3 Closure Modelling Near the Two-Component Limit

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1 Introduction

Most widely-used turbulence models have been developed and tested with reference to flows near local equilibrium, where there are only moderate levels of Reynolds stress anisotropy. The present contribution considers the development of models which are designed to give the correct behaviour in much more extreme situations, where the turbulence approaches a 2-component state.

To illustrate the type of flow situation to be considered, Figure 1 illustrates the flow in the vicinity of a wall. While all turbulent velocity components must vanish at the wall, the normal fluctuations, $v$, must vanish more rapidly since by continuity $\partial v / \partial y$ must always be zero there (as $\partial u / \partial x$ and $\partial w / \partial z$ both vanish), Figure 2. A similar two-component structure arises close to the free surface of a liquid flow where again fluctuating velocities normal to the free surface become negligible compared with fluctuations lying in the plane.

![Figure 1: Near-wall flow.](image1)

![Figure 2: Normal Reynolds stress components in plane channel flow, from the DNS of Kim et al. (1987).](image2)
of the free surface. Clearly, the turbulence structure in such a flow will be very different from that found in free flows, where the stress anisotropy is much smaller. Consequently, it might be expected that simple models developed and tuned for the latter flows are unlikely to give good predictions in near-wall or free-surface regions, or other flows which are close to the 2-component limit.

The importance of explicitly respecting this two-component limit in turbulence modelling originated from two papers from the 1970s. First, a short note by Schumann (1977) advocated that modelling proposals should make it impossible for unrealizable values of the turbulence variables to be generated (such as negative values for the mean square velocity fluctuations in any direction). Shortly thereafter, Lumley (1978) remarked that if such realizability was to be ensured one needed to focus on the behaviour of the model at the moment when one of the velocity components had just fallen to zero. When this two-component state has been reached one must ensure that, for the normal stress that has fallen to zero, its rate of change also vanishes. That is essential to prevent the stress field achieving unrealizable values at the next instant of time.

Shih and Lumley (1985) were the first to apply realizability constraints to the modelling of the pressure correlation terms in both the Reynolds stress and scalar flux transport equations. However, while the work initially adopted a rigorous analytical path, they later (Shih et al. 1985) had to include additional higher order correction terms to gain agreement with simple shear flow experiments. In later work at UMIST, Fu et al. (1987), Fu (1988), Craft et al. (1989) and Craft (1991) showed that by applying a slightly different constraint to the scalar flux model, a realizable model was obtained which gave good agreement with experiments for a range of shear flows. The model has since been extended further by Launder and Tselepidakis (1993), Launder and Li (1994), Craft and Launder (1996) to include viscous and inhomogeneity effects found in near-wall or surface regions. Consideration of these effects is, however, deferred until [11] on Impinging and Separated Flows. A further class of flows where turbulence approaches the two-component state is where a strongly stabilizing force field is applied, whether due to buoyancy, rotation, or electro-magnetic effects. The extension of the methodology to such cases is developed in [14].

2 A TCL closure of the Reynolds stress transport equations

From [2], equation (1), the stress transport equations, in the absence of any external force field, can be written symbolically as

$$\frac{Du_i u_j}{Dt} = P_{ij} + \phi_{ij} + d_{ij} - \varepsilon_{ij}. \quad (2.1)$$
The exact generation term \( P_{ij} \equiv -\overline{u_i u_k} \partial U_j / \partial x_k - \overline{u_j u_k} \partial U_i / \partial x_k \), whilst the remaining terms, which require modelling, are the pressure-strain correlation \( \phi_{ij} \), diffusion \( d_{ij} \) and dissipation rate \( \varepsilon_{ij} \). In the shear flows discussed here, diffusion is a relatively unimportant process, so the simple gradient diffusion model of Daly and Harlow (1970),

\[
d_{ij} = \frac{\partial}{\partial x_k} \left[ \left( \nu \delta_{lk} + c_s \frac{k}{\varepsilon} \overline{u_k u_l} \right) \frac{\partial \overline{u_i U_j}}{\partial x_l} \right],
\]

is often employed. The modelling of the remaining terms is made so as to ensure compliance with the two-component limit (TCL). Broadly, two strategies are adopted, which are exemplified in the modelling of the pressure-strain processes.

2.1 Pressure-Strain Processes

In many flows, it is the pressure-strain correlation \( \phi_{ij} \) which is the most important term requiring modelling, and consequently much effort has been put into developing improved models for this process. As reported in [2], equation (6) \textit{et seq.}, integration of the Poisson equation for pressure fluctuations for regions where the surface integral is unimportant leads to

\[
\phi_{ij} \equiv \frac{p}{\rho} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

\[
= \frac{1}{4\pi} \int_{Vol} \left( \frac{\partial^2 u_i'^* u_i'}{\partial r_i \partial r_j} + \frac{\partial^2 u_j'^* u_j'}{\partial r_i \partial r_j} \right) \frac{dV}{|r|}
\]

\[
- \frac{1}{2\pi} \int_{Vol} \left( \frac{\partial^2 u_i'^* u_i'}{\partial r_i \partial r_j} + \frac{\partial^2 u_j'^* u_j'}{\partial r_i \partial r_j} \right) \frac{\partial U_i'}{\partial r_i} \frac{dV}{|r|},
\]

where non-primed quantities are evaluated at the point where \( \phi_{ij} \) is being determined, whilst primed quantities are evaluated at positions within the integration volume at a displacement of \( r \) from this point.

From this, it can be seen that there are two distinct contributions to \( \phi_{ij} \): one involving interactions between fluctuating quantities and one dependent on mean strain rates. In buoyancy-affected flows there is a further contribution, which will be considered separately in [14].

The volume of integration in equation (2.3), although formally being the entire fluid domain, can in practice be regarded as the region where the time averaged two-point correlations are non-zero, corresponding to some relatively small region surrounding the point at which \( \phi_{ij} \) is being evaluated with a radius typically of the local integral lengthscale.
It is noted that equation (2.3) contains no surface integral discussed both in [2] and, in more detail, in [4]. This represents a fundamental difference of strategy between closure schemes which are otherwise of the same type. The philosophy explored at UMIST is that if one ensures compliance of the model of $\phi_{ij}$ with the two-component limit, there should, in many cases, be no requirement for any further wall correction - at least if one remains outside the buffer layer where the effects of very rapid spatial variations must be accounted for.

2.1.1 Mean-Strain (or ‘Rapid’) Part of $\phi_{ij}$

If the mean strain is assumed to vary much more slowly in space than the two-point correlation gradients in equation (2.3), it can be regarded as uniform over the volume of integration, so that $\phi_{ij2}$ can be modelled as

$$\phi_{ij2} = \left( X_{k ji}^{li} + X_{k ji}^{kj} \right) \frac{\partial U_k}{\partial x_l},$$

where the tensor $X_{k ji}^{li}$ represents the integral of the two-point velocity-derivative correlations:

$$X_{k ji}^{li} = -\frac{1}{2\pi} \int_{Vol} \frac{\partial^2 u'_i u'_l}{\partial r_k \partial r_l} \frac{dV}{|r|}.$$

This approach has been employed by Naot et al. (1973) and Launder et al. (1975) to derive models of $\phi_{ij2}$ in which $X_{k ji}^{li}$ is simply a linear function of the Reynolds stresses:

$$X_{k ji}^{li} = \alpha u_l u_i \delta_{kj} + \beta (u_i u_j \delta_{lk} + u_l u_k \delta_{ij} + u_i u_k \delta_{lj}) + \gamma u_k u_j \delta_{il} + \xi k (\delta_{ij} \delta_{ik} + \delta_{lk} \delta_{ij}) + \eta k \delta_{il} \delta_{kj},$$

where the Greek symbols are coefficients to be determined. One can note that the exact integral in equation (2.5) satisfies

- Continuity: $X_{k ji}^{li} = 0$
- Normalization: $X_{k kk}^{li} = 2 u_l u_i$.

By applying these two constraints, all the coefficients except one in equation (2.6) can be determined and the resultant model, known as the ‘Quasi Isotropic (QI) Model’, may be written as

$$\phi_{ij2} = -\frac{\gamma + 8}{11} (P_{ij} - \frac{1}{3} \delta_{ij} P_{kk}) - \frac{30 \gamma - 2}{55} k \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$$

$$-\frac{8 \gamma - 2}{11} (D_{ij} - \frac{1}{3} \delta_{ij} D_{kk}),$$

where $D_{ij} \equiv -u_i u_k \frac{\partial U_i}{\partial x_j} - u_j u_k \frac{\partial U_k}{\partial x_i}.$
However, it is not possible to choose a value of $\gamma$ that will enable this linear form to satisfy the 2-component limit, which requires $\phi_{\alpha\alpha} = 0$ if $u_{\alpha} = 0$.

An obvious extension of the approach is to recognise that the two-point correlations appearing in the integral of equation (2.5) will not depend linearly on the second-moments, and thus to allow $X_{kj}^{li}$ to be a nonlinear function of the Reynolds stresses. If one includes both quadratic and cubic terms, then the most general expression satisfying the required symmetry properties can be written as

$$\frac{X_{kj}^{li}}{k} = \lambda_1 \delta_{il} \delta_{kj} + \lambda_2 (\delta_{ij} \delta_{ki} + \delta_{lk} \delta_{ij})$$

$$+ \lambda_3 a_{ij} \delta_{kj} + \lambda_4 a_{kj} \delta_{li} + \lambda_5 (a_{ij} \delta_{ki} + a_{lk} \delta_{ij} + a_{kj} \delta_{lk} + a_{ki} \delta_{lj})$$

$$+ \lambda_6 a_{ij} a_{kj} + \lambda_7 (a_{ij} a_{ki} + a_{lk} a_{ij}) + \lambda_8 a_{lm} a_{mi} \delta_{kj} + \lambda_9 a_{km} a_{mj} \delta_{li}$$

$$+ \lambda_{10} (a_{im} a_{mj} \delta_{ki} + a_{lm} a_{mk} \delta_{ij} + a_{im} a_{mj} \delta_{lk} + a_{km} a_{mi} \delta_{lj})$$

$$+ \lambda_{11} a_{mn} a_{mn} \delta_{li} \delta_{kj} + \lambda_{12} a_{mn} a_{mn} (\delta_{ij} \delta_{ki} + \delta_{lk} \delta_{ij})$$

$$+ \lambda_{13} a_{li} a_{km} a_{mj} + \lambda_{14} a_{kj} a_{lm} a_{mi}$$

$$+ \lambda_{15} (a_{ij} a_{km} a_{mi} + a_{lk} a_{im} a_{mj} + a_{kj} a_{lm} a_{mk} + a_{ki} a_{ml} a_{mj})$$

$$+ \lambda_{16} a_{mn} a_{np} a_{pm} \delta_{li} \delta_{kj} + \lambda_{17} a_{mn} a_{np} a_{pm} (\delta_{ij} \delta_{ki} + \delta_{lk} \delta_{ij})$$

$$+ \lambda_{18} a_{mn} a_{mn} a_{mi} \delta_{lj} + \lambda_{19} a_{mn} a_{mn} a_{kj} \delta_{li}$$

$$+ \lambda_{20} a_{mn} a_{mn} (a_{ij} \delta_{ki} + a_{lk} \delta_{ij} + a_{kj} \delta_{lk} + a_{ki} \delta_{lj}). \tag{2.8}$$

The approach outlined below, following the analysis of Fu (1988), is to assume the coefficients $\lambda_1, \ldots, \lambda_{20}$ to be constants, and to apply the continuity, normalization and 2-component-limit constraints in order to determine as many of the coefficients as possible.

Applying the continuity constraint ($X_{kj}^{li} = 0$), and making use of the Cayley–Hamilton theorem, leads to six equations:

$$\lambda_1 + 4 \lambda_2 = 0 \tag{2.9a}$$

$$\lambda_3 + 4 \lambda_5 = 0 \tag{2.9b}$$

$$\lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + 5 \lambda_{10} = 0 \tag{2.9c}$$

$$\lambda_{10} + \lambda_{11} + 4 \lambda_{12} = 0 \tag{2.9d}$$

$$\lambda_{13} + \lambda_{14} + 4 \lambda_{15} + 2 \lambda_{18} + 2 \lambda_{19} + 10 \lambda_{20} = 0 \tag{2.9e}$$

$$\lambda_{16} + 4 \lambda_{17} + \frac{1}{3} (\lambda_{13} + \lambda_{14} + 2 \lambda_{15}) = 0 \tag{2.9f}$$

Similarly, the normalization constraint leads to a further six equations:

$$3 \lambda_1 + 2 \lambda_2 = 4/3 \tag{2.10a}$$

$$3 \lambda_3 + 4 \lambda_5 = 2 \tag{2.10b}$$

$$2 \lambda_7 + 3 \lambda_8 + 4 \lambda_{10} = 0 \tag{2.10c}$$

$$\lambda_9 + 3 \lambda_{11} + 2 \lambda_{12} = 0 \tag{2.10d}$$

$$\lambda_{13} + 2 \lambda_{15} + 3 \lambda_{18} + 4 \lambda_{20} = 0 \tag{2.10e}$$

$$4 \lambda_{15} + 9 \lambda_{16} + 6 \lambda_{17} = 0 \tag{2.10f}$$
The 2-component-limit constraint is most conveniently handled in principal axes of the Reynolds stresses, where \( \overline{u_i u_j} = 0 \) if \( i \neq j \). If \( \overline{u_3^2} \) is taken as the vanishing component, then the other two normal stresses can be written as \( \overline{u_2^2} = (1 + \delta)k \) and \( \overline{u_1^2} = (1 - \delta)k \). The 2-component limit requires that

\[
X_{k2} \frac{\partial \bar{U}_k}{\partial x_l} = 0,
\]

and substituting the above values for the stresses into this equation leads to a further four relations between the model coefficients:

\[
\begin{align*}
\lambda_1 + \lambda_2 - \frac{2}{3}(\lambda_3 + \lambda_4) - \frac{7}{3} & \lambda_5 + \frac{4}{9} \lambda_6 + \frac{10}{9} \lambda_7 + \frac{4}{9} (\lambda_8 + \lambda_9) + \frac{11}{9} \lambda_{10} \\
+ \frac{2}{3}(\lambda_{11} + \lambda_{12}) - \frac{8}{27} (\lambda_{13} + \lambda_{14}) - \frac{34}{27} \lambda_{15} - & \frac{2}{9} (\lambda_{16} + \lambda_{17}) \\
- \frac{4}{9} (\lambda_{18} + \lambda_{19}) - & \frac{14}{9} \lambda_{20} = 0
\end{align*}
\]

These equations can be solved, leaving four undetermined parameters:

\[
\begin{align*}
\lambda_1 &= \frac{8}{15} & \lambda_8 &= \frac{1}{5} + t - 2p & \lambda_{15} &= 3s - \frac{9}{4} t \\
\lambda_2 &= -\frac{2}{15} & \lambda_9 &= -\frac{15}{2} t - 2p & \lambda_{16} &= -2s + t \\
\lambda_3 &= \frac{14}{15} + \frac{4}{3} t & \lambda_{10} &= p & \lambda_{17} &= s \\
\lambda_4 &= \frac{4}{15} + \frac{11}{3} t & \lambda_{11} &= 3t + p & \lambda_{18} &= \frac{15}{4} t - r \\
\lambda_5 &= \frac{1}{5} - t & \lambda_{12} &= -\frac{3}{4} t - \frac{1}{2} p & \lambda_{19} &= r \\
\lambda_6 &= \frac{1}{10} + 8t - 2p & \lambda_{13} &= -6s - \frac{27}{4} t + 3r & \lambda_{20} &= 0. \\
\lambda_7 &= -\frac{3}{10} - \frac{3}{2} t + p & \lambda_{14} &= -6s + \frac{33}{4} t - 3r \\
\end{align*}
\]

However, it can be shown that the contributions to \( \phi_{ij2} \) arising from the terms with coefficients \( s \) and \( p \) are identically zero. The resulting model can thus be written as

\[
\phi_{ij2} = -0.6 (P_{ij} - \frac{1}{3} \delta_{ij} P_{kk}) + 0.3 a_{ij} P_{kk} \\
-0.2 \left[ \frac{u_k u_j u_i u_l}{k} \left[ \frac{\partial U_k}{\partial x_l} + \frac{\partial U_l}{\partial x_k} \right] - \frac{u_i u_k}{k} \left[ \frac{\partial U_j}{\partial x_l} + \frac{\partial U_l}{\partial x_j} \right] \right] \\
-c_2 [A_2 (P_{ij} - D_{ij}) + 3a_{mi} a_{nj} (P_{mn} - D_{mn})] \\
+c'_2 \left\{ \left( \frac{7}{15} - \frac{A_2}{4} \right) (P_{ij} - \frac{1}{3} \delta_{ij} P_{kk}) \right\} \\
+0.1 \left[ a_{ij} - \frac{1}{2} (a_{ik} a_{kj} - \frac{1}{3} \delta_{ij} A_2) \right] P_{kk} - 0.05 a_{ij} a_{ik} P_{kl}
\]
where $A_2 = a_{ij}a_{ij}$. There are two free coefficients, $c_2$ and $c_2'$, which can be set by tuning the model to simple shear flows. Fu et al. (1987) recommended values of $c_2 = 0.6$, $c_2' = 0$, which considerably simplifies the task of implementing the model in a computer code. Later, however, Fu (1988) concluded that slightly better agreement for free shear flows could be obtained with $c_2 = 0.55$ and $c_2' = 0.6$, values which greatly improved the performance in near-wall flows since in many cases if one remains outside the viscosity-affected sublayer, no wall corrections of the type described in [2] are then needed, Launder and Li (1994).

### 2.1.2 Turbulence (or ‘Slow’) Part of Pressure-Strain

No-one has so far devised a successful analytical route for modelling $\phi_{ij1}$ analogous to that for $\phi_{ij2}$. Thus the two-component limit is imposed empirically through stress invariants of which, for a second rank tensor, there are two independent parameters. One of these, $A_2$ has already appeared in the expression for $\phi_{ij2}$. The natural second parameter might be thought to be

$$A_3 \equiv a_{ij}a_{jk}a_{ki}.$$

(2.13)

However, Lumley (1978) showed that, for modelling purposes, a combined invariant $A$, defined as

$$A \equiv 1 - 9/8(A_2 - A_3),$$

(2.14)

was a particularly powerful choice because, in the limit of two-component turbulence, the parameter always goes to zero.\(^1\) By including the parameter $A$ in a model for $\phi_{ij1}$, one may thus arrange that the model of $\phi_{ij1}$ is consistent with the two-component limit.

Thus, for $\phi_{ij1}$, a nonlinear extension of the return to isotropy model of Rotta (1951) could be written as:

$$\phi_{ij1} = -c_1\varepsilon a_{ij} - c_1'\varepsilon(a_{ik}a_{kj} - 1/3A_2\delta_{ij}) - c_1''\varepsilon A_2a_{ij}$$

(2.15)

\(^1\)We can conveniently map the range of attainable states of the turbulent stress field as an $A_2$-$A_3$ plot (Lumley 1978), Figure 3a. All realizable states fall within or on the boundary of this triangle, the upper line corresponding to two-component turbulence while the two curved lines represent axisymmetric turbulence (that is, where two of the normal stresses are equal). The origin corresponds to isotropic turbulence. Alternatively, on an $A_2$-$A$ plot, two-component turbulence corresponds with states lying on the $A_2$ axis, Figure 3b.
Closure modelling near the two-component limit

(Although it might appear that the cubic term \( a_{ik}a_{kl}a_{lj} - \frac{1}{3}A_3\delta_{ij} \) should also be included in equation (2.15), the Cayley–Hamilton theorem means that the term is proportional to \( A_2a_{ij} \), and its inclusion would not thus add any more generality to the form shown).

![Figure 3: The anisotropy invariant map in \( A_2-A_3 \) and \( A_2-A_2 \) space.](image)

The approach followed is simply to make the coefficients functions of the invariants \( A \) and \( A_2 \), to ensure that they vanish in the 2-component limit. The UMIST group, for example, have employed the form

\[
\phi_{ij1} = -c_1\varepsilon \left[ a_{ij} + c'_1(a_{ij}a_{jk} - \frac{1}{3}A_2\delta_{ij}) \right] - f'_A\varepsilon a_{ij},
\]

(2.16)

where

\[
c_1 = 3.1(A_2A)^{1/2} \quad c'_1 = 1.1 \quad f'_A = A^{1/2}.
\]

2.2 Dissipation

Since the dissipative processes arise predominantly from the smallest scales of turbulence, \( \varepsilon_{ij} \) is normally considered to be essentially isotropic, even if the stress field is significantly anisotropic. From this assumption, \( \varepsilon_{ij} \) is often modelled as \( \frac{2}{3}\varepsilon\delta_{ij} \).

However, local isotropy is not consistent with the 2-component limit which requires \( \varepsilon_{22} \) to vanish at a wall. A simple way of ensuring compliance with this limit is to devise a model where \( \varepsilon_{ij} \propto (\bar{u}_i\bar{u}_j/k)\varepsilon \) close to a wall (or, indeed, in other circumstances where the stress field is near the two-component limit). A transition function, based on the ‘flatness’ parameter \( A \), can be employed to switch between the two forms:

\[
\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}f_f + \frac{\bar{u}_i\bar{u}_j}{k}\varepsilon(1 - f_f).
\]

(2.17)
If \( f_\varepsilon \) takes a value of unity in isotropic turbulence, far from walls (where \( A \approx 1 \)), but becomes zero when \( A \) vanishes, then \( \varepsilon_{ij} \) will switch between the two desired limits.

Whilst such a simple form does satisfy some of the conditions required of \( \varepsilon_{ij} \), it does not show the correct limiting behaviour for all components, nor does it behave correctly near a free surface. These aspects will be briefly considered in [11].

Of course, in practical calculations the dissipation rate \( \varepsilon \) also has to be modelled, and this is generally done by solving a separate transport equation for it. The most widely employed model can be written

\[
\frac{D\varepsilon}{Dt} = c_{\varepsilon 1}\frac{\varepsilon P_{kk}}{2k} - c_{\varepsilon 2}\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_k}\left[\left(\nu\delta_{lk} + \frac{k}{\varepsilon}\frac{u_k u_l}{\varepsilon}\right) \frac{\partial \varepsilon}{\partial x_l}\right],
\]

(2.18)

with coefficients \( c_{\varepsilon 1} = 1.44 \), \( c_{\varepsilon 2} = 1.92 \). At UMIST, workers have retained this general form, but have included some account of the effects of different stress anisotropy on \( \varepsilon \) by allowing \( c_{\varepsilon 2} \) to be a function of \( A \) and \( A_2 \), and reducing the value of \( c_{\varepsilon 1} \). The recommended form for the coefficients is:

\[
c_{\varepsilon 1} = 1.0 \quad c_{\varepsilon 2} = 1.92/(1 + 0.7A_2^{1/2}A).
\]

(2.19)

### 3 Applications to the computation of dynamic field

The performance of the model described in Section 2 is now considered, first for free flows, then for flows near a single plane wall or free surface and then, finally, for composite walls. To provide as accurate an impression as possible of the capabilities of the TCL approach, we limit attention to computations made with a single form of the model. Consequently, earlier TCL forms adopted by Tselepidakis (1991) (see Launder and Tselepidakis 1993) and Launder and Shima (1989) (see also Shima 1993, 1998) have not been included here even though the latter, in particular, has been successfully applied to a wide range of two- and three-dimensional boundary layers near walls. Nor do we include the very recent publications by Leschziner and his group of the TCL model applied to transonic and supersonic flows (Batten et al. 1999b,a).

#### 3.1 Free Shear Flows

The free coefficients in the model were originally assigned to secure satisfactory agreement in various homogeneous shear flows and plane strains. Thus one can hardly claim to be predicting these flows since they formed part of the overall optimization process. A typical example is the homogeneous shear flow considered in Figure 4. This particular example is a severe test as it starts from isotropic turbulence. The growth of the anisotropy is predicted broadly as the DNS of the four non-zero stress components indicates, although the
Figure 4: Development of stress anisotropies in homogeneous shear flow. Lines: TCL model predictions, Symbols: DNS of Matsumoto et al. (1991).

<table>
<thead>
<tr>
<th>Flow</th>
<th>Experimental value</th>
<th>Basic model</th>
<th>TCL model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane jet</td>
<td>0.110</td>
<td>0.100</td>
<td>0.110</td>
</tr>
<tr>
<td>Round jet</td>
<td>0.093</td>
<td>0.105</td>
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<tr>
<td>Plane wake</td>
<td>0.086</td>
<td>0.070</td>
<td>0.069</td>
</tr>
</tbody>
</table>

Table 1: Predicted and measured spreading rates of some self-preserving free shear flows.

The initial growth of the anisotropy of $u_1^2$ and $u_2^2$ is rather too weak. It is, however, considerably better than that achieved by the Basic Model, presented in [2].

Turning to inhomogeneous shear flows, Table 1 reports the computed and measured asymptotic growth rates of three free shear flows: the plane and round jets and the plane wake. These three flows collectively provide a severe test for any model. The table indicates that the TCL scheme again comes close to mimicking the growth rates of all three, whereas the Basic Model does badly for both the plane wake and the round jet. It is worth noting that these results were obtained with a full elliptic solution of the transport equations rather than the usual thin-shear-flow approximation. This practice led to a reduction in growth rates (compared with a thin-shear-flow treatment) of about 12% for the round jet and about 4% for the plane jet (El Baz et al. 1993). This difference reflected the rapid axial decay of the round jet. There was negligible difference between the two treatments for the wake. There are similar improvements in the prediction of growth rates for buoyantly-driven plumes, considered in [14].
Finally, Figure 5 compares the development of plane wakes created by two different bodies, thus providing different initial states of turbulence energy and dissipation, but with the same momentum deficit. The experiments show that the effects of the different initial conditions carry over very far downstream – well beyond the region of measurement. The Basic Model, however, quickly forgets about the different initial conditions, showing identical growth for the two cases. In contrast, the TCL scheme clearly displays a very similar development to that recorded.

Figure 5: Development of the centreline streamwise Reynolds stress and the half-width of the plane wake behind two different bodies. Left hand graphs: Basic Model, right hand graphs: TCL Model. From El Baz (1992).

3.2 Flows Near Plane Surfaces

If one applies a log-law boundary condition for velocity and analogous local-equilibrium conditions for the near-wall stresses it is possible to apply the model discussed so far to wall flows as well as free flows without introducing any form of ‘wall-reflection’ correction to $\phi_{ij}$. This is a very great benefit! The case of fully-developed flow in a plane channel is shown in Figure 6, where agreement with data is seen to be satisfactory. To integrate all the way to the
wall across the viscous sublayer does require a correction to account for the very rapid change of the mean velocity gradient within the ‘buffer region’ as well as the inclusion of viscous effects. These elaborations are discussed in [11] concerned with impinging and separated flows.

Figure 6: Reynolds stress profiles in fully developed channel flow at $Re = 20000$. From Li (1992). Solid line TCL model; broken line Basic Model with wall-reflection terms added; symbols experiments.

The flow in the vicinity of a free liquid surface (that is, a gas-liquid interface) traditionally requires the application of a ‘wall’ correction if the Basic Model is adopted (McGuirk and Papadimitriou 1985). Again, such corrections are dispensed with when the TCL closure is adopted. Figure 7 compares the development of a 3D surface jet adopting these two second-moment closures: the Basic Model, including ‘wall-reflection’ at the free surface and the TCL model (Craft et al. 2000). Quite clearly the development of the shear flow is much better captured by the latter scheme.

Finally, the corresponding case of a 3-dimensional wall jet (Craft and Lauder 1999, 2001) is summarized in Table 2. Firstly, it is noted from the experiments that the lateral spreading is markedly greater than that normal to the wall. This effect is due to an induced secondary flow that draws fluid down to the wall and ejects it parallel to the wall. The driving source for the secondary flow is the anisotropy of the turbulent stress field in the jet’s cross-section. Now, a linear eddy-viscosity model predicts isotropic normal stresses when there is negligible normal straining; consequently, this unequal growth rate is entirely missed at this level of modelling. Both second-moment closures, on the other hand, exhibit strongly anisotropic growth patterns – indeed appreciably too large (especially the Basic Model, whose growth rate is more than three times that reported experimentally). The reason appears to be that this flow takes much longer to reach full development than the experimenters had believed. If instead of the fully-developed value, one examines the spreading rate at around 70 jet diameters downstream (the downstream limit of the
experiments) the TCL model accords closely with the experimental growth (the corresponding developing-flow value for the Basic Model is not available, though there is no doubt that it would still be considerably too high).

Figure 7: Development of the three-dimensional free-surface jet half-widths normal to the free-surface \( (y_{1/2}) \) and in the lateral direction \( (z_{1/2}) \). Computations of Craft et al. (2000), ——: TCL model; ——: Basic model; Symbols: experiments of Rajaratnam and Humphries (1984). From Craft et al. (2000).

<table>
<thead>
<tr>
<th></th>
<th>( dy_{1/2}/dx )</th>
<th>( dz_{1/2}/dx )</th>
<th>( \hat{z}<em>{1/2}/\hat{y}</em>{1/2} )</th>
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<tbody>
<tr>
<td>Expt. (Abrahamsson et al, 1997)</td>
<td>0.065</td>
<td>0.32</td>
<td>4.94</td>
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<tr>
<td>Linear EVM</td>
<td>0.079</td>
<td>0.069</td>
<td>0.88</td>
</tr>
<tr>
<td>Basic model</td>
<td>0.053</td>
<td>0.814</td>
<td>15.3</td>
</tr>
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<td>TCL model</td>
<td>0.060</td>
<td>0.51</td>
<td>8.54</td>
</tr>
<tr>
<td>TCL model at 70 diameters</td>
<td>0.055</td>
<td>0.308</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 2: Spreading rates normal to the wall \( (dy_{1/2}/dx) \) and in the lateral direction \( (dz_{1/2}/dx) \) in the 3-dimensional wall jet.

### 3.3 Flow Over Complex Surfaces

Figure 8 shows axial velocity contours over the cross section of a straight rectangular sectioned duct where, over the lower wall, two regions, symmetrically located relative to the centre-plane of the duct, have been roughened. The
roughness creates a complex Reynolds stress pattern which, in turn, induces an appreciable secondary flow, with an upwelling of fluid in the vicinity of the mid-plane, which distorts the axial velocity contours as shown in the figure. Again, the source of this streamwise vorticity is the anisotropy of the in-plane Reynolds stresses. No linear eddy-viscosity turbulence model can create such streamwise vorticity. We see, however, that the TCL computations (Launder and Li 1994) mimic the measured distribution very closely. The Basic Model (with wall-reflection corrections) gets the correct sense of the secondary motion, but the detailed prediction is evidently not as successful as with the TCL model.

A similar story unfolds in the case of flow through a smooth square-sectioned U-bend (Iacovides et al. 1996). In this case, a strong secondary flow is induced by the bend curvature. Figure 9 shows the variation of shear stress between the inner and outer curved walls at 45° into the bend. On the symmetry plane both TCL and Basic models achieve reasonable agreement with the measured stress profile. As one moves progressively towards the top wall of the duct, however, the TCL scheme takes account of the influence of this upper wall much better than the Basic Model even though the former model has no explicit means (such as distance to the upper wall) of sensing the presence of that boundary.

Figure 8: Flow through a rectangular-sectioned duct with a partially roughened lower wall. Computations of Launder and Li (1994), experiments of Hinze (1973). (a) Contours of mean streamwise velocity. (b) Predicted secondary flow patterns. From Launder and Li (1994).
Figure 9: Turbulent shear stress profiles across the duct at 45° around a square-sectioned U-bend. Computations of Iacovides et al. (1996), ———: Basic model; – - -: TCL model; Symbols: experiments of Chang et al. (1983). From Iacovides et al. (1996).

4 Scalar flux modelling

Similar considerations relating to the two-component-limit behaviour can be applied to the modelling of the scalar-flux transport equations. The exact transport equations (see [2], equation (2.19)) can be written symbolically as

$$\frac{Du_i \theta}{Dt} = P_{i \theta} + \phi_{i \theta} + d_{i \theta} - \varepsilon_{i \theta},$$  \hspace{1cm} (4.1)
where the production \( P_{i\theta} \equiv -u_i u_k \frac{\partial \Theta}{\partial x_k} - \overline{u_k \theta} \frac{\partial U_i}{\partial x_k} \) which does not require modelling, whilst \( \phi_{i\theta} \) represents the pressure-scalar gradient correlation, \( d_{i\theta} \) the diffusion and \( \varepsilon_{i\theta} \) the dissipation rate of the scalar flux.

In this case, the assumption of isotropic dissipation leads to \( \varepsilon_{i\theta} = 0 \), and consequently the main modelling task is to approximate \( \phi_{i\theta} \).

An analytical expression similar to equation (2.3) can be obtained for \( \phi_{i\theta} \):

\[
\phi_{i\theta} = -\frac{1}{4\pi} \int_{\text{Vol}} \frac{\partial^2 \overline{u_i' \theta'}}{\partial r_i \partial r_i} \frac{dV}{|r|} - \frac{1}{2\pi} \int_{\text{Vol}} \frac{\partial \overline{u_i' \theta}}{\partial r_i} \frac{dV}{|r|},
\]

(4.2)

from which \( \phi_{i\theta} \) is traditionally modelled as

\[
\phi_{i\theta} = \phi_{i\theta 1} + \phi_{i\theta 2},
\]

(4.3)

where \( \phi_{i\theta 1} \) represents the turbulence interactions and \( \phi_{i\theta 2} \) depends on the mean strains. In buoyancy-affected flows there is a further contribution which will be discussed in [14].

4.1 Mean-Strain (or ‘Rapid’) Part of Pressure-Scalar Gradient Correlation: \( \phi_{i\theta 2} \)

Again, if the mean strain is assumed to be essentially constant over the volume of integration in equation (4.2), the \( \phi_{i\theta 2} \) process can be modelled as

\[
\phi_{i\theta 2} = 2b^l_{ki} \frac{\partial U_k}{\partial x_l},
\]

(4.4)

where the tensor \( b^l_{ki} \) represents the integral:

\[
b^l_{ki} = -\frac{1}{4\pi} \int_{\text{Vol}} \frac{\partial^2 \overline{u_i' \theta'}}{\partial r_k \partial r_i} \frac{dV}{|r|},
\]

(4.5)

Adopting the same approach to modelling this tensor as was done for \( \phi_{ij2} \), \( b^l_{ki} \) can be modelled in terms of the Reynolds stresses and scalar fluxes. However, the linearity principle (noting that the integral in equation (4.5) is linear in the scalar \( \theta \)) requires that an expansion for \( b^l_{ki} \), whilst possibly being nonlinear in the Reynolds stresses, should only depend linearly on the scalar fluxes. Including all possible terms which satisfy the required symmetry in \( i \) and \( k \), such an expansion up to cubic order can be written as

\[
b^l_{ki} = \alpha_1 \overline{u_i \theta \delta_{ik}} + \alpha_2 \overline{u_k \theta \delta_{li} + u_i \theta \delta_{lk}}
+ \alpha_3 \overline{u_i \theta a_{ik}} + \alpha_4 \left( \overline{u_k \theta a_{li} + u_i \theta a_{lk}} \right)
+ \alpha_5 \overline{u_k \theta a_{ml}a_{ij}} + \alpha_6 \overline{u_m \theta (a_{mk} \delta_{ij} + a_{mj} \delta_{ik})}
+ \alpha_7 \overline{u_k \theta a_{ml}a_{ik}} + \alpha_8 \overline{u_m \theta (a_{mk} a_{il} + a_{mi} a_{kl})}
+ \alpha_9 \overline{u_k \theta a_{ml} a_{mk} + \alpha_{10} a_{ml} \left( \overline{u_k \theta a_{im} + u_i \theta a_{km}} \right)}
+ a_{mn} a_{mn} \left( \alpha_{11} \overline{u_i \theta \delta_{ik}} + \alpha_{12} \left( u_i \theta \delta_{lk} + u_k \theta \delta_{li} \right) \right)
+ a_{mn} a_{nm} \left( \alpha_{13} a_{ml} \delta_{ik} + \alpha_{14} \left( a_{mk} \delta_{li} + a_{mi} \delta_{lk} \right) \right),
\]

(4.6)
Constraints similar to those applied in the modelling of $\phi_{ij2}$ can now be applied to determine as many of the model coefficients as possible. The equivalent continuity and normalization conditions give:

- Continuity: $b_{ki}^k = 0$
- Normalization: $b_{kk}^l = \frac{u_l}{\bar{u}} \theta$

and applying them leads to the eight relations:

\[
\begin{align*}
\alpha_1 + 4\alpha_2 &= 0 \quad (4.7a) \\
\alpha_3 + \alpha_4 + \alpha_5 + 4\alpha_6 &= 0 \quad (4.7b) \\
\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10} + \alpha_{13} + 4\alpha_{14} &= 0 \quad (4.7c) \\
\alpha_{10} + \alpha_{11} + 4\alpha_{12} &= 0 \quad (4.7d) \\
3\alpha_1 + 2\alpha_2 &= 1 \quad (4.7e) \\
2\alpha_4 + 3\alpha_5 + 2\alpha_6 &= 0 \quad (4.7f) \\
2\alpha_8 + 2\alpha_{10} + 3\alpha_{13} + 2\alpha_{14} &= 0 \quad (4.7g) \\
\alpha_9 + 3\alpha_{11} + 2\alpha_{12} &= 0. \quad (4.7h)
\end{align*}
\]

Before considering what 2-component limit constraint should be applied, note that if only the linear terms are retained (so only $\alpha_1$ and $\alpha_2$ are non-zero) the above equations yield $\alpha_1 = 0.4$, $\alpha_2 = -0.1$, leading to the linear QI model of Launder (1973) (see also, Launder 1975; Lumley 1975):

\[
\phi_{i\theta^2} = 0.8u_k \bar{\theta} \frac{\partial U_i}{\partial x_k} - 0.2u_k \bar{\theta} \frac{\partial U_k}{\partial x_i}. \quad (4.8)
\]

Returning to the question of satisfying the 2-component limit, Shih and Lumley (1985) applied a constraint that ensured the Schwarz inequality,

\[
(u_\alpha \theta)^2 \leq u_\alpha^2 \bar{\theta}^2, \quad (4.9)
\]

could not be violated. They did this by imposing the condition that the rate of change of the difference $(u_\alpha \theta)^2 - u_\alpha^2 \bar{\theta}^2$ should be zero when equality held or, mathematically,

\[
2u_\alpha \theta \frac{D u_\alpha \theta}{Dt} = u_\alpha^2 \frac{D \bar{\theta}^2}{Dt} + \bar{\theta}^2 \frac{D u_\alpha^2}{Dt}, \quad (4.10)
\]

when $(u_\alpha \theta)^2 = u_\alpha^2 \bar{\theta}^2$.

However, this relation links the models for $\phi_{ij2}$ and $\phi_{i\theta^2}$, and the outcome was that not only did Shih and Lumley (1985) determine all the coefficients in $b_{ki}^k$, but the above constraint also led to both free coefficients in the TCL model of $\phi_{ij2}$ being determined as zero.
Unfortunately, it was the $c_2$ and $c'_2$ terms that enabled good agreement with simple shear flow experiments. Without them, Shih and Lumley were forced to add some additional, arbitrary, higher-order correction terms to their model in order to get the correct stress levels in shear flow. There is, moreover, a further objection to the Shih and Lumley formulation, in that, if a genuine passive scalar is being considered, the thermal field modelling should not influence the modelling of the underlying dynamic field.

For these reasons, workers at UMIST have adopted an alternative approach, by ensuring that the nett contribution to the $u_i\theta$ transport equation, $\frac{\partial U_k}{\partial x_l} b^l_{k2} = \frac{1}{2} u_i\theta \frac{\partial U_2}{\partial x_l}$ (4.11)

when $u_2 = 0$.

By again considering the situation in principal axes of the stresses, this condition leads to the six equations

\begin{align*}
\alpha_1 - \frac{2}{3}\alpha_3 + \frac{1}{3}\alpha_5 - \frac{2}{9}\alpha_7 + \frac{4}{9}\alpha_9 + \frac{2}{3}\alpha_{11} + \frac{1}{9}\alpha_{13} &= \frac{1}{2} \quad (4.12a) \\
\alpha_5 - \frac{2}{3}\alpha_7 + \frac{2}{3}\alpha_{13} &= 0 \quad (4.12b) \\
2\alpha_{11} + \alpha_{13} &= 0 \quad (4.12c) \\
\alpha_2 - \frac{2}{3}\alpha_4 + \frac{1}{3}\alpha_6 - \frac{2}{9}\alpha_8 + \frac{4}{9}\alpha_{10} + \frac{2}{3}\alpha_{12} + \frac{1}{9}\alpha_{14} &= 0 \quad (4.12d) \\
\alpha_6 - \frac{2}{3}\alpha_8 + \frac{2}{3}\alpha_{14} &= 0 \quad (4.12e) \\
2\alpha_{12} + \alpha_{14} &= 0. \quad (4.12f)
\end{align*}

Solving these, together with the earlier continuity and normalization equations, leads to the result

\begin{align*}
\alpha_1 &= 0.4 & \alpha_8 &= 1/8 - 1/2\alpha_7 \\
\alpha_2 &= -0.1 & \alpha_9 &= -1/8 + \alpha_7 \\
\alpha_3 &= -1/6 & \alpha_{10} &= -1/2\alpha_7 \\
\alpha_4 &= -1/6 & \alpha_{11} &= 1/20 - 1/2\alpha_7 \\
\alpha_5 &= 1/15 & \alpha_{12} &= -1/80 + 1/4\alpha_7 \\
\alpha_6 &= 1/15 & \alpha_{13} &= -1/10 + \alpha_7 \\
\alpha_{14} &= 1/40 - 1/2\alpha_7.
\end{align*}

with, apparently, one free coefficient. However, the term multiplied by $\alpha_7$ can be shown to be identically zero, and hence the resulting model for $\phi_i\theta_2$ can be
written:

\[
\phi_{i\theta 2} = 0.8u_k\eta \frac{\partial U_i}{\partial x_k} - 0.2u_k\eta \frac{\partial U_k}{\partial x_i} + \frac{1}{3} \varepsilon u_i\eta P \varepsilon - 0.4u_k\eta a_{il} \left( \frac{\partial U_k}{\partial x_l} + \frac{\partial U_l}{\partial x_k} \right) \\
+ 0.1u_k\eta a_{ik}a_{ml} \left( \frac{\partial U_m}{\partial x_l} + \frac{\partial U_l}{\partial x_m} \right) - 0.1u_k\eta (a_{im}P_{mk} + 2a_{mk}P_{im}) / k \\
+ 0.15a_{ml} \left( \frac{\partial U_k}{\partial x_l} + \frac{\partial U_l}{\partial x_k} \right) (a_{mk}u_i\eta - a_{mi}u_k\eta) \\
- 0.05a_{ml} \left[ 7a_{mk} \left( u_i\eta \frac{\partial U_k}{\partial x_l} + u_k\eta \frac{\partial U_l}{\partial x_l} \right) \\
- u_k\eta \left( a_{ml} \frac{\partial U_l}{\partial x_k} + a_{mk} \frac{\partial U_k}{\partial x_l} \right) \right],
\]

(4.13)

where there are no free coefficients.

4.2 Turbulence (or ‘Slow’) Part of Pressure-Scalar Gradient Correlation: \( \phi_{i\theta 1} \)

To model \( \phi_{i\theta 1} \) in a similar manner to \( \phi_{ij1} \), the obvious extension to the linear model of Monin (1965), would be to employ an expression of the form

\[
\phi_{i\theta 1} = -c_{\theta 1} \varepsilon \frac{u_i\eta}{k} - c_{\theta 1}^{\prime} \frac{\varepsilon}{k} a_{ij} \frac{u_j\eta}{\theta} - c_{\theta 1}^{\prime\prime} \frac{\varepsilon}{k} a_{ik}a_{kj} \frac{u_j\eta}{\theta} - c_{\theta 1}^{\prime\prime\prime} \frac{\varepsilon}{k} A_2 \frac{u_i\eta}{\theta}.
\]

(4.14)

Considering this expression in principal axes of the stresses, it is clear that such a model satisfies the condition that \( \phi_{2\theta 1} \) should vanish when \( u_2^2 = 0 \) regardless of the values of the model coefficients. The UMIST group has therefore employed a form similar to this, allowing the coefficients to be functions of the stress invariants, and tuning them to a range of free shear flows.

The exact expression for \( \phi_{i\theta 1} \) does not depend explicitly on mean scalar gradients, and these have thus not traditionally appeared in the modelled process. However, Jones and Musonge (1983) argued that the fluctuating quantities do, nevertheless, depend on mean gradients, and thus included a term in their model for \( \phi_{i\theta} \) which did explicitly contain the mean scalar gradient. Craft (1991) also found it beneficial to include some explicit mean scalar gradient dependence in order to capture simple homogeneous shear flows at different strain rates. The form employed for \( \phi_{i\theta 1} \) in this latter work was

\[
\phi_{i\theta 1} = -c_{\theta 1} \varepsilon \left[ u_i\eta(1 + c_{\theta 1}^{\prime\prime\prime} A_2) + c_{\theta 1}^{\prime} a_{ik}u_k\eta + c_{\theta 1}^{\prime\prime} a_{ik}a_{kj}u_j\eta \right] - c_{\theta 1}^{\prime\prime\prime} \frac{r \kappa a_{ij}}{x_j} \frac{\partial \Theta}{\partial x_j},
\]

(4.15)
where

\[ c_{\theta 1} = 1.7 \left[ 1 + 1.2(A_2 A)^{1/2} \right], \quad c_{\theta 1}' = -0.8, \quad c_{\theta 1}'' = 1.1, \]

and the timescale ratio \( r \) is defined as \( r = (2\varepsilon_\theta / \overline{\theta^2})(k/\varepsilon) \). In this case the factor \( A^{1/2} \) in the coefficient \( c_{\theta 1}' \) ensures that this part of the model also satisfies the 2-component limit.

The parameter \( r \) represents the ratio of mechanical to thermal timescales, where \( 2\varepsilon_\theta \) is the dissipation rate of the scalar variance \( \overline{\theta^2} \). A common approach is to assume a constant value for \( r \), although available data shows that it takes significantly different values in different flows, and that such an approach does not, therefore, have a wide range of applicability. Craft et al. (1996) proposed modelling \( r \) as a function of the scalar flux invariant \( A_{2\theta} \equiv u_i \theta u_i \theta / (k \overline{\theta^2}) \), taking

\[ r = 1.5(1 + A_{2\theta}) \quad (4.16) \]

The above form was shown to give good predictions in a range of shear flows, including buoyancy-affected flows. Such a correlation does, nevertheless, have its limitations and the most reliable route for obtaining \( r \) would be to solve a suitable transport equation for the dissipation rate \( \varepsilon_\theta \). A number of such equations have been proposed (see, for example Newman et al. 1981; Jones and Musonge 1983; Shih et al. 1985; Craft and Launder 1989; Nagano et al. 1991) although it must be conceded that few of these have been applied over a very wide range of flows, and there is thus relatively little agreement on the exact form that such an equation should take. In the examples below, in order to focus attention on the modelling of the scalar fluxes, the timescale \( r \) has either been prescribed (from available data) or obtained from the correlation of equation (4.16). A comprehensive account of recent approaches to modelling the \( \varepsilon_\theta \) equation in near-wall heat transport is provided in [6].

5 Applications to the computation of the scalar field in free shear flows

Many important applications where scalar transport is of interest involve the prediction of heat or mass transfer rates to or from a solid surface. In such situations, however, the overall scalar transport is dominated by the flow behaviour in the near-wall sublayer, where viscous effects must be considered. Since, in this chapter, only high-Reynolds-number modelling has been considered, the examples presented relate only to free flows: in particular, the scalar field development in simple shear flows and in the plane and round jets.

\(^2r \) is the reciprocal of the timescale ratio \( R \) introduced in Chapter [2].
Figure 10: Thermal field development in weakly strained homogeneous shear flow. Solid line: TCL model, Broken line: Basic RSM. From Craft (1991).


Figures 10 and 11 show scalar field results in homogeneous shear flows, with a mean scalar gradient applied in the same direction to the shear. The figures plot the development of the ratio of streamwise to cross-stream scalar fluxes, and the turbulent Prandtl number, defined as $\sigma_t = (\overline{w\theta} d\Theta/dy)/(\overline{v\theta} dU/dy)$, against non-dimensional distance along the wind tunnel, $\tau = (x/U)dU/dy$.

Figure 10 corresponds to a case with a relatively low mean strain, resulting in turbulence not too far from local equilibrium, whilst Figure 11 relates to the case measured by Tavoularis and Corrsin (1981) at a higher mean strain rate. Although both the TCL and the widely used linear Basic Model give reasonable predictions when the flow is close to local equilibrium, the additional terms built into the TCL model clearly give much better predictions of the scalar fluxes at the higher strain rate.
As was seen in Section 3.1, the TCL model resulted in a better prediction of the hydrodynamic spreading rates of free jets than did the Basic Model. Table 3 shows the predicted scalar spreading rates in the plane and axisymmetric jets, obtained with both the TCL and the Basic models, together with experimental values. The predicted values are certainly not unreasonable, and the TCL model arguably returns slightly better predictions, although there is clearly room for further improvement. As discussed by Craft (1991), however, the TCL results can be improved by a more elaborate modelling of the timescale ratio $r$.

<table>
<thead>
<tr>
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<th>Round Jet</th>
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<tr>
<td>Experiment</td>
<td>0.140</td>
<td>0.110</td>
</tr>
<tr>
<td>Basic Model</td>
<td>0.145</td>
<td>0.131</td>
</tr>
<tr>
<td>TCL Model</td>
<td>0.132</td>
<td>0.127</td>
</tr>
</tbody>
</table>

Table 3: Scalar field spreading rates of free jets in stagnant surroundings.

Further applications of the models, to buoyancy-affected flows and to separated and impinging flows, will be presented in [14] and [11].

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