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Erratum to "Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson–Walker spacetimes". Math. Proc. Camb. Phil. Soc. (2012), 152, 365–383.

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The proof of Corollary 4·3 in our paper [**1**] is not correct because there is a mistake in the expression given for $\|X^* \wedge Y^*\|^2$ on page 374. In fact, the correct expression for this term is

$$\begin{aligned}\|X^* \wedge Y^*\|^2 &= \|X^*\|^2\|Y^*\|^2 - \langle X^*, Y^*\rangle^2 \\ &= 1 + \langle X, T\rangle^2 + \langle Y, T\rangle^2 \geqslant 1,\end{aligned}$$

and then the inequality (4·9) is no longer true. Observe that all the previous reasoning before the wrong expression for $\|X^* \wedge Y^*\|^2$ is correct.

The idea of Corollary 4·3 is to find simple and natural geometric hypothesis under which the sectional curvature of Σ is bounded from below and hence, by Lemma 4·2, one can guarantee that the Omori-Yau maximum principle holds on Σ for every semielliptic operator $L = \mathrm{Tr}(P \circ \mathrm{hess})$. When analyzing our (wrong) proof of Corollary 4·3, one realizes that the proof works with no problem if replacing the condition that the Riemannian fiber $\mathbb{P}^n$ has sectional curvature bounded from below by the condition that the Riemannian fiber $\mathbb{P}^n$ has non-negative sectional curvature. In that case, from (4·8) one sees that

$$\overline{K}(X, Y) \geqslant 0$$

and hence, since we are assuming that $\sup_\Sigma \|A\|^2 < +\infty$, one concludes from (4·7) that the sectional curvature of Σ is bounded below, as desired. Therefore, the following weaker version of Corollary 4·3 is true.

COROLLARY 1 (Correct statement of Corollary 4·3). *Let* $-I \times_\rho \mathbb{P}^n$ *be a generalized Robertson–Walker spacetime with warping function satisfying* $(\log \rho)'' \leqslant 0$ *and Riemannian fiber* $\mathbb{P}^n$ *having non-negative sectional curvature. Let* $f : \Sigma^n \to -I \times_\rho \mathbb{P}^n$ *be a* *complete spacelike hypersurface contained in a slab with* $\sup_\Sigma \|A\|^2 < +\infty$. *Then the sectional curvature of* Σ *is bounded from below and the Omori–Yau maximum principle holds on* Σ *for every semi-elliptic operator* $L = \mathrm{Tr}(P \circ \mathrm{hess})$ *with* $\sup_\Sigma \mathrm{Tr}P < +\infty$.

Corollary 4·3 was applied in the proofs of Theorem 4·5 and Theorem 4·6 in order to guarantee the validity of the Omori–Yau maximum principle on Σ. For that reason, in the correct statement of both results one needs to replace the condition that the Riemannian fiber $\mathbb{P}^n$ has sectional curvature bounded from below by the condition that the Riemannian fiber $\mathbb{P}^n$ has non-negative sectional curvature. We have then the following:

THEOREM 2 (Correct statement of Theorem 4·5). *Let* $-I \times_\rho \mathbb{P}^n$ *be a generalized Robertson-Walker spacetime whose warping function satisfies* $(\log\rho)'' \leqslant 0$, *with equality only at isolated points, and suppose that* $\mathbb{P}^n$ *has non-negative sectional curvature. Let* $f : \Sigma^n \to -I \times_\rho \mathbb{P}^n$ *be a complete spacelike hypersurface contained in a slab with* $H_k > 0$, *for some* $2 \leqslant k \leqslant n$, *and* $H_{i+1}/H_i = \mathrm{constant}$ *for some* $1 \leqslant i \leqslant k-1$. *Assume that* $\sup_\Sigma |H_1| < +\infty$ *and, for* $k \geqslant 3$, *that there exists an elliptic point in* Σ. *Then,* Σ *is a slice.*

THEOREM 3 (Correct statement of Theorem 4·6). *Let* $-I \times_\rho \mathbb{P}^n$ *be a generalized Robertson-Walker spacetime whose warping function satisfies* $(\log\rho)'' \leqslant 0$, *with equality only at isolated points, and suppose that* $\mathbb{P}^n$ *has sectional curvature bounded from below. Let* $f : \Sigma^n \to -I \times_\rho \mathbb{P}^n$ *be a complete spacelike hypersurface contained in a slab and assume that either*

(i) H_2 *is a positive constant, or*
(ii) H_k *is constant (with* $k \geqslant 3$*) and there exists an elliptic point in* Σ.

If $\sup_\Sigma |H_1| < +\infty$, *then* Σ *is a slice.*

On the other hand, Corollary 4·3 was used also in the proof of Theorem 5·6. However in this case the statement and the proof of Theorem 5·6 remains correct thanks to the following version of Corollary 4·3, which is stronger than Corollary 1 above:

COROLLARY 4. *Let* $-I \times_\rho \mathbb{P}^n$ *be a generalized Robertson–Walker spacetime and denote by* $K_\mathbb{P}$ *the sectional curvature of* $\mathbb{P}^n$. *Let* $f : \Sigma^n \to -I \times_\rho \mathbb{P}^n$ *be a complete spacelike hypersurface with* $\sup_\Sigma \|A\|^2 < +\infty$ *which is contained in a slab* $\Omega(t_1, t_2)$ *on which*

$$K_\mathbb{P} \geqslant c := \max_{[t_1,t_2]} \left(\rho^2(\log\rho)''\right).$$

Then the sectional curvature of Σ *is bounded from below and the Omori–Yau maximum principle holds on* Σ *for every semi-elliptic operator* $L = \mathrm{Tr}(P \circ \mathrm{hess})$ *with* $\sup_\Sigma \mathrm{Tr}P < +\infty$.

For the proof of Corollary 4 we follow the proof of Corollary 4·3 until equation (4·8). Using that

$$\begin{aligned}\|X^* \wedge Y^*\|^2 &= \|X^*\|^2\|Y^*\|^2 - \langle X^*, Y^*\rangle^2 \\ &= 1 + \langle X, T\rangle^2 + \langle Y, T\rangle^2 = 1 + \langle X, \nabla h\rangle^2 + \langle Y, \nabla h\rangle^2\end{aligned}$$

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we observe that (4·8) can be written as

$$\begin{aligned}\overline{K}(X, Y) &= \frac{1}{\rho^2(h)} K_{\mathbb{P}}(X^*, Y^*) \|X^* \wedge Y^*\|^2 \\ &\quad + ((\log \rho'))^2(h) - (\log \rho)''(h)(\langle X, \nabla h\rangle^2 + \langle Y, \nabla h\rangle^2) \\ &= \frac{1}{\rho^2(h)} K_{\mathbb{P}}(X^*, Y^*) + ((\log \rho'))^2(h) \\ &\quad + \left(\frac{1}{\rho^2(h)} K_{\mathbb{P}}(X^*, Y^*) - (\log \rho)''(h)\right)(\langle X, \nabla h\rangle^2 + \langle Y, \nabla h\rangle^2).\end{aligned}$$

Therefore, since $K_{\mathbb{P}} \geqslant c$ and

$$\frac{1}{\rho^2(h)} K_{\mathbb{P}}(X^*, Y^*) - (\log \rho)''(h) \geqslant 0,$$

we deduce

$$\overline{K}(X, Y) \geqslant \frac{1}{\rho^2(h)} K_{\mathbb{P}}(X^*, Y^*) \geqslant \frac{1}{\rho^2(h)}.$$

Finally, since h is a bounded function we conclude from (4·7) that $K_\Sigma(X, Y)$ is bounded from below by a constant.

REFERENCE

[**1**] L. J. Alías, D. Impera and M. Rigoli. Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson–Walker spacetimes. *Math. Proc. Camb. Phil. Soc.* **152** (2012), 365–383.