The tube in this form affords a blackboard method of describing an Ellipse.

The other end of the mercury thread describes an *Oval* of the fourth degree (Fig. 2).

When the length of the mercury thread is equal to the barometric height, the ellipse becomes a parabola and the oval asymptotic.

For the demonstration of Boyle's Law the lower end of the mercury column may be used as a pivot. In this case the end B describes a circle and the point O a horizontal straight line.

WILLIAM MILLER

The Reciprocal Polar of a Circle.—*The reciprocal polar figure of a circle s, with regard to another circle c, is a conic, one of whose foci is the centre of the reciprocating circle.* The polar reciprocal of s with regard to c is obtained by taking the polars of points on s with regard to c, and then constructing the envelope of these lines.

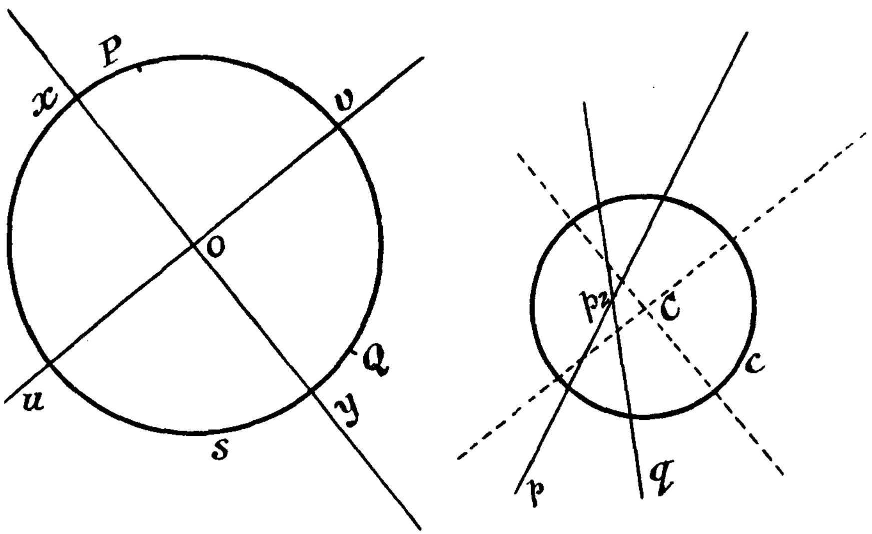

Fig. 1.

Let p, q (Fig. 1) be the polar of P and Q and p_2 their point of intersection. p_2 is then the pole of the line PQ, and the polar

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of any point other than P or Q cannot pass through p_2. From this we see that from any point in the plane of c, two and only two tangents can be drawn to the envelope, which therefore is a conic.

Also, the only lines through C (the centre of the reciprocating circle), connected with the reciprocal figure, are the polars of points at infinity.

Consider two conjugate diameters of the circle s, xy and uv. The polar of the point at infinity on xy is got by drawing through C a line perpendicular to xy. Similarly, for the polar of the point at infinity on uv. (These are the dotted lines through C). These two lines through C are obviously at right angles; they are also conjugate since the point at infinity on one is the pole of the other. Thus every pair of conjugate lines through C is a perpendicular pair, and by de la Hire's theorem C is a focus of the polar reciprocal.

We can immediately deduce a criterion for the nature of the conic. For the points at infinity on it are the poles of tangents which pass through C. If C be outside s, the two tangents are real, and so also are their poles. In this case the conic is a

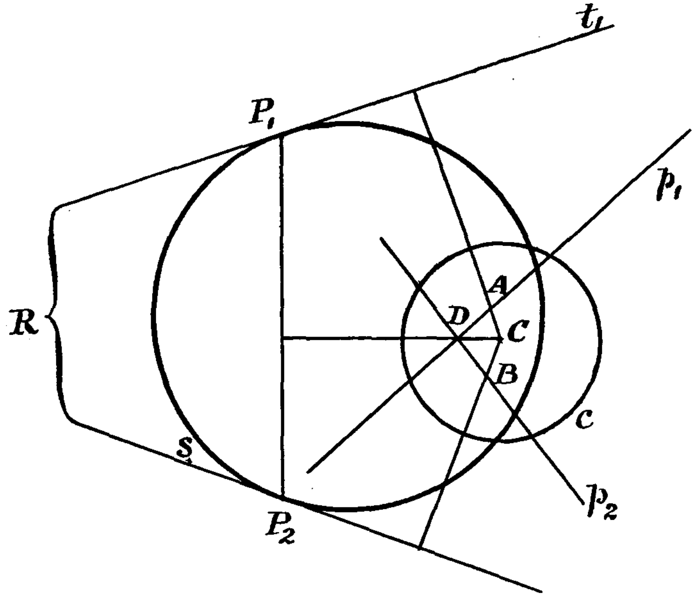

Fig. 2(a).

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hyperbola. If C is on s, the tangents are coincident and the conic is a parabola. When C is within s, we get an ellipse.

It is interesting to see how *we can establish the focal properties of the conic by this method.* Take as an example the well-known property that the angles subtended at a focus by two tangents drawn from a point are either equal or supplementary.

Let P_1, P_2 (Fig. 2) be two points on the original circle: t_1, t_2 tangents to s at these points: p_1, p_2 their polars with respect to c. Since the pole of t_1 must lie on the perpendicular from C to t_1, and also on p_1, we have that A is the pole of t_1 and therefore the point of contact of p_1 with the envelope. B is the point of contact of p_2. Also since D is the pole of P_1P_2, and C is the pole of the line at infinity, DC is the polar of the point at infinity on P_1P_2.

In Fig. 2(a) it is obvious that $\angle ACD = \angle RP_1P_2$

and that $\angle BCD = \angle RP_2P_1$.

But $\angle RP_1P_2 = \angle RP_2P_1$,

$\therefore \angle ACD = \angle BCD$.

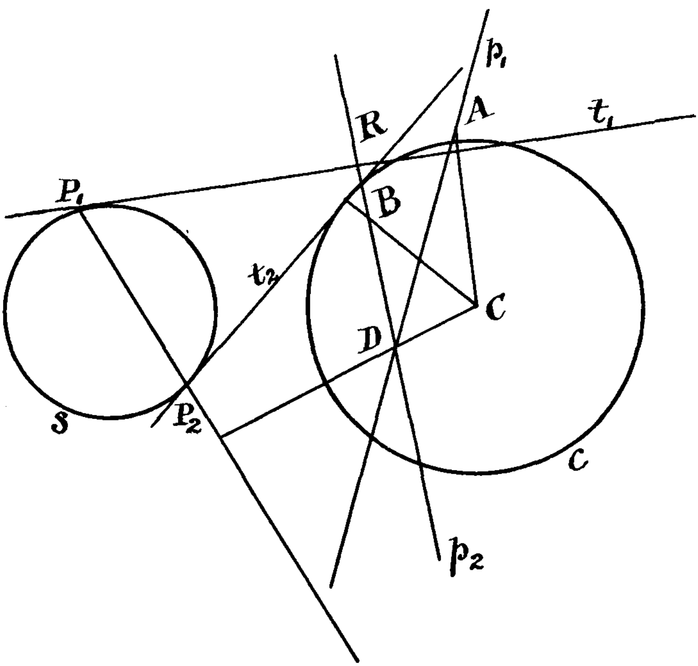

Fig. 2(b).

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In Fig. 2(*b*), which is the case of the hyperbola,

$$\angle \text{ACD is supplementary to } \angle \text{RP}_1\text{P}_2,$$

and $$\angle \text{BCD} = \angle \text{RP}_2\text{P}_1 = \angle \text{RP}_1\text{P}_2.$$

Therefore angles ACD and BCD are supplementary.

G. PHILIP

Note on Geometric Series.—The formula for the sum of n terms of the geometric series

$$a + ar + ar^2 + \ldots + ar^{n-1},$$

viz. $$s = \frac{a(r^n - 1)}{r - 1}, \text{ may be written } s = \frac{\text{T}_{n+1} - \text{T}_1}{r - 1},$$

where T_1 is the first term and T_{n+1} the term immediately succeeding the last to be summed This form is useful for finding the sum of a closed geometric series without first finding the number of terms. Thus

$$\tfrac{1}{8} + \tfrac{1}{4} + \tfrac{1}{2} + \ldots\ldots\ldots + 32 = \frac{64 - {}^1/_8}{2 - 1},$$

and $$a^{-5} + a^{-3} + a^{-1} \ldots\ldots + a^{19} = \frac{a^{21} - a^{-5}}{a^2 - 1},$$

since the terms of the series which succeed 32 and a^{19} are respectively 64 and a^{21}.

R. J. T. BELL

Distance of the Horizon.—The approximate rules given in this note may perhaps be new to some readers.

If an observer be at a height h above the Earth's surface, the distance of the horizon as seen by him is given by the formula

$$d = \sqrt{2rh},$$

where r is the radius of the Earth.

If h is given in feet, then

$$\sqrt{\left(\frac{h}{5280} \times 8000\right)}$$

will give d in miles.

Now $$\tfrac{8000}{5280} = \tfrac{100}{66} = \tfrac{3}{2}, \text{ nearly.}$$