

FIRST RESULTS OF THE INTEGRATION OF MOTION OF SHORT-PERIOD COMETS OVER 800 YEARS

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ABSTRACT. All the known short-period comets have been followed by numerical integration over a time span of 821 years, from 1585 to 2406. A preliminary survey of the results of these integrations has shown some interesting features, which become recognizable thanks to the length of the time interval covered, not negligible if compared with the typical evolutionary time scale of comets moving in short-period orbits. Interesting phenomena that have been recognized include: (1) captures from, or ejections into, very elongated ellipses, with perihelia of the parking orbits close to the orbit of Jupiter and aphelia within or beyond the region of outer planets; (2) passages of comets from the control of Saturn to that of Jupiter; (3) orbital evolutions controlled mainly by Saturn; (4) librations of comets around low-order resonances; (5) repeated close approaches of comets to Jupiter, often with the comet being captured as a temporary satellite; (6) an almost perfect coincidence of two comet orbits just before a close approach to Jupiter, suggesting their genetic relationship.

1. INTRODUCTION

The orbital evolution of all 126 known short-period comets has been followed, by integrating their motion over a long time span, from 1585 February 1.0 (JD 2300000.5) to 2406 June 17.0 (JD 2600000.5). This work (Long-Term Evolution Project, see Carusi et al., 1985a) has provided an atlas of potential cometary orbital evolutions, that is described in more detail in Carusi et al. (1984).

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Here we only want to make some comments about the LTEP, its precision and its reliability as a sample of the dynamical evolutions of real objects. First of all, not all the orbits that have been used to start the integrations are of high quality. Many of them - especially those of one-apparition comets - are known with limited accuracy and the corresponding evolutions simply represent possible behaviours of objects in specific regions of the phase space. Moreover, the objects were supposed to move in a purely gravitational dynamical system, where the action of nongravitational forces was completely ignored. For this reason, even the reconstructed motion of some comets with excellent starting orbits cannot be considered reliable before or after very close approaches to Jupiter that took place far from the starting date (see, for example, Carusi et al., 1985b). It should be clear to the user that, while the accuracy of the integrations is the same throughout, the extent of coincidence with the motion of the real objects is quite different and has to be estimated in each case individually.

Although these inaccuracies - and also those coming directly from the integration method used - render our sample of cometary evolutions not perfectly identical to the real sample, it can be used as a powerful mean for investigating the principal dynamical processes governing the motion of comets under the perturbing action of planets.

In this paper we will review some of the most interesting features singled out during a first analysis of the LTEP data. Some of them have already been pointed out and discussed in more detail by other authors, and are included with the relevant quotations for the sake of completeness. Some others will require further detailed studies, in order to depict a coherent scenario of the motion of comets captured into short-period orbits.

2. GENERAL CHARACTERISTICS

Owing to the different initial dynamical status, individual objects have widely different histories, including comets staying all the time very close to the starting orbit; objects oscillating for the whole integration, or only a part of it, about some resonance with Jupiter; and objects crossing different and widely separated regions. In order to overview the internal mobility of the sample, we have collected all the 126 osculating orbits computed at every 5,000 days, extrema included. This represents a total of 7,686 comet orbits: their distribution is shown in fig. 1, where the distribution parameter is the ratio between the comet's osculating period and the mean period of Jupiter. In fig. 1a all the orbits with periods between 0 and 2 times that of Jupiter are included. The distribution shows remarkable Kirkwood gaps, the

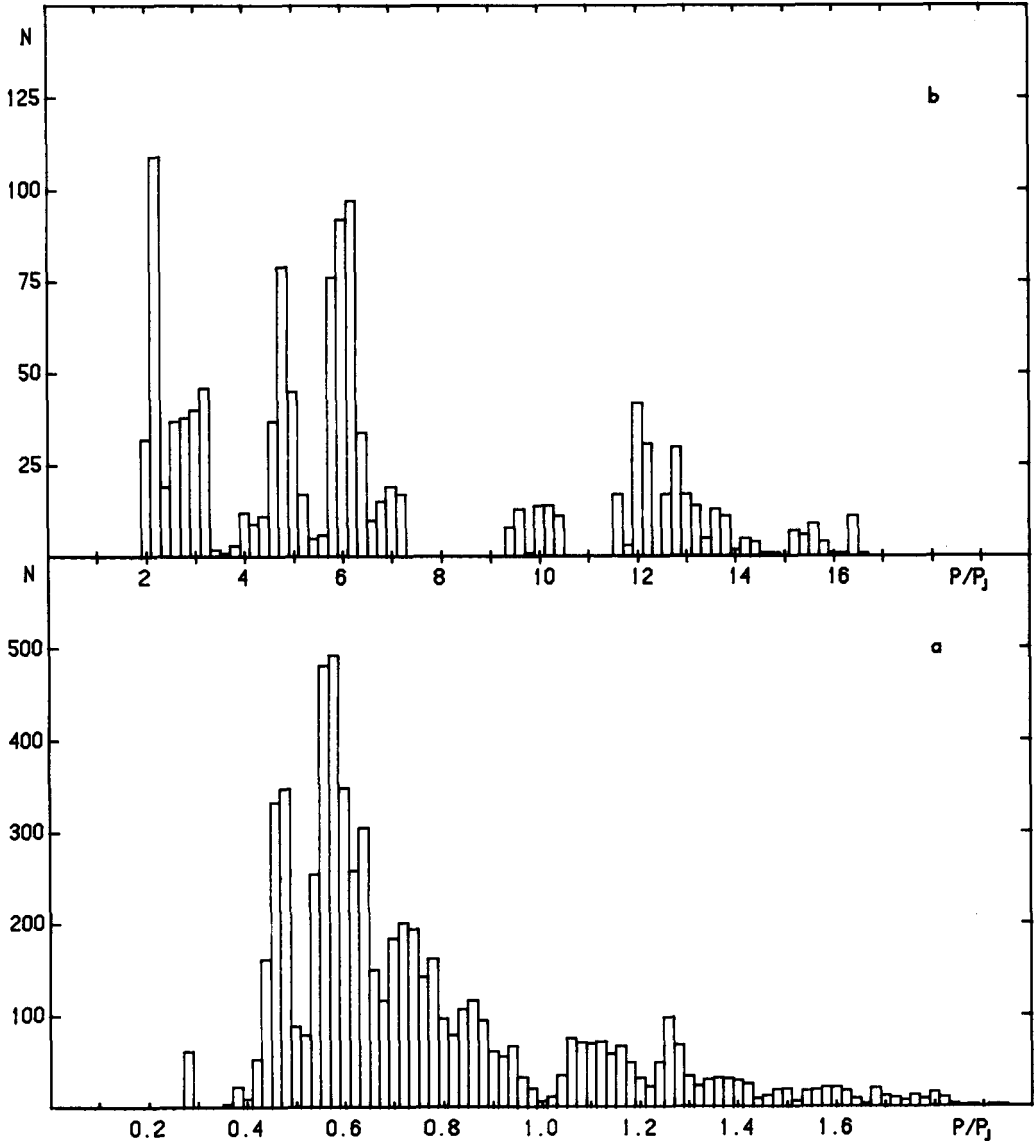


Figure 1. a: histogram of ratios of comet periods to Jupiter's period between 0 and 2. b: same as in plot a, for P/P_J between 2 and 20.

most prominent corresponding to the $1/2$, $3/5$, $2/3$, $3/4$, $4/5$ and $1/1$ resonances with Jupiter's motion. The single 61-points column on the left of the $1/3$ resonance is due to P/Encke. The gaps are at least partially produced by comets temporarily librating about the corresponding resonances, especially that of $1/2$. On the contrary, a concentration of orbits at 1.25 ($5/4$ resonance) is evident, representing mainly

the contribution of the orbits of P/du Toit and P/Gehrels 1.

Fig. 1b shows the same distribution for ratios of the periods between 2 and 20. Only two comets (P/Lexell after the ejection by Jupiter in 1779, and P/Wild 2 before the capture in 1702) move temporarily along orbits with periods longer than 20 Jupiter's periods. The long tail of the distribution beyond 2 is almost entirely due to 17 comets of intermediate periods (Halley type). Their contribution to the distribution,

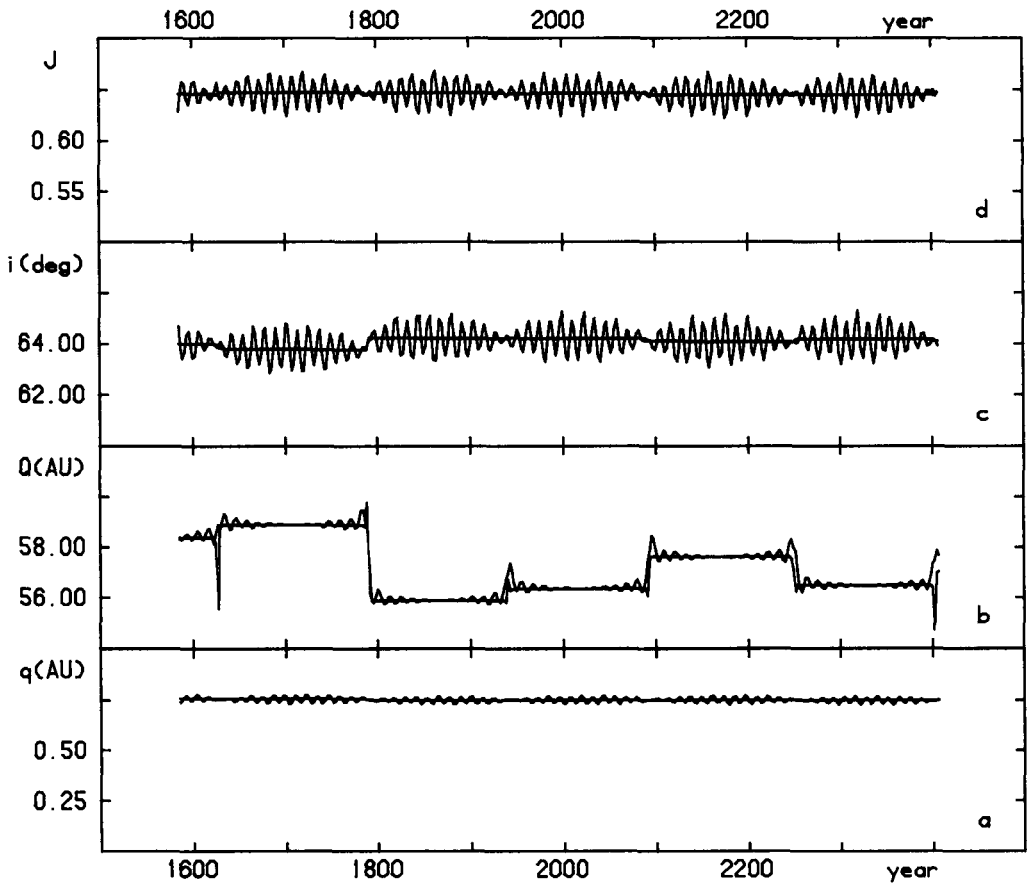


Figure 2. Time evolution of perihelion distance (a), aphelion distance (b), inclination (c), and Tisserand invariant with respect to Jupiter (d) of comet P/Herschel-Rigollet. For explanation see text.

however, is somehow peculiar: just as an example, only four comets of this type produce the wide-spread portion between 11 and 17, with a large overlapping of the points corresponding to different comets. The spread is maximum for P/Wilk, whose points range from 12 to 16.5, implying a variation of semimajor axis exceeding 6 AU.

The mechanism responsible for the spread is connected with the motion of the Sun about the barycentre of the solar system. A common feature of comets with aphelia well outside the orbit of Jupiter is a marked modulation of their heliocentric orbital parameters, which disappears when the orbit is computed with respect to the barycentre instead of the Sun. In fig. 2, for example, the evolution of four relevant orbital parameters of P/Herschel-Rigollet is shown. The quantities computed in the heliocentric frame exhibit the mentioned modulation, while their tracks become almost flat from perihelion to perihelion when computed in the barycentric frame. The sudden changes in the aphelion distance are not due to encounters with the major planets, since this comet does not approach any planet within 1 AU for the whole time span of the integration. They reflect, in fact, the different relative positions of the Sun and the major planets (mainly Jupiter) each time the comet enters

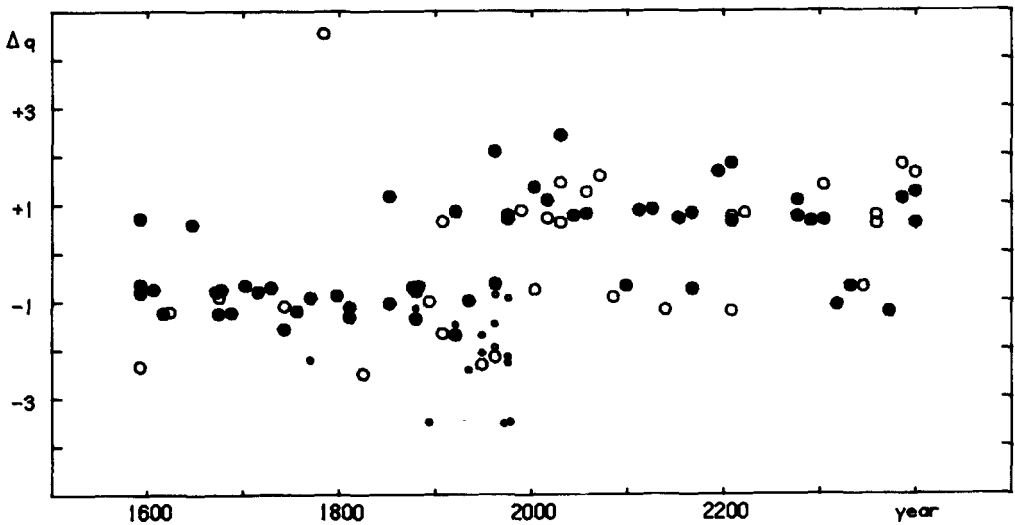


Figure 3. The 100 absolutely largest variations of perihelion distance Δq . Open circles: one-apparition comets; circles: more-than-one-apparition comets; dots: variations immediately before discovery.

the planetary region, and its motion becomes for a while heliocentric instead of barycentric.

Fig. 3 lists the 100 largest variations (both positive and negative) of the perihelion distance in the 821-year interval. One-apparition and more-than-one-apparition comets are marked by open and solid circles, respectively; small dots indicate that the corresponding variation has taken place immediately before the discovery of the comet, irrespectively if now of one or more apparitions. Obviously the latter variations are all negative, and concentrate in the XXth century, when most of these comets were discovered. Remarkable is a lack of strong positive variations in the past, reflecting a deficiency of accelerating encounters which would have rendered the discovery of such a comet essentially impossible. On the other hand, decelerating encounters are less frequent than the accelerating ones in the future. This is also understandable, because for a comet that has already suffered a strong deceleration it is more probable to undergo encounters leading to an increase of the perihelion distance instead of a further decrease (that could not be very large, anyway, because most of these comets are already close to Jupiter's barrier).

Only 42 comets contribute to the 100 strongest variations. Among them 16 are present only once; 9 twice; 9 three times; 3 four times (P/Gehrels 3, P/Reinmuth 2, and P/Wolf-Harrington); 4 five times (P/Bus, P/Smirnova-Chernykh, P/Ashbrook-Jackson, and P/Whipple). P/Shajn-Schal-dach contributes as much as 7 times. The four largest variations are due to P/Lexell (4.5 AU, the largest and the only positive among the four) and to P/West-Kohoutek-Ikemura, P/Brooks 2 and P/Wild 2 (-3.5 AU all the three).

A preliminary survey of close encounters with Jupiter, Saturn and Uranus has already been done. No comet encounters Neptune or Pluto, while a statistics of close approaches to the inner planets has not yet been performed. It is certainly possible that some event has remained hidden among the ten million numbers of the core files of LTEP, but we think that the more detailed analyses we are planning will not change the general statistics we are presenting here.

Table 1 lists the number of encounters per century within a sphere of 0.5 AU around each planet. The only encounter with Uranus is due to P/Tempel-Tuttle in 2317 (minimum distance from the planet: 0.29 AU). The eleven encounters with Saturn are listed in table 2.

As regards the encounters with Jupiter, we only list in table 3 the ten closest approaches: note that four of them are due to P/Gehrels 3. In Section 3.5 we will examine the history of these encounters in greater detail.

In fig. 4 the distribution of encounters with Jupiter to within 0.5 AU between 1601 and 2400 is shown (dates correspond to minima of dis-

TABLE 1. Number of encounters within a sphere of radius 0.5 AU

Period	Jupiter	Saturn	Uranus
1585-1600	9	0	0
1601-1700	67	2	0
1701-1800	63	2	0
1801-1900	82	1	0
1901-2000	91	1	0
2001-2100	78	0	0
2101-2200	80	1	0
2201-2300	76	1	0
2301-2400	73	2	1
2401-2406	3	1	0
<hr/>			
1585-2406	622	11	1

TABLE 2. Encounters with Saturn

Comet	Date	Minimum distance (AU)
P/IRAS	(1) 1950	0.06
		Orbit of Phoebe
P/IRAS	(2) 1627	0.17
P/Wild 1	(1) 2260	0.26
P/Tempel-Tuttle	(1) 1630	0.34
P/Chernykh	(2) 1749	0.36
P/Bowell-Skiff	(2) 1816	0.37
P/IRAS	(2) 2333	0.40
P/Wild 1	(1) 2113	0.41
P/Oterma	(2) 1770	0.41
P/Kowal-Vávrová	(2) 2324	0.43
P/Bowell-Skiff	(3) 2403	0.48

(1) Quite reliable (2) Less reliable (3) Unreliable

TABLE 3. The ten deepest encounters with Jupiter

Comet	Date	Minimum distance (AU)
P/Brooks 2	(1) 1886	0.0010
P/Gehrels 3	(1) 1970	0.0014
P/Lexell	(1) 1779	0.0015
Orbit of Io		
P/Gehrels 3	(3) 2305	0.0055
P/Wild 2	(1) 1974	0.0061
Orbit of Callisto		
P/West-Kohoutek-Ikemura	(1) 1972	0.0138
P/Gehrels 3	(2) 2062	0.0159
P/Gehrels 2	(2) 2029	0.0182
P/Brooks 1	(3) 1739	0.0184
P/Gehrels 3	(3) 2400	0.0190

(1) Quite reliable (2) Less reliable (3) Unreliable

tance). The highest concentration corresponds to the period 1951–1975, and is apparently due to the high percentage of comets discovered in the

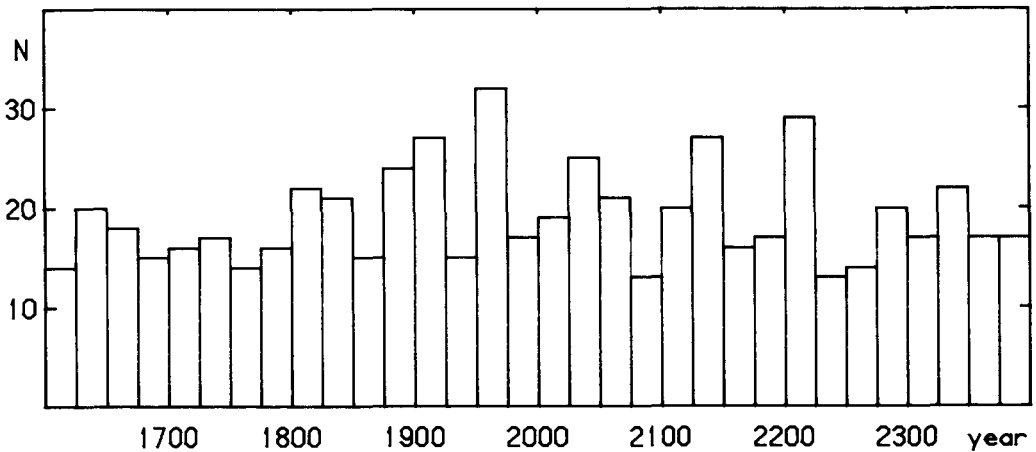


Figure 4. Time distribution of the encounters with Jupiter within a sphere of 0.5 AU around the planet.

sixties and seventies after a considerable reduction of their perihelion distance (see also fig. 3).

On the average there are 0.76 encounters with Jupiter per year: since in our sample the mean duration of an encounter within 1 AU from Jupiter is about 1.6 years, it appears that, between 1601 and 2400, there were, on the average, more than one of the known short-period comets within that distance from the planet at any time. For comparison, in the period 1951-1975 there were, on the average, two or three of these comets within 1 AU from Jupiter at a time.

3. SPECIAL FEATURES

In this Section we will examine some special features recognizable in the LTEP data that are relevant to the study of the dynamical evolution of short-period comets. They are:

- 1) captures of comets from, or ejection into, very elongated ellipses caused by the gravitational influence of Jupiter;
- 2) comets passing from the complete control of Saturn to that of Jupiter;
- 3) comets mainly governed by Saturn;
- 4) comets with persisting librations around a resonance with Jupiter;
- 5) comets undergoing repeated, long and deep encounters with Jupiter, leading often to temporary satellite captures;
- 6) comets with orbits almost coincident before a close approach to Jupiter.

3.1. Captures and ejections

Only three from among the 126 integrated comets have been captured by Jupiter from, or ejected into, orbits of very high eccentricity, with aphelia well outside the planetary region. The first of them was the well known case of P/Lexell, ejected in 1779 from an orbit of period 5.6 yr (close to the 1/2 resonance with Jupiter) into an orbit of period around 280 yr. P/Lexell is the only case in our sample of a comet that, still remaining bound to the Sun, reaches an orbit of period exceeding the conventional limit of 200 years for short-period comets: in this sense it should not be regarded as of short-period anymore. Before 1767 P/Lexell moved on a somewhat larger orbit, with a period of 9.2 years; it is unique by three close planetary encounters in rapid succession: 1767 Jupiter - 1770 Earth - 1779 Jupiter. The unique dynamical history of P/Lexell was already recognized and studied in detail by Lexell (1778), Leverrier (1857) and Kazimirchak-Polonskaya (1961). An investigation of the fate of the possible meteor stream associated with it (Ca-

rusi et al., 1982) has shown an extreme sensitivity of the orbital parameters after 1779 to the initial conditions: a variation of -1 m/sec in the velocity of the comet at the perihelion passage before the encounter caused a final orbit with a period of only 23.6 years, whereas a variation of $+1$ m/sec led to a hyperbolic orbit.

P/Kearns-Kwee (fig. 5a) starts its evolution in an elongated orbit with a period of 170 yr and aphelion 57 AU from the Sun. The comet is then captured by Jupiter into a parking orbit of about 50 yr period around 1700, and finally transferred inside the orbit of Jupiter in 1962. The final orbit, the one in which P/Kearns-Kwee is moving at present, has a period of only 9 years. The previous 50-yr period was quite

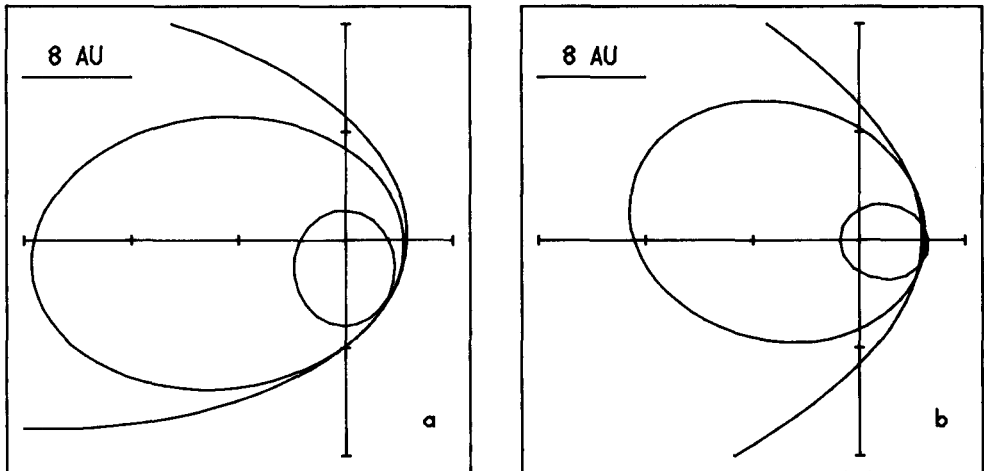


Figure 5. a: successive orbits of P/Kearns-Kwee. Inner orbit: after 1962; intermediate orbit: between 1700 and 1962; outer orbit: before 1700. b: successive orbits of P/Wild 2. Inner orbit: after 1974; intermediate orbit: between 1702 and 1974; hyperbolic orbit: before 1702.

evident from the one-apparition orbit; Kazimirchak-Polonskaya (1967)

gave a different scenario that produced it (an earlier capture from a definitely hyperbolic orbit); but Marsden and Aksnes (1967) wisely concluded that it was then unknowable. Even after the later orbit improvement, the circumstances before the 1700 encounter remain rather indeterminate.

The third case of this type is that of P/Wild 2 (fig. 5b). At the beginning of our integration period, in 1585, the comet moves along a hyperbolic orbit of eccentricity 1.03. Given the uncertainty of the orbital elements, which are based on the discovery apparition alone, it can only be said that this comet may have come from an orbit of very long period. The value of the eccentricity of P/Wild 2, 1.03, is comparable with the one of the fictitious Lexell already mentioned: 1.05. A velocity increment of +1 m/sec at perihelion reflects, in the case of Lexell, into a variation of semimajor axis on the fourth decimal digit, which - in turn - causes a relative variation of the orbital period of the order of 0.0001. This is also the uncertainty in the present period of P/Wild 2. In this respect, however, it should be noted that the capture of P/Wild 2 into the present orbit proceeds, like that of P/Kearns-Kwee, through an intermediate parking orbit, with a period of 38 years, where the comet is placed by Jupiter in 1701. The second deep encounter, leading to the present orbit, takes place immediately before the discovery in 1974. During this last encounter, as we have already shown in table 3, the comet passes Jupiter at only 0.006 AU - one of the deepest encounters on record. This fact, together with the orbital uncertainty, makes the evidence of a capture from a hyperbolic orbit entirely unreliable.

It may appear strange that integrations over a grand total of over 100,000 years have not revealed any other case of comparably strong perturbation, all the three extreme cases falling within two revolutions of the comet's discovery, and of the osculating date of its starting elements. In interpreting this finding from the statistical point of view, two effects have to be taken into account. First, it is just a close decelerating encounter which can render a comet observable for the first time, by reducing appreciably its perihelion distance. And second, the limited accuracy of the starting orbits would make the encounter conditions far from the osculating date rather uncertain. The computation may reveal a deep encounter which in fact did not occur, but it can also fail to identify a real one. While the probability of positive and negative errors in the comet's position on the orbit is the same in principle, the window of strong interactions may become much narrower than the error dispersion (see, e.g., Carusi et al., 1981a). This is why more real encounters may remain hidden than fictitious encounters found. Therefore, the rate of very close encounters resulting from our long-term integrations must be taken with due precautions.

3.2. The case of P/Oterma

Comet P/Oterma has been studied by many authors (Oterma, 1958; Fokin, 1958; Marsden, 1961; Kazimirchak-Polonskaya, 1967; Carusi et al., 1981a) due to its high dynamical interest. It has been the first recognized case of a real comet undergoing temporary satellite captures by Jupiter, and the study of its two encounters with the planet in 1937 and 1963, separated by a residence in the 2/3 resonance with Jupiter, has opened a number of new investigations about the dynamical behaviour of comets of moderate eccentricity and very high value of the Tisserand invariant (i.e., a low encounter velocity) with respect to Jupiter.

The LTEP has further emphasized the relevance of this object for the study of comet dynamics. At the beginning of the integration P/Oterma moves along an orbit of very high perihelion distance (6.33 AU), that does not allow the comet to approach Jupiter to less than 0.87 AU. The

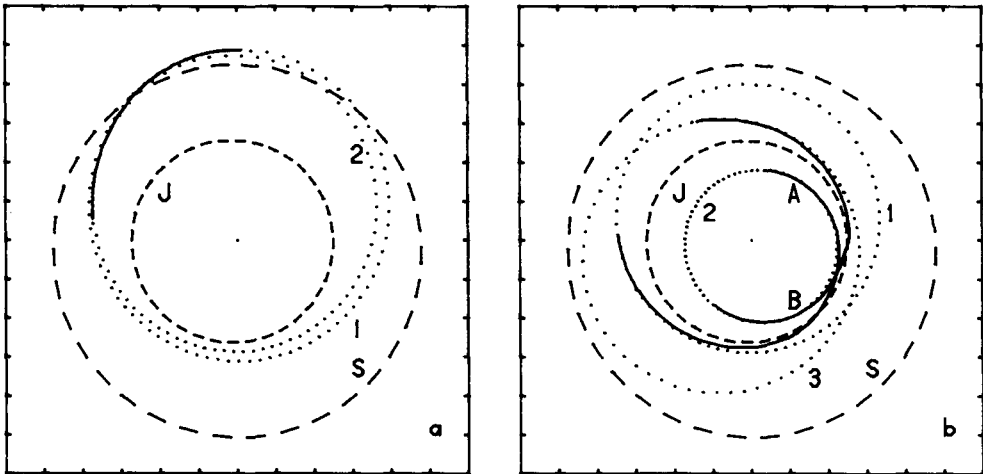


Figure 6. Transfer of P/Oterma from Saturn to inside Jupiter. Dashed orbits in both plots refer to Jupiter and Saturn. a: orbit 1: before 1770; solid line: path of the comet inside a 2 AU sphere around Saturn in 1770; orbit 2: after 1770. b: orbit 1: before 1933; solid line A: 1933-1941 encounter; orbit 2: between 1941 and 1960; solid line B: 1960-1967 encounter; orbit 3: after 1967.

motion of the comet is, at that stage, completely dominated by Saturn; then, an approach to this planet in 1770, with a minimum distance of 0.4 AU, reduces the perihelion distance to 5.77 AU (see fig. 6a). During the XIXth century two rather shallows encounters with Jupiter reduce the aphelion distance by almost 1.5 AU, detaching the comet from the control of Saturn. Finally, the mentioned encounter with Jupiter around 1937 puts P/Oterma in the inner orbit, exactly in the 2/3 resonance with the planet, so that a symmetric encounter takes place three revolutions later, sending the comet back to the original place (see fig. 6b). In the present orbit, after the return into the region between Jupiter and Saturn, P/Oterma can encounter both planets, but no approaches within 0.5 AU are found until the end of the integration.

3.3. Comets mainly governed by Saturn

The orbit of P/Oterma before 1770 represents in the LTEP the only case of avoidance of encounters with Jupiter because of a large perihelion distance. Among the other comets, however, there are many cases in which Saturn shares with Jupiter the control of the orbital evolution. Only in two cases, namely P/Wild 1 and P/IRAS (discovered after the last edition of Marsden's Catalogue, 1982), Saturn totally controls their perturbed motion. Jupiter cannot be encountered because of the unfavourable geometrical configuration of the orbits. Table 4 lists the

TABLE 4. Geometrical data on comets controlled by Saturn

	P/Wild 1		P/IRAS	
	min	max	min	max
Distance from the Sun at				
Ascending node	8.4	9.3	1.5	2.2
Descending node	1.9	2.1	9.3	9.6
Distance from the ecliptic at				
r = 5.2 AU	1.0	1.3	2.6	2.9
r = 5.2 AU	1.4	1.7	2.8	3.5
r = 9.5 AU	-	-	0.02	0.92
r = 9.5 AU	-	-	0.6	1.7

distances from the Sun when the two comets cross the ecliptic plane, and the vertical distances from the ecliptic when the radius vectors are equal to 5.2 AU (orbit of Jupiter) and 9.5 AU (orbit of Saturn). Note that the latter data are not reported for P/Wild 1, since its aphelion is always less than 9.539, the mean semimajor axis of Saturn.

3.4. Librators between Jupiter and Saturn

The LTEP has shown that many short period comets spend at least part of the 821 years librating about a low-order resonance with Jupiter. This behaviour was discovered by Marsden (1970), and discussed also by Kresák (1974), Franklin et al. (1975), and Vaghi and Rickman (1982).

As an example, comets spending a part or the whole time span librating about the 1/2 resonance with Jupiter are listed in table 5, together with the number of cycles performed. As shown in the table, 12% of the comets of our sample are, at least temporarily, librating about the 1/2

TABLE 5. Comets temporarily librating about the 1/2 resonance with Jupiter

Comet	Number of cycles
P/Forbes	1
P/de Vico-Swift	1
P/Tsuchinshan 1	1
P/Tsuchinshan 2	1
P/Kopff	2
P/Tempel-Swift	2
P/Clark	2
P/du Toit-Neujmin-Delporte	2
P/Harrington-Wilson	2
P/Pons-Winnecke	3*
P/Tempel 1	3
P/Pigott	3*
P/Howell	3
P/Haneda-Campos	4*
P/Tritton	4*

* Comets librating from 1585 to 2406

resonance, and 3% during the whole 821-years interval; these numbers increase, of course, if we take into account librations about other low-order resonances.

Of particular interest are the objects with semimajor axes between those of Jupiter and Saturn which, as it is well known, have a ratio of orbital periods close to $2/5$.

The most impressive case is that of P/du Toit, which librates all the time about the $5/4$ resonance with Jupiter. This, in turn, is also very close to the $1/2$ resonance with Saturn. Figs. 7a,b show the joviocentric and saturnocentric patterns of the comet for the whole time interval: the frames are rotating with the corresponding planet, which is in the centre, so that the Sun is always located on the negative x-axis. While P/du Toit is almost perfectly locked in the mentioned resonance with Jupiter, the 8-shaped pattern with respect to Saturn rotates slowly, but continuously, in the clockwise direction. The integration covers only about one fourth of this last cycle, and the possible influence by Saturn on the joviocentric pattern remains open.

P/Gehrels 1 is another example of this type of libration: in this

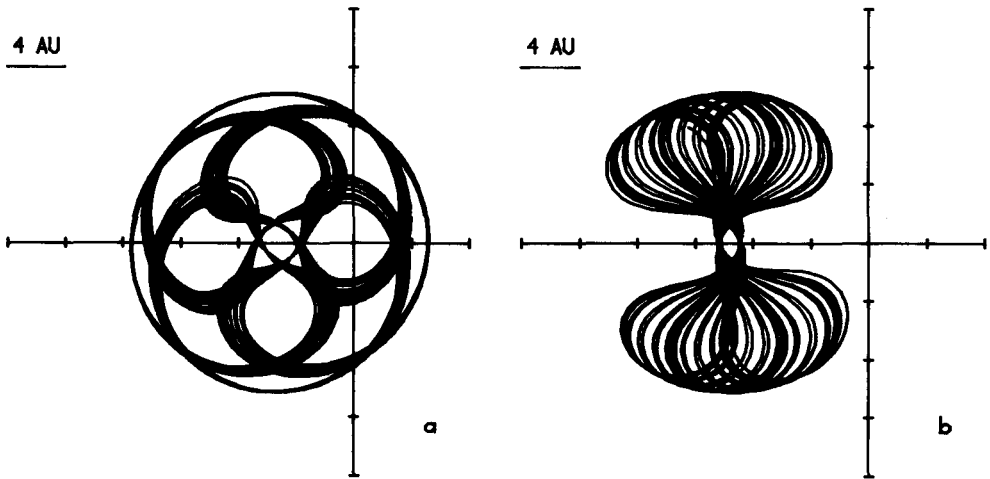


Figure 7. a: path of P/du Toit in a frame centred on Jupiter and rotating with the angular velocity of Jupiter's revolution. b: path of P/du Toit in a similar frame centred on Saturn. Both paths are between 1585 and 2406. For further explanations see text.

case, however, the joviocentric pattern (see fig. 8a) is much less regular and there are indications that the dynamical situation is changing in the last century of the integration.

A third example is the well known case of P/Neujmin 1, librating for the whole period about the $3/2$ resonance with Jupiter, which is also quite close to the $3/5$ resonance with Saturn. The joviocentric pattern of P/Neujmin 1 is also much less regular than that of P/du Toit (see fig. 8b), while the comet is definitely circulating across the $3/5$ resonance with Saturn. The libration of P/Neujmin 1 was discovered by Marsden (1970) who found it to persist over 4,000 years.

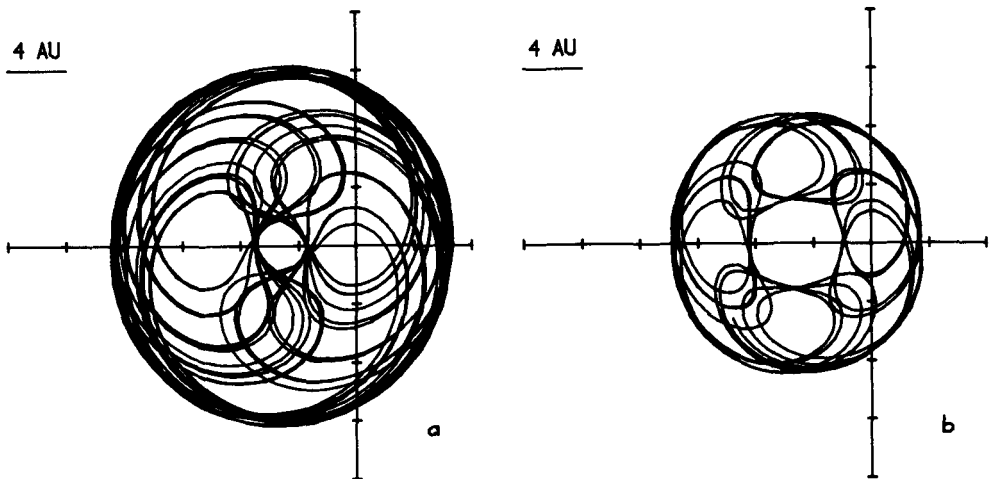


Figure 8. a: same as fig. 7a for P/Neujmin 1. b: same as fig. 7a for P/Gehrels 1.

3.5. Close encounters and satellite captures

As we have already shown, during the 800-years period from 1601 to 2400, the 126 short-period comets undergo a total of 610 encounters with Jupiter at a distance of less than 0.5 AU. Only 75% of the comets are responsible for all the encounters: 25% of them do not approach Jupiter within that distance, or do not have even moderate approaches, as is the case of P/IRAS and P/Wild 1; 40% of the comets have more than 5 encoun-

ters, and 13% more than ten. The highest number of encounters is due to P/Pons-Winnecke (21) which, as already mentioned, librates about the 1/2 resonance with Jupiter. P/Gehrels 3 alone accounts for 40% of the ten closest approaches (see table 3). This comet is exceptional in many respects: it has one of the highest Tisserand invariants (3.02) and, hence, a very low encounter velocity; it experiences five approaches with minimum distances from Jupiter smaller than those of the direct irregular satellites (one of them inside the orbit of Io, and another one well inside the orbit of Ganymede); it is captured as a temporary satellite at each encounter, spending a total of 31.4 years bound to the planet (3.8% of the total time of the integration, 7% of the time from the first to the last encounter). It is also noteworthy that the only close encounter with Jupiter that occurred in the backward part of the

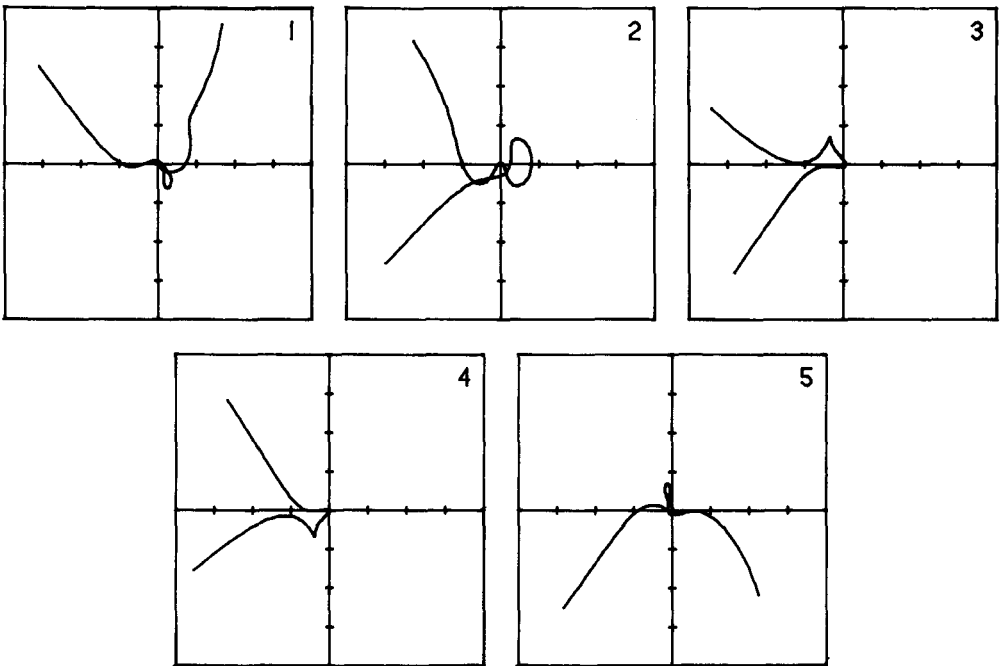


Figure 9. Jovicentric trajectories for the five very close encounters of P/Gehrels 3 with Jupiter. The dates (corresponding to minimum distances) are: 1-1970, 2-2062, 3-2203, 4-2305, 5-2400. Tick marks: 0.5 AU.

integration is the one that transferred P/Gehrels 3 from outside to inside the orbit of the planet. In the forward part the other four encounters occur, the last one placing the comet again beyond Jupiter.

The five joviocentric patterns corresponding to the five encounters are shown in fig. 9; all of them are typical of low-velocity encounters with Jupiter (see, for comparison, Carusi et al., 1981b, objects CAP 21, OTE 14, 61 and 37). Quite impressive is the similarity between the encounters of 2200 and 2300, which can be almost exactly obtained from each other by a rotation of π about the x-axis. Symmetries of this type are quite common among low-velocity encounters, as can easily be noted in the last cited reference.

As already noted (see also Carusi et al., 1985b), very close encounters invalidate to a large extent the integrations beyond them: in the present case only the two events nearest to the starting date (one backwards and one forwards) can be accepted with a sufficient confidence. Nevertheless, this evolution is interesting in itself, as a potential behaviour of a comet with a high value of the Tisserand invariant with respect to Jupiter.

P/Gehrels 3 is fairly unique in our sample; other comets undergoing less frequent or more distant encounters, sometimes being captured as

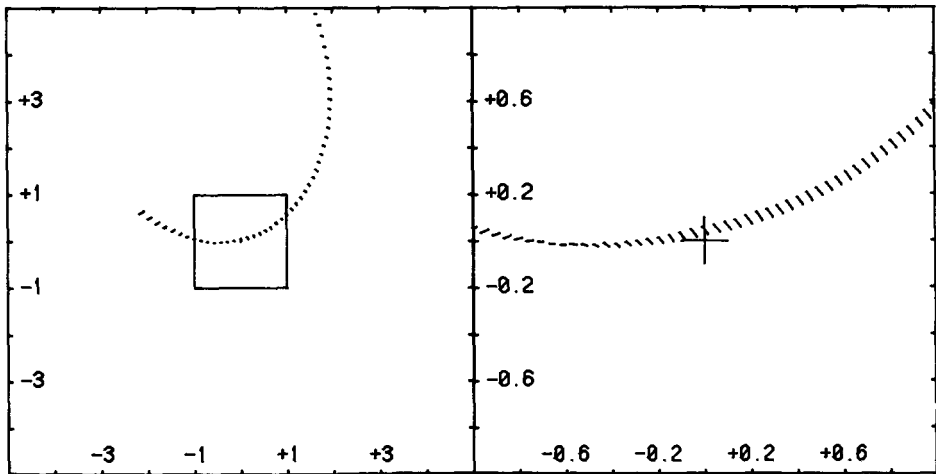


Figure 10. Jovicentric trajectories of P/Neujmin 3 and P/Van Biesbroeck in a sphere of 5 AU radius (left plot) and 1 AU radius (right plot) centred on Jupiter during the encounter of 1850. The frame is rotating with the planet. The positions of the two comets are connected by a segment, whose upper end in the right-hand plot is P/Neujmin 3.

temporary satellites, are P/Smirnova-Chernykh and P/Oterma.

3.6. P/Neujmin 3 and P/Van Biesbroeck

P/Neujmin 3 and P/Van Biesbroeck, two comets with very well determined orbits - currently not very similar to one other - have rather complex and interesting histories when traced backward in time. Both of them, in fact, approach Jupiter in 1850, passing it quite closely, and almost perfectly join prior to this encounter. The striking similarity between the orbits prior to 1850, and the physical proximity of the two comets indicate that, very probably, they represent fragments of a parent body that split before the mentioned encounter, as already announced by the authors in the IAUC No. 3940 (1984). The event is shown graphically in figs. 10 and 11. The two plots of fig. 10 are the joventric trajectories of the two comets during the approach to Jupiter. The po-

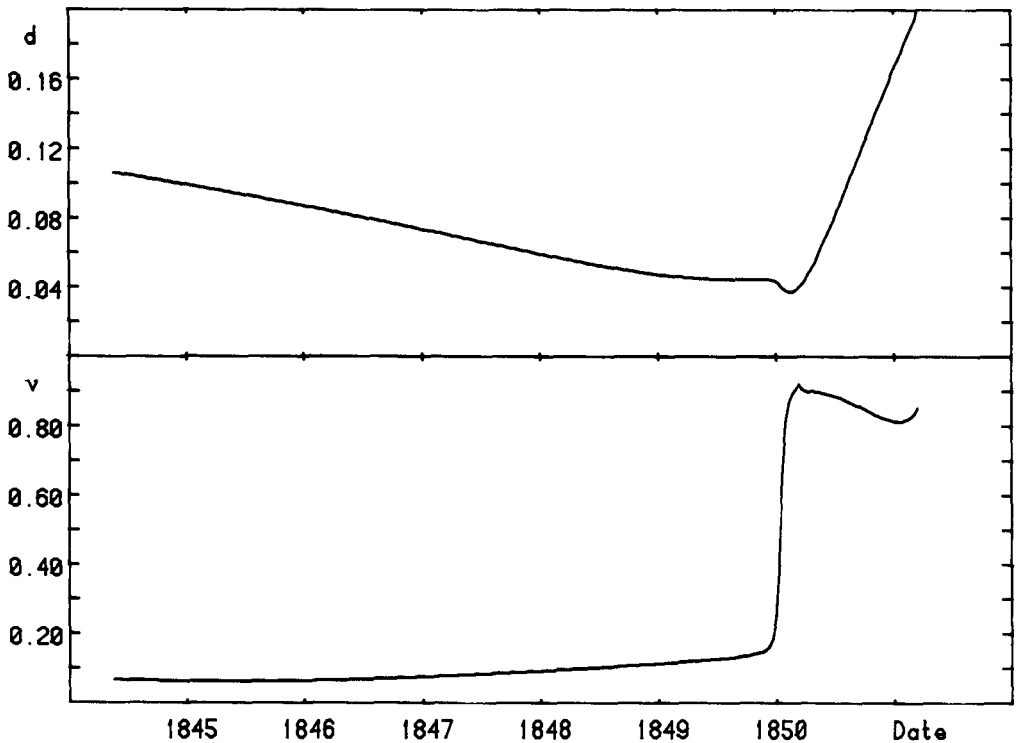


Figure 11. Time evolution of the distance (upper plot) and relative speed (lower plot) of P/Neujmin 3 and P/Van Biesbroeck between 1844 and 1851. Units are AU and km/sec.

sitions of the objects, projected onto the ecliptic plane, are connected by a segment, whose upper end before the encounter represents P/Neujmin 3. As the segment rotates counter-clockwise, just after the closest approach the two trajectories do cross. Fig. 11 gives the temporal variation of the mutual distance and of the relative velocity of the two objects: they remain for a long time much closer than 0.1 AU, with a relative speed less than 0.1 km/sec.

TABLE 6. Orbital data of P/Neujmin 3 and P/Van Biesbroeck

Date		v	a	e	i	ω	Ω	D
1845 03 10.0	N3	185.71	6.1046	0.5494	4.71	290.90	348.54	0.013
	VB	186.03	6.1352	0.5504	4.11	289.96	349.14	
1849 10 05.0	N3	240.09	6.1477	0.5615	4.74	291.14	348.45	0.017
	VB	240.20	6.1860	0.5621	4.13	290.05	349.08	
1850 05 03.0	N3	239.05	5.1978	0.5814	3.14	131.85	162.66	0.285
	VB	246.46	5.6038	0.5642	7.04	123.09	163.98	
1858 11 17.0	N3	164.16	5.1259	0.5756	3.35	136.49	159.13	0.344
	VB	175.24	5.5443	0.5483	7.05	125.57	162.52	

Orbits referred to equinox and ecliptic 1950.0

Owing to the inherent uncertainty of the starting data, and to the noise introduced by the 100 years of integration after 1850, it is quite surprising that the orbits before that date do agree so closely; a further refinement of starting data is needed, in order to try to eliminate some residual differences, mainly in the inclination. Table 6 lists the orbital elements of the two comets for selected dates before and after the encounter, together with the value of the D-parameter introduced as a quantitative measure of orbital similarity by Southworth and Hawkins (1963; for definition see also Lindblad, this volume).

P/Neujmin 3 and P/Van Biesbroeck are the only comets for which a clear indication of a genetic relationship has been found in our first analysis of LTEP data. Nevertheless, another pair of comets would deserve, in this respect, a deeper study. It is the pair constituted by P/Kearns-Kwee and P/Wild 2, whose evolution is shown graphically in fig. 5. The two objects experienced the most effective captures by Jupiter of all comets examined here (see section 3.1). The timing of the two events

agrees to within 14 months (1700 December - 1702 February), a coincidence which is striking in itself when compared with the interval of 821 years covered by our integrations.

At first glance, the pre-capture elements do not appear similar at all because the nodes are reversed. However, the perihelion longitudes differ only by 1.8° , the perihelion latitudes by 1.4° , and the small inclinations make the orbital planes deviate by only 7.2° . The largest difference is that in the eccentricity, the orbit of P/Wild 2 being formally hyperbolic and that of P/Kearns-Kwee elliptic with a period of 170 years. One must bear in mind, however, that the one-apparition orbit of P/Kearns-Kwee (Kazimirchak-Polonskaya, 1967) was also indicative of an originally hyperbolic orbit.

Further difficulties are given by the 1974 encounter of P/Wild 2 with Jupiter, that renders the computed orbit prior to that date very uncertain; by the probable presence of nongravitational forces (Forti, 1983); by the intrinsic uncertainty of the orbit of P/Wild 2, still based on one apparition only. The comet, however, has already been recovered as 1983s and will pass the perihelion this August. A further improvement of its orbital elements and integration of a manifold of neighbouring orbits, both for P/Wild 2 and P/Kearns-Kwee, will be needed to settle the question of their possible genetic relationship with some confidence. We intend to do this when more observations from the current return of P/Wild 2 become available. First tentative computations work against this hypothesis but, owing to the numerous uncertainties, there is still room for a remote possibility that needs a more careful investigation. An identification would probably imply that the XIIth century progenitor comet did not come directly from the Oort cloud, and that there is not a single case of that kind in our sample.

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REFERENCES

- Carusi, A., Kresák, L., Valsecchi, G.B.: 1981a, Astron. Astrophys., 99, 262-269.
- Carusi, A., Kresák, L., Valsecchi, G.B.: 1981b, IAS-Internal Report 12.
- Carusi, A., Kresáková, M., Valsecchi, G.B.: 1982, Astron. Astrophys., 116, 201-209.
- Carusi, A., Kresák, L., Perozzi, E., Valsecchi, G.B.: 1984, IAS-Internal Report 12.
- Carusi, A., Kresák, L., Perozzi, E., Valsecchi, G.B.: 1985a, this volume

- Carusi, A., Perozzi, E., Pittich, E., Valsecchi, G.B.: 1985b, this volume.
- Fokin, A.V.: 1958, Byull. ITA Leningrad, 7, 89-119.
- Forti, G.: 1983, Astron. Astrophys., 126, 307-310.
- Franklin, F.A., Marsden, B.G., Williams, J.G., Bardwell, C.M.: 1975, Astron. J., 80, 729-746.
- Kazimirchak-Polonskaya, E.I.: 1961, Trudy ITA Leningrad, 7, 19-190.
- Kazimirchak-Polonskaya, E.I.: 1967, Astron. Zh., 44, 439-460.
- Kresák, L.: 1974, Asteroids, Comets, Meteoric Matter (C. Cristescu, W.J. Klepczynski eds.), Edit. Acad. Bucharest, 193-203.
- Leverrier, U.J.J.: 1857, Ann. Obs. Paris, 3, 203-270.
- Lexell, A.I.: 1778, Acta Acad. Sci. Petropol., 1, 317-352.
- Marsden, B.G.: 1961, Astron. J., 66, 246-248.
- Marsden, B.G.: 1970, Astron. J., 75, 206-217.
- Marsden, B.G.: 1982, Catalogue of Cometary Orbits, IAU Central Bureau for Astron. Telegrams, Cambridge, Mass.
- Marsden, B.G., Aksnes, K.: 1967, Astron. J., 72, 952-954.
- Oterma, L.: 1958, Turku Informo, 17, 1-3.
- Southworth, R.B., Hawkins, G.S.: Smithsonian Contr. Astrophys., 7, 261-285.
- Vaghi, S., Rickman, H.: 1982, Sun and Planetary System (W. Fricke, G. Teleki eds.), Reidel, Dordrecht, 391-394.

DISCUSSION

F. Mignard: What about using analytical ephemerides elaborated at Bureau des Longitudes, to make an atlas in terms of a fixed number of revolutions for each comet, instead of a preassigned time span?

A. Carusi: I think it is a good idea; the philosophy of the work, however, and its statistical meaning would change, since in that case one would have integrations of very different length.