the central theorem resting on such shaky foundations. There is indeed a case for presenting Cauchy's theorem at different levels, but one of them should be a version that is free of dependence on these topological notions. As is by now widely recognised, the version involving a closed path in a star-shaped set satisfies this condition, and is sufficient for all the applications required in a first course.

There is a plentiful supply of exercises, and the technical presentation by Oxford University Press is superb.

G. J. O. JAMESON

ROSENBLUM, M. and ROVNYAK, J., *Hardy classes and operator theory* (Oxford University Press, USA, 1985), xii + 161 pp., £37.50.

This book treats a mature and substantial branch of classical function theory, centring on analytic functions on a disc or half-plane, using the insights and techniques of the theory of operators on Hilbert space. This particular conjunction of ideas has had a recent triumph with the proof of the Bieberbach conjecture by L. de Branges (the second-named author's sometime collaborator), but the present monograph addresses itself to quite other matters, barely touching on coefficient problems. The two main strands of the book are interpolation problems and factorization questions—for instance, does a given positive function on $\mathbb{R}$ admit a representation as the boundary value function of $|g|^2$ for some analytic function g in the upper half-plane? Such questions arise in a diversity of investigations of a "pure mathematical" nature, and they have been long studied. However, much of the impetus for present developments comes from applications. Factorization problems play a central role in prediction theory and in several areas requiring the solution of linear integral equations on a half-line, while variants of the interpolation problem for complex functions solved by Nevanlinna and Pick are highly topical in the design of control systems.

Classical studies concentrated on scalar analytic functions, whereas many of the applications deal in matrix- or operator-valued functions. Refinements of the old results are thus required, and operators are inescapably involved. However, even for scalar functions operator theory plays an important role: it is implicit in the early classical work. A bounded analytic function acts by multiplication on a suitable Hilbert space of analytic functions, and so is accessible to the highly developed concepts and techniques of operator theory. This approach does to some extent unify the theory, though it has to be used in conjunction with older methods: the book contains plenty of solid hard analysis.

Although the authors are aware of the applications, their own interest is in the fundamental underlying mathematical issues. They have made many contributions to the field themselves, and the choice of material in the book reflects the predilections evident in their research. They present a wealth of material from Nevanlinna–Pick and Loewner interpolation theory, the factorization of Toeplitz operators, Nevanlinna and Hardy classes of operator-valued functions, inner and outer functions and the factorization of operator functions. The style of the book is rigorous and economical to the point of terseness, and is marked by a consistent adherence to the strict logical progression from general to particular. Theorems are stated and proved in the fullest possible generality, with more concrete conclusions being relegated to somewhat hurried sections of "Examples and addenda". This policy allows the authors to cover a great deal of ground, but does not always bring out the aesthetic appeal of the results. It therefore places a burden on the reader's knowledge and experience and makes the book most suitable for a mathematician who is familiar with the scalar theory and wishes to learn about the analogue for operator-valued functions, rather than one who is trying to build up intuition from scratch. There is not much motivation, but there are historical notes pointing to a substantial bibliography.

There is one feature of the authors' treatment which, I believe, sets them apart from Schur, Nevanlinna and the many mathematicians who apply the theory: they place a strong emphasis on

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existence results and show no interest in methods of calculating the functions whose existence is asserted. They do achieve thereby some very short and slick proofs, but I question whether this is a worthwhile gain. Despite this, I unhesitatingly recommend this work of great scholarship in a fundamental and still active branch of analysis.

N. J. YOUNG

WEEKS, J. R., *The shape of space: how to visualize surfaces and three-dimensional manifolds* (Pure and Applied Mathematics Vol. 96, Marcel Dekker, New York, 1985), x + 324 pp., $59.50.

In this fascinating book the author's aim is to give the reader an intuitive understanding of three-dimensional manifolds with particular reference to the geometry and shape of the universe. It is also his hope that the book will be accessible to the "interested non-mathematician" though, recognising the dearth of nontechnical accounts of these topics, he suggests that mathematicians may also find it a useful general introduction. How accessible the book will be to the wider audience is perhaps a matter of definition but certainly all mathematicians will get from it a clear, nontechnical and intuitive exposition of current developments in the topology and geometry of 3-manifolds.

The author's starting point is E. A. Abbott's classic *Flatland: A Romance of Many Dimensions* first published in 1884. From there he proceeds to describe the construction of surfaces and 3-manifolds by glueing and takes care to distinguish between their topology and geometry. This occupies the first two-thirds of the book and this part ends with an account of the Gauss–Bonnet formula for surfaces with constant curvature. The final part starts by producing examples of 3-manifolds that admit elliptic, Euclidean or hyperbolic geometry and then contains a description of the eight homogeneous geometries that can arise on closed 3-manifolds. In the chapter the author explores possible geometries and global topologies for the universe.

The book is well supplied with clearly drawn diagrams and contains an instructive collection of exercises. At the end there is a useful set of solutions to the exercises as well as a bibliography. The latter refers the reader to a number of the standard texts for which the author has written an admirable complement.

R. M. F. MOSS

GROSSWALD, E., *Representations of integers as sums of squares* (Springer-Verlag, 1985) 251 pp., DM 148.

The study of the representations of a number as a sum of squares has a long history. Diophantus concerned himself with several problems of this type sixteen hundred years ago, but it was only towards the end of the seventeenth century that notable advances were made and valid proofs published. For a positive integer k denote by $r_k(n)$ the number of representations of the non-negative integer n as a sum of k squares of integers; thus $r_k(n)$ is the number of solutions of the Diophantine equation

$$x_1^2+x_2^2+\cdots+x_k^2=n \quad (x_i\in\mathbb{Z},\ 1\leq i\leq k). \tag{1}$$

The order of the x_i is taken into account so that, for example, $r_2(1)=4$, the solutions being $(x_1,x_2)=(\pm 1,0)$ and $(0,\pm 1)$. In his treatment the author follows the historical development of the subject, beginning with the case $k=2$.

Euler proved that, for $k=2$, (1) is soluble if and only if each prime divisor p of n, for which $p\equiv 3\,(\mathrm{mod}\,4)$, occurs in n to an even power. Later, the formula

$$r_2(n)=4\left\{\sum_{\substack{d|n\\ d\equiv 1\,(\mathrm{mod}\,4)}}1-\sum_{\substack{d|n\\ d\equiv 3\,(\mathrm{mod}\,4)}}1\right\}$$

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