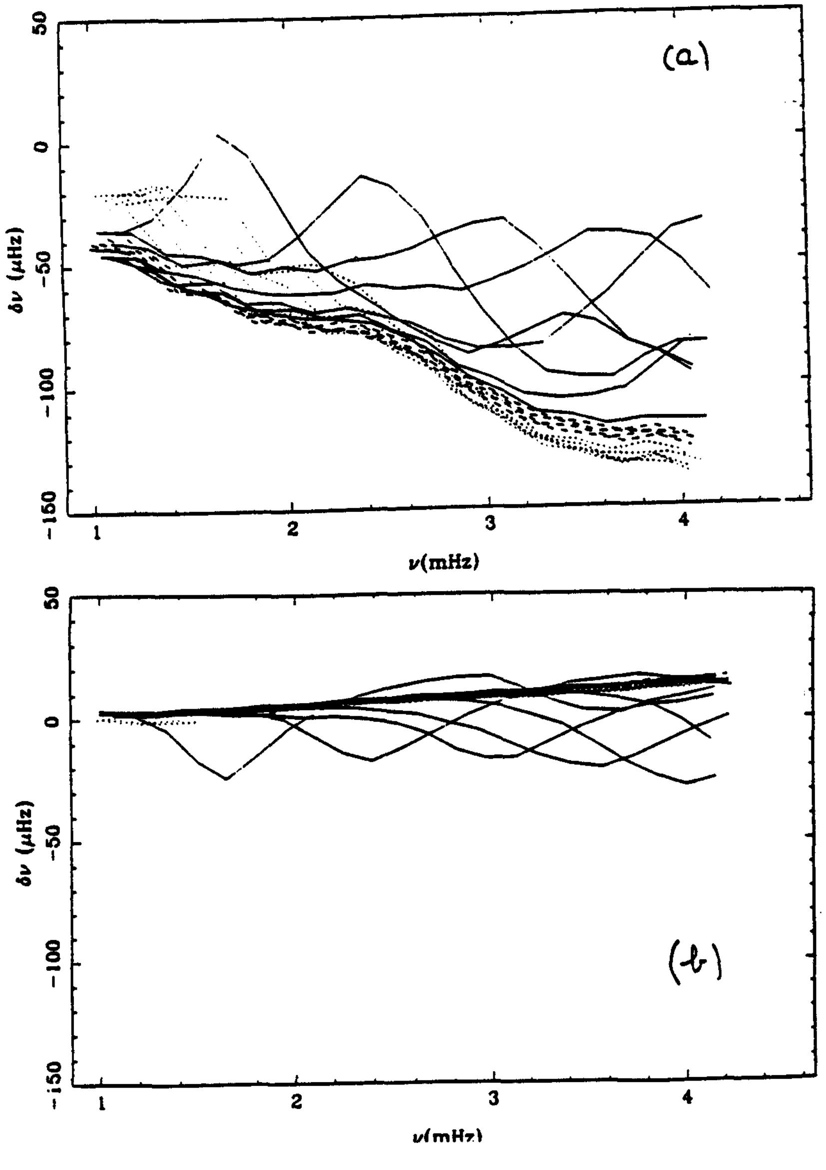

Fig. 39.3 Difference between the computed theoretical frequencies of two models from Chabrier et al. (1992) as a function of the frequency for degrees ℓ =0, 1, 2, 3, 4, 5, 6 (full line), ℓ = 7, 8, 9, 10 (dashed line), ℓ = 12, 14, 16, 18 and 20 (dotted line): (a) difference between models including or not the PPT ((1) - (2)); (b) difference between models with a small difference in the description of the PPT ((2) - (3)).

determined by the sound speed in the cavity of the mode, the observations of oscillations of various degrees will enable us to test different regions of the interior of the planet.

The frequencies of linear, adiabatic, global acoustic modes of the Jovian

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models (1), (2) and (3) have been numerically computed for modes of degree ℓ from 0 to 20. In order to accurately compare the models from the seismological point of view, the frequency differences between two models with different EOS are plotted as a function of the frequency in Figure 3. We have neglected the rotation, which strongly affects the frequencies (Lee 1993), but not too much their differences.

The frequency differences between models (1) and (2) (Figure 3 a) are very large, up to 100 μHz. For the low degree modes, which penetrate deep into the core of the planet, the curves have an oscillatory behavior. This is mainly due to the difference of structure in the core and to its signature, as asymptotically analysed by Provost et al. (1993). For the higher degree modes, less sensitive to the core, the frequency differences are larger, but do not oscillate any more: they are dominated by the difference of structure at the PPT.

The frequency differences between models (2) and (3), which have a slightly different description of the external layers, are shown in Figure 3 b. As expected, they are essentially due to the differences of structure in the core: for the low degree modes, one finds again an oscillatory behavior with values up to 30 μHz, while the higher degree modes are almost unaffected by the different assumptions in the planet modelling.

Thus, when the PPT is taken into account, the frequencies are substantially modified and the observation of oscillation modes of different degrees would provide a strong constraint on the physics of the interior of giant planets.

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