Banach manifold, 24 Bastiani calculus, 7, 203 Boman's theorem, 202 bornology, 202 bump function, 195 Butcher group, 185 Butcher-Connes-Kreimer algebra, 182 canonical manifold of mappings, 41, 219 carrier of a function, 195 chain rule, 11, 19, 21 change of charts, 16 character group of a Hopf algebra, 178 character of an algebra, 177 Chen's relation, 160, 167, 168 Christoffel symbol, 91 CIA see continuous inverse algebra, 49 compact open C^{∞} -topology, 4, 30, 192, 198, 221 compact open topology, 13, 208 continuous inverse algebra, 49 unit group, 49 continuous projection, 194 convenient calculus, 202 convenient smooth map, 202 coordinate projection, 11 cotangent bundle, 21, 81 current group, 72 curvature, 96, 100 Bianchi identity, 101 sectional, 151 curve c-parallel, 99 energy of a, 84, 222 length of a, 84

passing through a point, 19 piecewise C^1 , 84 principal part of a, 99 smooth, 2 derivation of an algebra, 227, 230 derivative covariant, 94 covariant, along a map, 109 directional, 7 iterated directional, 8 left logarithmic, 62, 238 Lie derivative, 227, 236 metric derivative, 97 of a manifold-valued curve, 22 diffeomorphism group, 35, 50, 120 acting *n*-transitive, 121 canonical action, ix, 51 contactomorphisms, 52 stabiliser of a point, 121 symplectomorphisms, 52 volume-preserving, 52, 144, 242 differential form, 231, 240 (locally) integrable, 237 Lie derivative, 236 preserved by a diffeomorphism, 52 pullback, 235 volume form, 241 dual space, 4, 83, 193 embedding, 23 set of, 34 equation of Lie type, 60 Euler equations Eulerian form, 143 Lagrangian form, 145

264

Euler-Arnold equation, 152 Euler-Poincaré equation, 149 evaluation map, 39, 210 evolution map, 62 exponential law, 39, 211 exterior differential, 232 fibre product (of manifolds), 27 final topology, 206 flip of the double tangent bundle, 215 formal power series, 183 Fréchet calculus, 14 Fréchet space, 3, 198 Fundamental theorem of calculus, 5 geodesic, 88, 100, 139 of a spray, 92 of the L^2 -metric, 112 geodesic distance, 85 non-vanishing, 86 geodesic equation, 92, 99 gradient, 143, 242 Grossman's ellipsoid, 103 group of bisections, 126 vertical bisections, 130 group of gauge transformations, 77 Hölder continuity, 169 Hahn-Banach theorem, 4 half-Lie group, 155 Hilbert sphere, 16, 83 homogeneous space, 77, 183 Hopf algebra, 177 Hopf-Rinow theorem, 102 Hunter-Saxton equation, 153 immersion, 23 naïve, 24 set of, 33 infinitesimally injective (or surjective), 24 initial topology, 206 Kolmogorov's normability criterion, 192 Lie algebra, 55 associated to a Lie group, 57 current algebra, 72 Jacobi identity, 55 locally convex, 56 of (left-)invariant vector fields, 56 of divergence-free vector fields, 243 of vector fields, 228

Lie bracket, 55 of vector fields, 56 trivial, 56 Lie group, 48 Bourbaki construction principle, 70 exponential, 65 exponential coordinates, 66 left- (right-)translation, 48 Lie subgroup, 52 locally exponential, 66 regular (in the sense of Milnor), 62 semidirect product, 54 semiregular, 62 split exact sequence, 54 tangent Lie group, 53 Lie group action, 51 adjoint action, 60 Lie groupoid, 125 action groupoid, 127 bisection action groupoid, 134 current groupoid, 128 enough bisections, 135 gauge groupoid, 128 pair groupoid, 127 tangent groupoid, 132 unit groupoid, 127 vertex group, 125 Lie polynomial, 164 Lie series, 164 Lie theorems, 60 local addition, 43, 213 normalised, 218 local-to-global argument, 196 loop group, 74 Lyons lift, 170 Lyons' lifting theorem, 170 Mackey complete, 5, 187, 202 manifold atlas, 16 manifold chart, 16 Maurer-Cartan form, 237 mean value theorem, 9 Minkowski functional, 83, 192 multiplicative functional, 168.179 Neumann inversion formula, 161 Omori's theorem, ix, 155 orbifold, 124, 127 partition of unity, 195 pre-shape space, 114

principal bundle, 76, 129 gauge group, 76 structure group, 76 pro-Lie group, 166 product manifold, 17 pullback, 31, 45, 209 pushforward, 13, 31, 43, 209 Riemann-Stieltjes sum, 157, 158 Riemannian metric H^1 -metric, 113 \dot{H}^1 -semimetric, 153 L^2 -metric, 81, 108, 140 co-orthogonal structure, 102 elastic metric, 116 invariant L^2 -metric, 83, 88, 89, 144 pullback metric, 116 right/left invariant, 87 robust, 101 strong, 81 weak, 80 rough norm, 197 rough path, 170 branched, 181, 183 Brownian motion, 171 set of weakly geometric, 170 weakly geometric, 170 rule on partial differentials, 9, 22 Schwartz' theorem, 8, 12 seminorm, 3 basis condition, 191 fundamental system of, 191 generating family of, 3 separating family of, 190 shape analysis, 106 shape space, 114 shuffle algebra, 176, 183 signature of a smooth path, 162 smooth variation, 138 right-shifted, 146 space C^k -paracompact, 196 C^k -regular, 196 complemented subspace, 194 convenient vector space, 202 locally convex, 3, 192 metrisable, 3 of Hölder continuous functions, 169 of smooth p-forms, 232

sequentially closed, 12 short exact sequence, 25 topological vector space, 1, 186 spray, 91 associated covariant derivative, 95 connector, 96 metric, 92 metric spray, 97 square root velocity transform, 116 SRVT see square root velocity transform, 116 Stacey-Roberts Lemma, 45 standard simplex (Δ), 168 submanifold, 17 split, 17 submersion, 23 naïve. 24 set of. 33 support of a function, 195 tangent bundle, 19 tangent map, 21 tangent space, 19 tangent vector (geometric), 19 tensor algebra, 160 topology of compact convergence, 208 transversal, 26 unit group (of an algebra), 49 vector bundle, 215 direct product, 217 dual bundle, 21, 81 pullback, 216 set of all sections, 215 smooth section, 215 Whitney sum, 217, 220 vector bundle morphism, 215 vector field 225 (left) invariant, 56, 64 along a smooth map, 109 complete, 64 divergence, 143, 242 flow, 226 Helmholtz decomposition, 242 integral curve, 226 Lie bracket, 226 local representative, 225 principal part, 225

266

related, 226 second-order, 91 vector space of, 225 vector topology, 1 generated by seminorms, 3 Volterra series, 63, 162 weak integral, 4 wedge product, 234 Young integral, 158

zero-section, 43, 213, 220