relative leftmost path principle is, not the same as, but very close to a variant of β-model reflection.

In Chapter 6, we introduce a hierarchy dividing $\{\sigma \in \Pi^1_2 : \Pi^1_1\text{-CA}_0 \vdash \sigma\}$. Then, we give some characterizations of this hierarchy using some principles equivalent to Π^1_1-CA_0: leftmost path principle, Ramsey's theorem for Σ^0_n classes of $[\mathbb{N}]^{\mathbb{N}}$ and the determinacy of Gale–Stewart game for $(\Sigma^0_1)_n$ classes. As an application, our hierarchy explicitly shows that the number of applications of the hyperjump operator needed to prove Σ^0_n Ramsey's theorem or $(\Sigma^0_1)_n$ determinacy increases when the subscript n increases.

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Abstract

Arithmetic and logic seem to enjoy an especially close relationship. Frege once wrote that arithmetic is reason's nearest kin. To deny any of the basic laws of arithmetic seems tantamount to denying a basic law of logic. My dissertation is concerned with two great attempts to make something more of this informal idea. In one direction, Frege tried to reduce arithmetic to nothing but quantificational logic and definitions. *Neologicists* continue to follow in Frege's footsteps, pursuing a version of this program today. In the other direction, Gödel tried to reduce certain applications of quantificational logic to nothing but arithmetic and definitions, by means of his *Dialectica translation*.

In the first half of my dissertation, I prove new theorems (with Jeremy Avigad) that shed a surprising light on the prospects for neologicism. An important objection against neologicism is that it makes use of allegedly stipulative definitions that are not *conservative* over pure logic, i.e., definitions that yield new consequences expressible in old vocabulary. This violates a basic requirement on stipulative definitions. I argue that by passing to a richer logical and definitional framework, it is possible to overcome the conservativeness objection. However, there is a subtlety: the strategy succeeds only if conservativeness is understood semantically rather than deductively. This suggests that the viability of neologicism is highly sensitive to the way in which epistemic commitments are represented in formal theories.

In the second half of my dissertation, I argue that Gödel's Dialectica translation succeeds in assigning a constructive meaning to quantificational theories of arithmetic. Virtually all commentators have objected that Gödel's translation makes use of definitions which presuppose the very quantificational logic that Gödel was trying to eliminate. This would render the translation philosophically circular. Gödel was adamant that there was no circularity here. He attempted to explain the matter in a page-long footnote, which, however, no one has been able to understand. I vindicate Gödel, showing that there is no circularity and answering a longstanding exegetical question in Gödel scholarship.

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