STATISTICAL PROPERTIES OF SOLAR-TYPE CLOSE BINARIES

J.-L. Halbwachs, 1 M. Mayor, 2 S. Udry, 2 and F. Arenou 3

RESUMEN
Con el propósito de investigar las propiedades estadísticas de las binarias con períodos hasta de 10 años se combinan dos estudios de velocidades radiales efectuados con Coravel, y dedicados a las estrellas de campo F7-K y a los cúmulos abiertos. Gracias a las paralajes trigonométricas exactas proporcionadas por Hipparcos, se escoge una muestra no sesgada de binarias espectroscópicas (SB). Después de corregir las incertidumbres de las mediciones, se obtienen los siguientes resultados. La distribución de cocientes de masas muestra un pico para binarias con masas iguales (gemelas), el cual es mayor para binarias de corto período que para las de largo período. Además de las "gemelas", la distribución de cocientes de masa muestra otro pico ancho entre 0.2 y 0.6. Las excentricidades orbitales de las "gemelas" son un poco menores que las de las otras binarias. Se observa un exceso de SB con períodos más cortos que 50 días en relación a la distribución log-normal de Duquennoy y Mayor. Estas características sugieren que las binarias cerradas se generan por dos procesos distintos. Una posible diferencia podría deberse a la aceleración hacia la binaria, por ejemplo, procedente de un envolvente común, o de un disco circumbinario. Por otra parte, las "gemelas" podrían deberse a la evolución dinámica de sistemas múltiples. No queda claro si los modelos de formación han alcanzado ya el refinamiento necesario para reproducir nuestras estadísticas.

ABSTRACT
Two Coravel radial velocity surveys dedicated to F7-K field dwarfs and to open clusters are merged in order to investigate the statistical properties of binaries with periods up to 10 years. Thanks to the accurate trigonometric parallaxes provided by Hipparcos, an unbiased sample of spectroscopic binaries (SB) is selected. After correction for the uncertainties of the measurements, the following results are obtained: 1. The distribution of mass ratios exhibits a peak for equal-mass binaries (twins), which is higher for short-period binaries than for long-period binaries. 2. Apart from the twins, the distribution of mass ratios exhibits a broad peak from 0.2 to 0.6. 3. The orbital eccentricities of twins are slightly smaller than those of other binaries. 4. An excess of SB is observed with periods shorter than about 50 days in comparison with the Duquennoy and Mayor log-normal distribution of periods. These features suggest that close binary stars are generated by two different processes. A possible difference could come from the accretion onto the binary, for instance from a common envelope or from a circumbinary disk. Alternatively, twins could come from dynamic evolution of multiple systems. It is not clear whether the formation models are already sufficiently elaborated to reproduce our statistics.

Key Words: BINARIES: GENERAL — BINARIES: SPECTROSCOPIC — STARS: FORMATION

1. INTRODUCTION
The formation of binary stars is still poorly understood, and observational constraints are helpful in this investigation. Relevant clues may be derived from the statistical properties of binaries. The distributions of periods and of mass ratios are the most important: they may indicate which processes are efficient in forming binaries, and they help in understanding how these processes work. Major progress was achieved when Duquennoy & Mayor (1991) investigated an unbiased sample of G-type stars in the solar neighbourhood. Nevertheless, the number of binaries involved in their statistics was not very large: the properties of binaries with periods less than 10 years were derived from their so-called complete sample, which contained only 25 spectroscopic binaries (SB) with known orbital elements. A study of the detailed shape of the distributions was then difficult, and it was not possible to find correlations between the parameters. For this reason, the statistics of Duquennoy & Mayor has been revised, including more stars in the analysis (Halbwachs et al. 2003). We take this opportunity to improve the selection of the sample and the derivation of the mass ratios, thanks to the Hipparcos astrometric mission. Nevertheless, with the aim of homogeneity, only surveys carried out with the spectrovelocimeter.
CORAVEL are taken into account.

2. THE CORAVEL SURVEYS OF SOLAR-TYPE BINARIES

Two different surveys were dedicated to solar-type spectroscopic binaries: the first one concerned stars of the solar neighbourhood, and the second one stars in two nearby open clusters.

2.1. The solar neighbourhood survey

About 20 years ago, the stars in the catalogue of nearby stars (hereafter CNS) were put in the observation runs of the Swiss telescope operated at the Haute-Provence Observatory. This programme started on the basis of the second edition of the CNS (Gliese 1969), supplemented by Gliese & Jahreiss (1979), which became the third edition (Gliese & Jahreiss 1991) after a few modifications. 623 stars with spectral types F, G and K were observed 8 or 9 times each, over about 10 years. The errors of the radial velocity (hereafter RV) measurements were about 0.3 or 0.4 km/s. Each time a SB was found, it received additional observations in order to derive its orbital elements. The programme was prolonged more than 10 years after the completion of the detection survey, in order to get the orbital elements of all the SB with periods less than 10 years.

2.2. The Pleiades and Praesepe survey

Stars in the Pleiadias and in the Praesepe cluster were observed at least 4 times over about 8 years in order to detect SB (Mermilliod et al. 1992, Mermilliod, Duquennoy & Mayor 1994, Mermilliod & Mayor 1999). Again, the SB were observed until the orbital elements of the systems with periods up to 10 years were obtained. This programme involved 167 main sequence stars with B magnitudes brighter than 12.5 mag.

2.3. Merging both surveys: the extended and the unbiased samples

All of the SB found in the two surveys are taken into account in our statistics. However, their origins are kept in mind, since we need them to correct the counts for incomplete selection. An extended sample is defined with the aim of deriving the distribution of periods, but also the distribution of mass ratios for $q > 0.5$, nor the binary frequency. The reason is that a field binary with nearly equal-mass components, and therefore nearly equal-luminosity components, looks brighter than the primary component alone. If the selection of the sample is based on the apparent magnitudes of the stars, these systems are taken within a larger volume than those with small $q$. In order to avoid this bias, a sample defined on the basis of the true distances, or of the trigonometric parallaxs, is required. It appears from the Hipparcos catalogue that, although the CNS3 is far from exhaustive for $\pi \geq 46$ mas, a sample of field stars based on this limit is not significantly biased in favor of the intrinsically bright stars. Therefore, the unbiased sample includes 240 nearby stars with $\pi \geq 46$ mas. In the open clusters, when the secondary component contributes significantly to the magnitude of the system, the difference of magnitudes between the components is evaluated from the position of the SB above the main sequence. Therefore, the magnitudes of the primary components in the $B$ color, $B_1$, are derived for all SB, and an unbiased sample is obtained from the condition $B_1 < 12.5$ mag. The unbiased sample finally contains 407 stars, including 52 SB with periods up to 10 years.

3. THE SELECTION EFFECTS

Our ability to detect a SB and to derive its orbital elements depends on several parameters: the mass ratio, the period, the eccentricity, the orientation of the orbit in space, etc. As a consequence,
a SB survey is never complete, but, for each interval in mass ratio and period, a fraction of binaries are missed. Since we are interested in the properties of binaries as a whole, it is necessary to model the obtainability of a SB orbit in order to correct the statistics for incompleteness.

The detection of a SB, and the derivation of the orbital elements are two distinct processes. The detection of the SB was an objective operation. The $\chi^2$ of the RV measurements were computed, as well as the probabilities to get values so large or even larger assuming constant RV. The stars with $P(\chi^2) < 1\%$ were considered as SB candidates and were regularly observed.

After a SB is detected, the probability to get its orbital elements is still difficult to evaluate, since the process is rather complicated. It depends on the period, $P$, on the semi-amplitude of the radial velocity of the primary component, $K_1$, and on the eccentricity of the orbit, $e$. It comes from the distribution of the SB in the $(P, K_1/\sqrt{1-e^2})$ diagram that the orbits are obtained when the following conditions are satisfied:

- **When $q < 0.9$,**
  \[ K_1/\sqrt{1-e^2} > c \epsilon_{RV}, \]  \( (3) \)
  where $\epsilon_{RV}$ is the error of the RV measurements, and $c$ is a coefficient depending on the period, $P$. For the nearby stars,
  \[ c = \max(3, 0.647 \times (P_{\text{days}})^{0.222}); \]  \( (4) \)
  but, in the open clusters,
  \[ c = \max(4, (P_{\text{days}})^{0.848}/87.53). \]  \( (5) \)
- **When $q > 0.9$,** the condition is
  \[ K_1/\sqrt{1-e^2} > 23\epsilon_{RV}, \]  \( (6) \)
  since it is very hard to separate the spectra when the lines are blended and have similar shapes and intensities.

Moreover, the maximum mass ratio for a field SB1 seems to be around $q = 0.65$, since no SB1 have $q$ certainly larger than this limit, and, in the same time, this limit is very close to the smallest mass ratio found for a SB2.

The consequence of the conditions above are illustrated in Figure 1. It appears that our survey is efficient even in the brown dwarf range, as long as periods shorter than about one year are considered.

![Fig. 1. The probability to detect a SB and to derive its orbital elements, as a function of its period and of its mass ratio.](image)

4. THE DISTRIBUTION OF THE MASS RATIOS

4.1. Derivation of the mass ratios

The mass ratios of the SB2 are immediately derived from the orbital elements, since $q = K_1/K_2$. The treatment of the SB1 is more difficult, since it is not possible to calculate the mass ratio from the spectroscopic elements alone. We only know the mass function, $f_M$, which is converted in the mass ratio function, $Y$:

\[ Y = f_M/M_1 = \frac{q^3}{(1+q)^2 \sin^3 i}. \]  \( (7) \)

For 18 nearby SB1 among 34, it is possible to derive the inclinations of the orbits, $i$, from the astrometric measurements performed by the Hipparcos satellite. The mass ratios of these binaries are then directly obtained from Equation 7. Moreover, we have 3 SB1 for which the astrometric signature is too small to derive the inclination, but this small size is a relevant limitation on the mass ratio. This constraint is also taken into account (see below).

In the open clusters, the binaries with large mass ratios are above the main sequence in the $(B - V, V)$ diagram. For 7 SB1 among 22, this deviation is sufficient to derive $q$. For the 15 others, upper limits of $q$ are obtained.

We finally have 31 SB1 for which $q$ is not fixed. These binaries are taken into account in the derivation of the distribution of the mass ratios, using the Richardson–Lucy algorithm (Mazeh & Goldberg 1992, Boffin, Cerf & Paulus 1993). For each star, the probability density of the mass ratio is computed, taking into account the possible constraints coming from astrometry or photometry.
4.2. The corrected distribution of mass ratios

The distribution of mass ratios of the binaries in the sample is plotted in Figure 2. The left-hand side of the distribution, up to $q = 0.5$, comes from the extended sample, and the distribution for $q > 0.5$ is taken from the unbiased sample. The bias toward bright secondaries is thus avoided, but the distribution is still altered by incompleteness. For this reason, we implement the model used to derive Figure 1 in the calculation code. The distribution of mass ratios thus computed is no longer altered by selection effects. It is also plotted in Figure 2, with error bars derived by a Jackknife calculation.

Two main features appear on this figure. From left to right we first see a broad peak from $q \approx 0.2$ to $q \approx 0.7$. The rising part on the left wing is not due to incompleteness. The frequency is really low for $q$ corresponding to secondary masses in the brown dwarf range, as has already been pointed out from a slightly different sample and with another approach (Halbwachs et al. 2000). The gap around $q = 0.55$ doesn’t look significant, in contrast to the gap around $q = 0.75$. The second feature is the rising part from $q = 0.75$ to $q = 1$, which makes what we call the “peak of the twins” hereafter. This name was already given by Tokovinin (2000) to a high frequency of binaries with nearly equal-mass components, but our definition is slightly different since Tokovinin referred to mass ratios larger than 0.95. It is worth noticing that the frequency of twins increases when the selection effects are taken into account, but the peak is already significant in the raw data.

4.3. Correlation with other parameters

We consider whether the distribution of mass ratios is correlated with other parameters. The field nearby SB are compared to the SB in the open clusters, and it appears that both subsamples have the same distribution. We also compare the F7-G SB to the K-type SB, and, again, no discrepancy is found. The gap around $q = 0.75$ is clearly present for both subsamples. Moreover, it is also visible in the sample of M-type SB observed by Marchal et al. (2004). This indicates that the distribution of mass ratios doesn’t depend on $M_1$, in contrast to the distribution of $M_2$.

When the short period binaries are compared to the binaries with long periods, the peak of the twins is significantly smaller for the latter. This effect appears clearly in Figure 3, where a limit of period of 50 days is applied. The limit of 50 days is chosen since it is close to the median of the sample. Transition between a high twin peak and a low twin frequency regime seems to be rather smooth in reality, since the proportion of twins among SB gradually decreases when longer periods are considered.

The distributions of mass ratios for short and for long period binaries are also given in Table 1, since they may be useful for simulations involving binary stars.

5. THE DISTRIBUTION OF PERIODS

The distribution of periods is plotted in Figure 4. The distribution corrected for incompleteness was derived at the same time as the corrected distribution of mass ratios, and it is also represented in this figure. The log-normal distribution proposed by Duquennoy & Mayor (1991) is plotted for comparison.
TABLE 1
THE DISTRIBUTIONS OF MASS RATIOS\(^a\)

| \(q\) | \(f_q(q|P < 50\text{d})\) | \(f_q(q|P > 50\text{d})\) |
|---|---|---|
| 0. | 0.422 | 0.496 |
| 0.083 | 0.422 | 0.496 |
| 0.25 | 1.065 | 1.544 |
| 0.417 | 0.994 | 1.094 |
| 0.583 | 0.962 | 1.466 |
| 0.75 | 0.245 | 0.604 |
| 0.917 | 2.312 | 0.796 |
| 1. | 2.312 | 0.796 |

\(^a\)We make a distinction between the binaries with periods shorter than 50 days and those with periods longer than 50 days. This is for practical reasons, but the transition is rather smooth in reality.

Fig. 4. The distribution of periods. The histogram in light dashes refers to the observed SB of the solar neighbourhood, and the histogram in heavy dots represents all the extended sample, including the SB in open clusters. The solid line is the distribution corrected for selection effects, and the big dots refer to the log-normal distribution of Duquennoy & Mayor.

The corrected distribution of periods exhibits two peaks, but none is really significant. Nevertheless, the former is a broad peak around 20 days, and it is rather tempting to relate it to the large frequency of twins which is expected in this range of periods. The second peak corresponds to periods between 1 and 2 years. It is due to an excess in the count of field SB in one bin, which probably comes from random variations. In conclusion, the true distribution of \(\log P\) could be constant from 12 days to 10 years.

The agreement with the log–normal distribution is rather bad, with a rejection threshold a bit smaller than 5 %. Again, this comes from the two peaks mentioned above. Therefore, the rising part from 1 to 20 days could be due to the contribution of twins, but the low frequency of periods longer than about 1000 days looks questionable. In the very last bin of the distribution, it could come from the cluster SB, which could be less frequent among the long periods than field SB. However, we have verified that the differences between the field binaries and the cluster binaries are not significant. Anyway, the shape of the distribution of periods around 10 years will appear clearly when the visual binaries will be added to our statistics (Eggenberger et al. 2003).

6. BINARY FREQUENCY

The raw binary frequency in the unbiased sample is 51 SB among 405 stars. When the selection effects are taken into account, the number of SB which are omitted is 4.8. This estimation doesn’t include the companions in the planetary range, but only the mass ratios larger than about 0.03. The frequency of binary and multiple systems with periods shorter than 10 years is finally \(13.5 \pm 1.8 \%\).

When field stars and open cluster stars are considered separately, the corrected frequencies are 11.2 and 16.9 %, respectively. However, the difference is not statistically significant.

7. THE PERIOD–ECCENTRICITY DIAGRAM

Since the twins are clearly separate from the other binaries in the mass ratio distribution, and since they are more frequent among short period binaries, it is suspected that they are generated by another formation process. Therefore, it would not be surprising if they were also different from the other binaries in the period–eccentricity diagram. For this reason, they are tagged in the diagram plotted in Figure 5.
It is visible that the mass ratios larger than 0.8 are preferably in the lowest part of the \((P-e)\) diagram. In order to calculate the significance of this effect, we draw the median eccentricity and we test if the excess of twins among the low-eccentricity SB is significant. For that purpose, we calculate the probability to randomly get an excess at least as large as the observed one: we have 24 twins among 80 SB with \(P > 5\) days, and 17 twins have eccentricities smaller than the median. When the eccentricities are not correlated with the mass ratios, the number of twins below the median, \(k_t\), should obey an hypergeometric law. The probability to get exactly \(k_t\) twins with \(e < \epsilon_{\text{median}}\) among a total of \(N\) SB is:

\[
P(k_t) = \frac{C^k_{k_t} C^{N-k_t}_{N-k_t}}{C^k_N}.
\]

It comes from Equation 8 that, with \(N = 80\) and \(k_t = 24\), the probability of having \(k_t\) as large as, or larger than the excess actually obtained is \(P(k_t \geq 17) = 1.35\%\). However, since we have an a priori no reason to expect an excess of twins among the low eccentricities rather than among the high eccentricities, we conclude that the rejection threshold of the null hypothesis is twice \(1.35\%\), i.e. \(2.7\%\). This is still enough to conclude that the eccentricities and the mass ratios are statistically correlated. When the test is repeated on the unbiased sample alone, we count 45 SB with \(P > 5\) days. Since we want to split the sample in two equal parts, one with \(e > \epsilon_{\text{median}}\) and the other with \(e < \epsilon_{\text{median}}\), we need an even number of SB. Therefore, we discard the SB which has exactly the median eccentricity in the last bin. We consider the remaining 44 SB, including 12 twins. Since 10 twins have eccentricities lower than the median, the rejection threshold is now 1.6\%. Therefore, the significance is still better than for the extended sample, although the number of SB involved in the test is smaller.

We verify this effect still adding the SB found in a survey of proper-motion stars (Goldberg et al. 2002, Latham et al. 2002). Only 6 SB are common to both samples (GJ 171, GJ 292.2, GJ 469.1, GJ 629.2A, GJ 793.1, and GJ 886), and we then obtain a sample of 273 SB. The period—eccentricity diagram of these stars is shown in Figure 6. Again, the twins are preferably found among the low eccentricities. The significance of this effect is derived, ignoring the SB with periods shorter than 10 days since these close orbits are often circularized. Therefore, the median is drawn for the 249 remaining SB. We count 43 with \(q > 0.8\), and 29 have eccentricities below the median. According to the hypergeometric law, the probability to get this excess, or an excess still larger, is only 0.9\%. This confirms the tendency of SB with large mass ratios to have smaller eccentricities than the other SB within the same period range. It is worth noticing that this effect is not restricted to the range of the short periods, where twins are the most frequent.

8. DISCUSSION AND CONCLUSION

We have investigated the properties of F7 to K main—sequence binaries with periods shorter than 10 years, and we obtained the following results:

- The binaries in the solar neighbourhood have the same properties as the binaries in open clusters.
- The frequency of binaries, corrected for selection effects, is \((13.5 \pm 1.7)\%\). This may be compared to the frequency of stars harbouring planets. Assuming that the G-type stars of the unbiased sample were the best observed in planet searches, the raw frequency of planets is \(7/102 = (6.9\pm1.8)\%\). Therefore, since the selection effects against planet detection are not taken into account, planetary systems could be as frequent as stellar systems.
- The distribution of \(\log P\) is steeply rising from 1 to 20 days, and smoothly rising afterward.
- The distribution of mass ratios is bimodal, with a low frequency of brown dwarf companions, and a minimum at \(q \approx 0.75\). As a consequence of the scarcity of brown dwarfs among short—period SB, the proportion of binaries which
could be confused with stars harbouring planets due to low orbital inclinations is only 0.3 %.

• Binaries with large mass ratios have periods shorter and orbits less eccentric than the other binaries.

Our sample is mainly constituted of disk stars, and it may be compared to the SB found among the proper-motion stars, which come from the halo and from the old disk. It is worth noticing that Goldberg, Mazeh & Latham (2003) also found a peak of twins in the distribution of mass ratios, but the gap between this peak and the main broad peak is shifted to the left: it is around \( q = 0.65 \) for the old disk, and around \( q = 0.55 \) in the halo. Therefore, the peak of twins is broader, more extended toward small mass ratios, when the age of the sample increases (or when the metallicity decreases).

These findings are important indications to the formation of binary stars. First at all, it seems that binaries are generated by two processes. The first one could be efficient on the complete range of periods, including the very wide pairs (see the distribution of mass ratios of wide pairs in Halbwachs et al. 1997). The other process, producing mass ratios larger than 0.8 (0.55 for low-metallicity stars), is particularly efficient among short periods. Moreover, the eccentricities are then slightly smaller than with the first process.

Although it would be hard to explain all these features with the current formation models, we may try to interpret the general trends. Twins could be generated by accretion on a seed binary with very low-mass close components (Bate 2001). Since each component nearly receives the same amount of mass from the accreting common envelope, the final mass ratio is then very close to one. Meanwhile, the close binaries with low mass ratios would then come from migration of components embedded in an accretion disk. However, the formation of a close seed binary is now considered as questionable, and, moreover, the expected mass ratios are then much larger than the observed limits (0.75, and even 0.55 for halo SB). Alternatively, large mass ratios and close separations may be produced by dynamical evolution of small clusters (Bate, Bonnell & Bromm 2002). However, it is not clear if this model can really explain all the observed properties.

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Jean-Louis Halbwachs: Observatoire Astronomique de Strasbourg, UMR 7550, 11 rue de l’université, 67000 Strasbourg, France (halbwachs@astro.u-strasbg.fr).
Michel Mayor and Stéphane Udry: Observatoire de Genève, 51 chemin des Maillettes, 1290 Sauverny, Switzerland (Michel.Mayor,Stephane.Udry@obs.unige.ch).
Frédéric Arenou: Observatoire de Paris-Meudon, bat. 11, 5 place Jules Janssen, 92195 Meudon Cedex, France (Frederic.Arenou@obspm.fr).
DISCUSSION

Udry — To follow up on Andrei’s and Cathy’s questions on the shape of $f(q)$, we have a similar study for M dwarfs using both precise RV measurements and adaptive optics imaging that allows us to get $q$ for all binaries without the need of any deconvolution process. We get the same distribution.

Hummel — Can you provide the reference for the Richardson-Lucy algorithm.

Halbwachs — The first application of the Richardson-Lucy algorithm to the derivation of the distribution of mass ratios in SB was presented simultaneously in 1992 by Boffin, Paulus, & Cerf and by Mazeh & Goldberg, on pp. 26 and 170, respectively, of Binaries as Tracers of Stellar Evolution, ed. A. Duquennoy & M. Mayor, Cambridge U. Press.

Zwitter — Can a small number of binaries with $q < 0.2$ be interpreted as a scarcity of secondaries with $M < 0.2 M_{\text{Sun}}$, so that earlier type binaries would have more commonly extreme mass ratios?

Halbwachs — The comparison F7-G to K does not show a difference in $f(q)$ for small $q$. However, the error bars are rather large, and we cannot rule out the hypothesis that, for the range $q \leq 0.2$, the distribution of $M_2$ is the same for both samples (instead of the distribution of $q$).

Valtonen — Are the twins more common among early type binaries? If not, it seems less likely that the twins-peak is dynamically produced.

Halbwachs — No, we found exactly the same frequency among F7-G binaries and among K-type binaries.

Tokovinin — The minimum of the mass-ratio distribution at $q = 0.75$ could result from not including the SB1/SB2 detection in the statistical model used for inverting $f(q)$; I made such a mistake in the past and later detected it by simulations.

Halbwachs — I took care of this kind of effect. I used various limits for the transition between SB1 and SB2, but the position of the gap was always at $q = 0.75$, even when the limit was much larger. Thanks to Hipparcos and to the $V - (B - V)$ sequences in the clusters, we have only few SB1 with large mass functions for which $q$ was not derived.

Abt — You compute secondary masses relative to F-K primaries. However, the primaries range by a factor of 2 in mass. What would happen to your mass distribution if you plotted against secondary mass instead of $q$?

Halbwachs — When we are using $M_2$ in place of $q$, the gap at $q \sim 0.75$ corresponds to values of $M_2$ varying with the spectral types of the primaries, and it is not visible any more. We just get a very broad “peak”.

Clarke — Have you tested the statistical significance of this minimum in the $q$ distribution? It would be interesting to test the data over the whole $q$ range against e.g., a uniform or linearly increasing distribution (with a K-S test) to assess if it’s a 2-sigma or 3-sigma result.

Halbwachs — Yes I did. The hypothesis $f(q) = \text{constant}$ from 0.7 to 1 was rejected by a Kolmogorov-Smirnov test at the 3% level.