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INTRODUCTION

Introduction to the special issue: Unification

In the days of its foundation, the field of science covered by *UNIF* – a series of annual international workshops on unification – was still in its infancy. With the advent of automated reasoning, term rewriting, logic programming, natural language processing, and program analysis, the areas of computer science concerned by unification were seething with excitement. With the coming out of researches in constraint solving and admissibility of inference rules and with the breaking out of applications, such as type checking, query answering, and cryptographic protocol analysis, the development of unification was not long in going at full speed.

The *raison d'être* of a workshop is to promote the progress of a field of science by reporting new ideas and trends. Over the last 34 years, *UNIF* has covered most aspects – theoretical and practical – of research in the field of unification. And most of the prominent researchers in unification have already presented in its successive editions some of their most important contributions.

The six articles presented in this issue of *Mathematical Structures in Computer Science* have been carried out by renowned researchers. Their contents have been presented during the 32nd edition of *UNIF* that took place at Oxford University within the 3rd International Conference on *Formal Structures for Computation and Deduction*, part of the 2018 *Federated Logic Conference*. They cover many aspects of the different areas of computer science concerned by unification.

Using duality techniques and bounded bisimulation ranks, Silvio Ghilardi and Luigi Santocanale give a semantic proof of Ruitenburg's Theorem saying that every endomorphism of a finitely generated free Heyting algebra is ultimately periodic if it fixes all the generators but one. They also show that arbitrary endomorphisms between free algebras are not, in general, ultimately periodic. Finally, they demonstrate that when arbitrary endomorphisms between free algebras are ultimately periodic, their period can be explicitly bounded as a function of the cardinality of the set of generators.

Franz Baader, Pavlos Marantidis, Antoine Mottet, and Alexander Okhotin investigate two extensions of *ACUI* – the theory of an associative, commutative, and idempotent binary function symbol with the unit. On one hand, they consider approximate *ACUI*-unification where they use appropriate measures to express how close a substitution is to be a unifier. On the other hand, they extend *ACUI*-unification to *ACUIG*-unification, that is, unification in equational theories obtained adding to *ACUI* a finite set G of ground identities. Finally, they combine the two extensions, that is, they consider approximate *ACUIG*-unification and determine the exact worst-case complexity of the unification problem.

David Cerna and Temur Kutsia consider anti-unification for simply typed lambda terms in theories defined by associativity, commutativity, and identity axioms. They develop a sound and complete algorithm that receives a pair of lambda terms and computes their equational generalizations in the form of higher-order patterns – the minimal complete set of such generalizations containing finitely many elements, the problem is finitary. Also, they investigate special restrictions of the problem for which an optimal solution can be computed in linear or polynomial time.

The equational theory *ACh* consists of a function symbol that is homomorphic over an associative-commutative operator. Since unification modulo *ACh* is undecidable, Ajay Kumar Eeralla and Christopher Lynch define a bounded version of *ACh* unification and give a (terminating) sound and complete algorithm for solving it. Thanks to the implementation in Maude of their algorithm, they argue that it is useful in cryptographic protocol analysis.

The intruder deduction problem and the static equivalence problem are knowledge problems critical to the verification of security protocols that can be related to particular forms of matching and unification. They are defined regarding an equational theory and are known to be decidable when the equational theory is given by a subterm convergent term rewrite system. In their work, Serdar Erbatur, Andrew Marshall, and Christophe Ringeissen consider a subterm convergent term rewrite system defined modulo an equational theory. They present two pairs of solutions to these important problems. The first solves the deduction and static equivalence problems in rewrite systems modulo shallow theories. The second provides a general procedure that solves the deduction and static equivalence problems in subterm convergent systems modulo syntactic permutative theories provided a finite measure is ensured.

As for Yunus Kutz and Manfred Schmidt-Schauß, they consider the Knuth–Bendix confluence test on rewrite rules in a nominal setting extended by atom-variables. They present a constrained form of nominal expressions allowing the abstraction of both atoms and meta-variables together with its semantics for which notions of equality, validity, matching, unification, and rewriting are carefully formalized. The most elaborated aspect is the adaptation of a Knuth–Bendix confluence criterion for such nominal systems. As an interesting example of application, the paper presents a convergent rewriting system for a restricted version (using just alpha-equivalence) of the monad laws.

We would like to thank all authors for their enthusiasm and their support. The task of fairly presenting with rigor the latest developments in a field of science such as unification is not easy. Over and above the editorial work of the journal administrators, it requires the benevolent contributions of anonymous referees. On the occasion of this special issue on unification, let us thank them all for the willingness and the competence they put in ensuring that accepted articles meet the highest standards of quality and validity.

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