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CALCULATION OF MAXIMUM SNOW-AVALANCHE RUN-OUT DISTANCE BY USE OF DIGITAL TERRAIN MODELS

by

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ABSTRACT

Digital maps and terrain models are used to calculate maximum snow-avalanche run-out distance based on topographic parameters. Maps of 1 : 50 000 scale are found to be accurate enough for the purpose. 113 well-known avalanches are discussed in this paper. A computer system is used to calculate terrain parameters such as rupture area (A), avalanche-path length (L), avalanche-track lengths (L_1, L_2), and run-out lengths (L_3). Maximum run-out angle (α), avalanche-track angle (β), and average angle of rupture zone (γ) are also found by computer. The use of computer and terrain model reduces subjective judgement of parameters to a minimum. Run-out distance was found to be best expressed by the regression equation:

$\alpha = 0.91\beta + 0.08\gamma - 3.5^\circ$. $R^2 = 0.94$, $S = 1.4^\circ$.
$L = 0.93L_1 + 0.97L_2 + 0.61A + 182$ m. $R^2 = 0.96$, $S =$ 137 m.

INTRODUCTION

This paper discusses how maximum snow-avalanche run-out distance may be calculated by a combination of topographic parameters and use of a digital terrain model. Digitized maps are being made for extensive areas of Norway. By use of digital terrain models it is possible to identify a set of objective topographic parameters which can be applied in an avalanche run-out model. These topographic parameters are calculated by computer, thereby reducing subjective judgement of parameter values. By means of the computer, numerical values of the parameters are found quickly, and data from a great number of avalanche paths may be handled in a short time.

TOPOGRAPHIC PARAMETERS

Parameters used in earlier works

The main ideas and the basic topographic parameters applied in this model were first described by Lied and Bakkehøi (1980), and later by Bakkehøi and others (1983). Terrain parameters shown in Figure 1, worked out on the basis of data from 206 avalanches for which the maximum reach is assumed to be known, were used to calculate the run-out distance. The parameters were defined in the following way:

α = angle of straight line between observed outer end of avalanche debris and starting point.
β = angle of straight line between point on terrain profile, where slope angle equals 10°, and starting point.
H = vertical distance from starting point to low point in parabola that best fits the longitudinal profile.
θ = slope angle of top 100 vertical metres of starting zone.
y'' = second derivative of the polynomial function $y = ax^2 + bx + c$ best fitted to terrain profile.

Based on these parameters, the following regression equation was found:

$$\alpha = 0.92\beta - 7.9 \times 10^{-4} H + 1.4 \times 10^{-2} Hy''\theta + 0.04.$$

Standard deviation $S = 2.28^\circ$; correlation coefficient $R =$ 0.92.

Further statistical analysis of the data showed that the variable β was the dominating factor of the equation. Using β as the only predictor variable, the regression equation obtained was:

$$\alpha = 0.96\beta - 1.4^\circ; \qquad S = 2.3^\circ; \qquad R = 0.92.$$

Later work (Martinelli, 1986; McClung and Lied, 1987) has confirmed both the principle of using terrain variables as predictors for maximum snow-avalanche run-out distance, and that the β-angle is the dominating variable in the prediction of α. In the work of Lied and Bakkehoi (1980), other topographic parameters were also tried out in a search for a good α-angle correlation. These parameters were maximum width of rupture in the starting zone (R), minimum width of track (T), and maximum width of run-out zone (D).

Altogether 26 combinations of the above seven predictor variables were tried out. The best correlation was found between α and the variables in the first equation. The effect of confinement in the avalanche track was especially studied by applying the parameters R, T, and D. Basically, it was presumed that an avalanche with a wide rupture zone, which is channelled into a narrow track, has a lower α value (longer reach) than an avalanche following an unconfined path. However, no such tendency was found, and it was concluded that avalanches running in unconfined paths, with a constant width from the starting point to the stopping position, obtain equal α values as confined avalanches when other variables are constant.

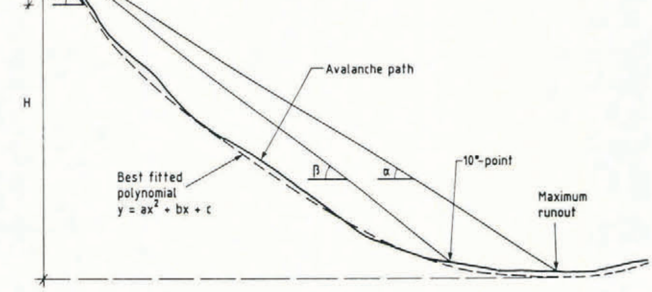

Fig. 1. Topographic parameters from Bakkehøi and others (1983).

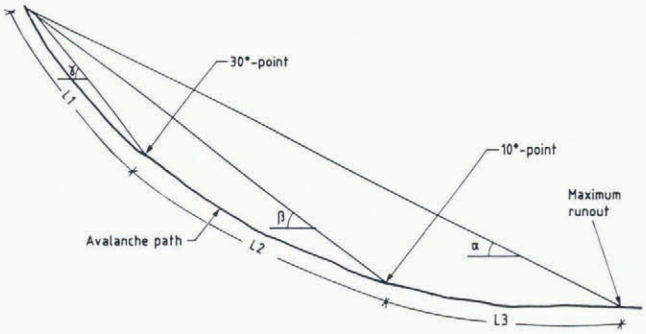

Fig. 2. Topographic parameters in the present paper.

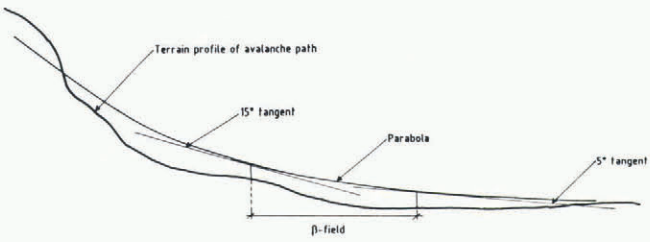

Fig. 3. Definition of β field by best-fit parabola.

Topographic parameters used in the present paper

The α and β angles as they are defined in the previous section are used in this paper (Fig. 2).

Area of the starting zone

One of the factors that may influence run-out distance is the volume of the avalanche. The volume is difficult to calculate in a model where only *terrain* parameters are included. A correlation between the area of the starting zone and the maximum mass of snow in the avalanche probably exists, as described by Schaerer (1975). Starting zone area (A) is therefore one of the parameters treated in this paper. The same parameter was studied by Bovis and Mears (1976), but their definitions both of the starting zone and the run-out zone are different from the definitions used in this paper and cannot therefore be directly compared. In the present paper, the area of the starting zone is defined as the part of the path lying between the starting point of the avalanche and the 30° point. The lateral extension of the starting zone is bounded by the local topography from where *one single* avalanche can be launched, and must be evaluated subjectively.

Average inclination of the starting zone

Another parameter for describing the characteristics of the starting zone is the angle of the line connecting the lowest point of the starting zone with the starting point (γ angle) (Fig. 2). This parameter describes the average slope inclination of the potential starting zone. In the work of Lied and Bakkehøi (1980), the inclination of the starting zone was introduced as a parameter in run-out calculations (θ angle). The idea behind the introduction of this parameter was that magnitude of an avalanche partly depends on the terrain inclination in the starting zone. On gentle slopes, avalanches were thought to accumulate more snow before rupture occurs than on steep slopes. If avalanche mass is of importance for the run-out distance, there should be a tendency for low-angle rupture zones to create avalanches with low values of α angles. The analysis showed that the θ parameter was of minor importance for the prediction of α, a result which was confirmed by Bakkehøi and others (1983). By introducing γ as a parameter for the inclination of the starting zone, a more logical way of representing the starting zone is achieved, as the terrain slope in the whole of the zone, and not only that in the uppermost 100 m, is included (Mears, 1985).

Length parameters

Four different length parameters, L, L_1, L_2, and L_3 are also introduced. These are:

L = total length of avalanche path ($L_1 + L_2 + L_3$) (Fig. 2).
L_1 = length of the starting zone, defined as distance between starting point and point on the path where slope inclination = 30°.
L_2 = length of avalanche track, defined as distance between the 30° point and 10° point.
L_3 = length of run-out zone, defined as distance from the 10° point to the outer end of avalanche debris.

DIGITAL MAPS AND THE TERRAIN MODEL

In Norway the topographic map series on a scale of 1 : 50 000, with contour intervals 20 m, has been available in digital form since 1982. The digitized version of the maps has made possible computations based on the map. The terrain-model system, TERMOS, was developed by Stabell and Toppe for the purpose of snow- and rock-slide hazard zoning (Toppe, 1987). The model has a bi-curved surface, built by overlapping cubic spline functions. The spline functions are controlled by a regular grid with a cell size of approximately 30 m × 30 m, when the map scale 1 : 50 000 is concerned, and a total of about one million cells. The elevation and the normal vector are stored for each grid cell.

In this model the terrain surface is mathematically determined. Elevations can quickly be computed anywhere on the surface based on the stored grid. Profiling and computation of real lengths and areas is therefore easily done. The terrain model is accurate and well within the quality demands set by the U.S. Geological Survey. The calculations of the avalanche parameters are performed on a minicomputer (Prime or VAX). Tektronix 4115 or 4125 graphical terminals with puck and tablet are used as work stations. The terminals have built-in zooming and panning functions and all the map handling is performed locally in the work station.

COMPUTATION OF THE AVALANCHE PARAMETERS

Avalanche path

As a first step, all avalanches are drawn on a map at the work station; this is done manually using puck and tablet. Each avalanche is drawn to its maximum known extent, after which the centre line of the avalanche is drawn, from the start to the stop position, and the system computes the terrain profile along the path. The best-fitting parabola approximating the profile is computed by a least-squares algorithm.

α, β, and γ parameters

The 10° point on the avalanche path is itself identified on the *natural* profile. To avoid that the model chose small segments with inclination less than 10° higher up in the path, a β point is accepted only if it is inside the section of the profile shown in Figure 3. This section is limited by the points where the angle between the tangent of the best-fit parabola and the horizontal plane is between 5° and 15°. A similar procedure is performed to identify the 30° point on the avalanche path. The tangent angle of acceptance for this point is between 25° and 35°. Note that the purpose of using the parabola is to ensure that the β and γ points are placed in the correct sections of the real profile. The α angle is then computed, and all three points for α, β, and γ respectively are marked on the profile (Fig. 2). The respective values of the angles are written on the screen.

Area of starting zone

All areas steeper than 30° are automatically identified by the computer using the stored normal-vector information. The boundaries of the starting zone have already been described and within these boundaries the computer calculates the real area using a multiple-direction narrow-band algorithm; the lengths of profile lines running across the starting zone are calculated and these lengths are

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multiplied by the distance between the lines to form narrow bands whose width varies with the size of the area. A typical distance is about 50 m. Both the horizontal projection of the area and the real area of the rupture zone are calculated and their values are written on the screen.

RESULTS AND DISCUSSION

The data set used for the present analysis consisted of topographic parameters derived from 113 avalanche paths in different regions of western Norway. Extreme run-out positions were measured in the field, based on knowledge amongst the local population of the avalanches over a time period of at least 100 years. All of the avalanches have terminated on open flat land with no major topographic obstructions in the run-out zone. Most of these avalanches have also been studied by Bakkehøi and others (1983).

For all parameters shown in Table I the range is large, indicating that avalanches differ considerably in length, steepness, and potential rupture area. (A complete list of avalanches and variables is presented in Table II.) Linear regression analyses were performed using α as the dependent variable, and the other listed parameters in Table I as predictor variables. Many different combinations of the predictor variables have been tried and the results clearly demonstrate that β is the dominant predictor for α. The following equations for α are presented:

$$\alpha = 0.96\beta - 1.7^\circ \qquad R^2 = 0.93 \qquad S = 1.4^\circ \qquad (1)$$

$$\alpha = 0.99\gamma - 10.3^\circ \qquad R^2 = 0.56 \qquad S = 3.8^\circ \qquad (2)$$

$$\alpha = -0.0092A + 30.3^\circ \qquad R^2 = 0.05 \qquad S = 5.6^\circ \qquad (3)$$

$$\alpha = 0.0036L_1 + 24.91^\circ \qquad R^2 = 0.03 \qquad S = 5.7^\circ \qquad (4)$$

$$\alpha = 0.006L_1 - 0.008L_2 + 28.0^\circ \qquad R^2 = 0.35 \qquad S = 4.7^\circ \qquad (5)$$

$$\alpha = 1.0\gamma + 0.046L_1 - 13.7^\circ \qquad R^2 = 0.60 \qquad S = 3.7^\circ \qquad (6)$$

$$\alpha = 0.91\beta + 0.08\gamma - 3.5^\circ \qquad R^2 = 0.94 \qquad S = 1.4^\circ \qquad (7)$$

In Equation (1), both R and S values are high, and these values give better correlations between α and β than those obtained in earlier work in which the same parameters have been studied (Fig. 4). The reason for this is probably that the avalanches studied in the present work all exceed the β point, which is not the case for all the avalanches examined by earlier workers (Lied and Bakkehøi, 1980; Bakkehøi and others, 1983).

As seen from Equation (2), a correlation between α and γ exists ($R^2 = 0.56$). The equation has an S value of 3.8°, which is too high for practical applications in run-out distance calculations (Fig. 5).

Except for Equation (1) and (7), the highest R values are found in Equation (6), which is a combination of the average slope of the rupture zone (γ) and the length of this zone (L_1). Correlating (γ) and L_1 results in $R^2 = 0.000$, demonstrating that the two variables are statistical independent. The R^2 value of 0.60 in Equation (6) is a fairly high correlation, but as $S = 3.7^\circ$ the variation is too high for the handling of run-out problems.

The correlations of α and A, L_1, and L_2 all show very low values, as can be seen from Equations (3), (4), and (5). This is not too surprising because α, unlike the above-mentioned predictors, is a dimensionless number. It is interesting that no relationship seems to exist between the maximum area of a rupture zone and the run-out distance expressed by α.

Applying the combination $\alpha = f(\beta,\gamma)$, Equation (7) gives $R^2 = 0.94$ and $S = 1.4^\circ$. This is a small improvement on the R value in Equation (1). The two predictors are not statistically independent, as $\beta = f(\gamma)$ gives $R^2 = 0.56$ and $S = 2.9^\circ$.

The possibility of using the path segment L_3 as a dependent variable has been examined, but no combination of predictor variables seems to give R and S values which would enable sufficiently accurate calculation of run-out distance:

$$L_3 = 0.467A + 126\text{ m} \qquad R^2 = 0.20 \qquad S = 135\text{ m} \qquad (8)$$

$$L_3 = 0.098L_1 + 148\text{ m} \qquad R^2 = 0.03 \qquad S = 147\text{ m} \qquad (9)$$

$$L_3 = 0.070(L_1 + L_2) + 118\text{ m} \qquad R^2 = 0.07 \qquad S = 117\text{ m} \qquad (10)$$

$$L_3 = 0.065L_1 + 0.074L_2 + 119\text{ m} \qquad R^2 = 0.08 \qquad S = 119\text{ m} \qquad (11)$$

$$L_3 = 0.601A - 0.04(L_1 + L_2) + 168\text{ m} \qquad R^2 = 0.21 \qquad S = 135\text{ m} \qquad (12)$$

The total length of an avalanche path may also be used as a predictor, and regression analysis resulted in these equations:

$$L = 3.56A + 1080\text{ m} \qquad R^2 = 0.65 \qquad S = 382\text{ m} \qquad (13)$$

$$L = 1.07L_1 + 1.08L_2 + 115\text{ m} \qquad R^2 = 0.95 \qquad S = 147\text{ m} \qquad (14)$$

$$L = 0.93L_1 + 0.97L_2 + 0.61A + 182\text{ m} \qquad R^2 = 0.96 \qquad S = 137\text{ m} \qquad (15)$$

From Equation (13), it can be seen that there is a correlation between the total length of the avalanche path (L) and the size of the rupture area, although the S value is high. A strong correlation also exists with L, L_1, and L_2, as is seen from Equation (14). Combined with A, the prediction is slightly improved (Equation (14)). The prediction of path lengths according to Equations (14) and (15) will give constant lengths independent of the steepness of the avalanche path. The α/β relation (Equation (1)) shows that low values of β will create long run-out lengths, which is more realistic and in accordance with what is really found in Nature.

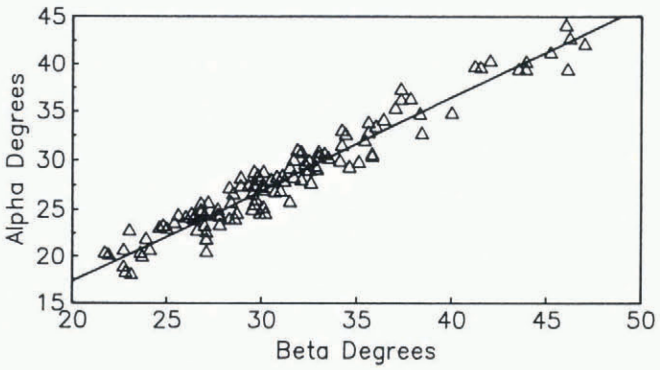

Fig. 4. Observed α angles related to measured β angles, with regression line.

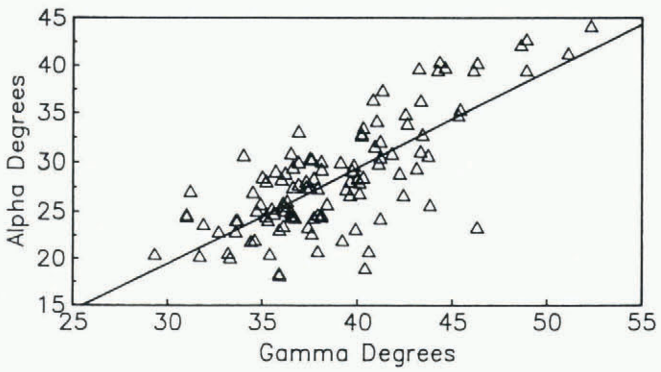

Fig. 5. Observed α angles related to measured γ angles, with regression line.

McClung and Lied (1987) used the relation $\delta x/x_B$ as a predictor for the run-out distance; x_B was defined as the horizontal distance from the avalanche starting point to the 10° point, and δx as the horizontal distance from the 10° point to the outer end of the avalanche debris. It was shown that the avalanches with the 50 highest values of this relation gave a very good fit to an extreme-value distribution.

The relationship $L_3/(L_1 + L_2)$ for the 113 avalanche paths which are analysed in the present paper are plotted as a histogram (Fig. 6). The range of observations is from 0.027 to 0.503, standard deviation S = 0.092, median and mean values are 0.130 and 0.148, respectively.

Forty-five of the avalanches considered have a reach longer than the mean value, and 15 avalanches reach beyond the mean value + 1*S*. When 2*S* is added to the mean, seven avalanches have a longer reach. Whether the data fit a normal or an extreme value distribution has not been further analysed.

CONCLUSION

The basic conclusions drawn from this work are that digital terrain models are well suited to the study of avalanche-terrain parameters. Maps at a scale of 1 : 50 000, with contour-line separation of 20 m, seem accurate enough for calculation of maximum snow-avalanche run-out distances. Maps of larger scale would of course add more detail, but would also vastly increase the amount of data and the need for computer resources.

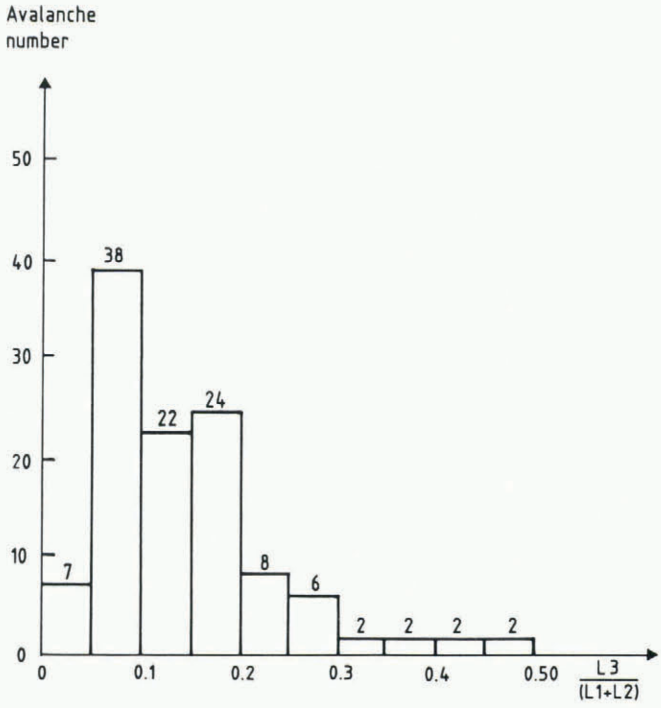

Fig. 6. Histogram of $L_3/(L_1+L_2)$, and avalanche numbers.

TABLE I. PARAMETERS USED IN ANALYSIS

Parameter	Mean	Median	Standard deviation (*S*)	Min. value	Max. value
α(°)	28.2	27.3	5.8	18.0	44.0
β(°)	31.3	30.1	5.8	21.7	47.0
γ(°)	38.8	37.9	4.4	29.3	52.3
L_1(m)	896	914	284	342	1610
L_2(m)	773	631	577	103	1943
L_3(m)	235	195	151	48	886
L(m)	1905	1802	639	848	3654
A(1000 m^2)	232	196	145	47	762

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TABLE II. LIST OF AVALANCHES AND VARIABLES

No.	α	β	γ	L_1	L_2	L_3	L	A(1000 m^2)
1	30.1	33.5	37.6	885	246	178	1309	109
2	32.5	34.4	40.2	907	304	48	1259	79
3	32.7	35.6	40.2	874	335	97	1306	80
4	34.6	38.3	45.3	519	220	138	877	61
5	20.3	23.6	29.3	596	1250	370	2216	174
6	30.5	35.8	43.7	704	414	229	1347	96
7	32.6	38.4	43.4	757	175	247	1179	80
8	30.3	35.8	41.2	690	368	231	1289	47
9	25.7	31.5	36.1	1194	447	370	2011	177
10	25.6	27.2	34.9	596	580	71	1247	85
11	27.8	31.2	40.1	656	548	96	1300	130
12	27.9	30.5	35.2	550	654	156	1360	120
13	24.5	30.2	37.9	384	345	196	925	60
14	22.5	27.1	37.6	527	570	260	1357	130
15	29.7	35.1	41.1	820	472	251	1543	87
16	25.9	29.7	36.3	825	575	206	1606	143
17	25.1	30.1	35.5	915	445	343	1703	180
18	24.2	27.8	37.7	1144	828	306	2278	333
19	25.4	29.6	36.1	996	860	299	2155	187
20	21.7	27.1	34.4	999	511	184	1694	221
21	20.6	22.7	37.9	914	1547	309	2824	257
22	23.9	26.4	33.6	785	491	384	1660	156
23	20.4	27.1	33.2	931	829	886	2646	377
24	22.7	26.6	32.7	1180	899	545	2624	410
25	23.3	27.8	36.1	975	432	642	2049	339
26	21.8	27.1	34.6	827	558	506	1891	232
27	23.2	24.8	37.4	515	928	131	1574	154
28	23.9	28.6	33.7	944	1001	186	2132	291
29	24.2	26.5	36.7	1023	1599	407	3029	323
30	19.9	23.7	33.3	1604	1386	664	3654	584
31	26.5	28.5	42.4	475	626	209	1310	76
32	24.5	29.9	38.1	1222	1835	169	3226	159
33	28.3	29.8	40.3	842	1072	152	2066	116
34	30.7	32.1	41.8	1041	921	128	2090	151
35	29.8	34.1	39.1	1024	560	173	1757	140
36	31.4	34.2	40.9	1610	923	354	2887	490
37	23.9	28.3	35.3	952	1187	368	2507	505
38	28.9	32.5	35.7	675	281	141	1097	91
39	29.3	32.8	36.6	712	424	137	1273	90
40	29.2	34.6	43.1	768	609	225	1602	301
41	27.9	29.9	39.6	899	1765	327	2991	762
42	25.5	26.8	43.8	785	1603	163	2551	333
43	36.1	37.3	43.3	808	462	143	1413	106
44	40.1	43.9	46.3	704	498	113	1315	92
45	39.3	43.9	46.1	745	454	118	1317	90
46	39.5	41.5	43.2	757	448	109	1314	123
47	21.8	23.9	39.2	1057	1644	301	3002	300
48	27.1	28.3	39.4	465	491	385	1341	302
49	23.2	27.0	46.3	878	1079	357	2314	345
50	18.2	22.8	35.9	653	1213	646	2512	318
51	18.8	22.7	40.4	431	655	305	1391	146
52	24.9	29.5	36.2	414	246	188	848	54
53	25.6	28.4	38.4	429	516	62	1007	72
54	28.7	29.6	36.2	1005	756	56	1817	305
55	23.0	24.6	39.9	1316	1853	300	3469	485
56	24.3	25.6	38.1	967	1267	286	2520	290
57	24.3	27.7	36.5	1354	1293	459	3106	387
58	22.9	25.0	35.9	1173	1555	231	2959	684
59	18.0	23.1	35.9	836	1777	193	2806	327
60	37.2	37.3	41.3	808	601	64	1473	122
61	29.9	31.7	38.1	1288	920	189	2397	406
62	27.6	32.6	36.9	1190	906	369	2465	423
63	24.4	26.3	36.6	958	854	630	2496	450
64	23.5	25.4	31.9	1520	1736	204	3460	758
65	24.5	28.7	31.0	1125	853	147	2125	253
66	39.3	43.5	44.2	1168	611	205	1984	257
67	40.2	42.0	44.3	1012	971	68	2051	385
68	26.9	30.1	31.2	948	1313	358	2619	292
69	30.9	31.9	43.3	1288	1943	97	3328	479
70	42.6	46.2	48.9	1222	155	96	1473	204
71	41.1	45.2	51.1	1229	204	92	1525	164
72	42.0	47.0	48.6	1142	103	522	1767	253
73	39.6	41.2	44.6	1155	950	74	2179	256
74	39.3	46.1	48.9	958	927	425	2310	387
75	24.3	27.0	31.0	1296	885	477	2658	372

TABLE II *continued*

No.	α	β	γ	L_1	L_2	L_3	L	A(1000 m^2)
76	24.6	26.8	35.6	770	592	156	1518	242
77	20.6	24.1	40.6	447	902	210	1559	167
78	20.1	21.9	31.7	342	1371	250	1963	107
79	26.8	31.0	34.5	1066	829	289	2184	265
80	27.3	29.6	37.3	963	584	143	1690	173
81	24.9	27.7	34.7	1029	1095	305	2429	528
82	27.2	30.0	37.9	998	1089	188	2275	234
83	26.7	30.7	40.1	914	980	328	2222	210
84	27.3	29.3	37.5	839	907	254	2000	384
85	22.7	23.0	33.6	761	1405	58	2224	271
86	20.3	21.7	35.4	361	737	182	1280	106
87	27.9	32.1	37.3	556	318	189	1063	73
88	26.5	29.5	39.6	843	476	224	1543	98
89	30.7	33.0	36.5	1135	318	145	1598	203
90	30.2	33.0	37.5	1000	392	118	1510	161
91	27.3	28.9	36.6	653	596	97	1346	108
92	30.5	33.3	34.0	544	1071	314	1929	194
93	28.3	31.1	35.0	558	1090	306	1954	164
94	36.2	37.8	40.8	1297	290	114	1701	172
95	33.3	36.0	40.3	1239	395	136	1770	278
96	29.8	32.5	36.9	1117	499	195	1811	261
97	24.3	27.7	35.2	1211	896	239	2346	208
98	29.1	31.5	38.1	1352	849	195	2396	247
99	29.5	32.3	39.8	1168	756	180	2104	196
100	29.0	32.9	39.7	931	760	294	1985	167
101	32.9	34.2	36.9	1472	231	99	1802	206
102	31.9	35.4	41.2	1126	531	162	1819	196
103	24.8	26.8	36.5	360	631	145	1136	90
104	24.1	26.0	41.2	479	902	106	1487	152
105	28.7	30.1	42.2	496	574	72	1142	158
106	35.2	37.0	45.4	671	273	67	1011	123
107	28.2	31.7	37.7	753	443	218	1414	165
108	33.7	35.6	42.6	681	256	122	1059	123
109	34.0	36.4	41.0	899	332	99	1330	197
110	44.0	46.0	52.3	934	198	56	1188	142
111	34.7	40.0	42.5	1012	171	205	1388	216
112	28.1	28.9	36.0	1023	1306	124	2453	240
113	28.2	30.8	40.0	764	561	174	1499	200

The analysis presented has confirmed that the prediction of maximum avalanche distance can be based on a two-dimensional terrain profile. Area of rupture zone (A) seems to have no correlation with run-out distance expressed as α angle. When run-out distance is expressed in metres (L_3), the correlation to rupture area is present, but still weak, as $R = 0.45$. A similar correlation is found when L_3 is predicted by rupture area A, in combination with length of avalanche track, $L_1 + L_2$. S values for both these combinations are about 130–150 m.

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