

Smoothness and rotundity in Banach spaces

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This thesis studies the structure of a Banach space by considering its geometry; that is, by considering the "shape" of the unit sphere. The first chapter deals with various types of smoothness and the associated rotundity conditions. Each type of smoothness is related to a type of differentiability of the norm, and each type of differentiability is related to a continuity property of the support mappings.

Two new types of smoothness - extreme smoothness and very smoothness - are studied extensively. It is shown that very smooth spaces have many of the properties of Fréchet differentiable spaces. Extremely smooth spaces are compared with spaces which have an equivalent uniformly Gâteaux differentiable norm.

Next the concept of weak rotundity is introduced. One version characterizes the duals of very smooth spaces; another characterizes the duals of extremely smooth spaces. Weak rotundity methods are used when considering the Fréchet differentiable and uniformly Gâteaux differentiable analogs of very smoothness and extreme smoothness, and are used to obtain some results about quotient spaces. These results are contained in Yorke [1].

A type of uniform differentiability which lies "between" uniform Gâteaux differentiability and uniform Fréchet differentiability is investigated, and several equivalent formulations of this concept are given. It is still unknown whether or not this type of differentiability is strong enough to imply reflexivity.

Received 3 May 1977. Thesis submitted to the University of Newcastle, New South Wales, December 1976. Degree approved, April 1977. Supervisor: Associate Professor J.R. Giles.

The second chapter develops some renorming theorems. First, the method of "averaging" norms due to E. Asplund is discussed, and extended to apply to weakly locally uniformly rotund and weak-* locally uniformly rotund norms. Several applications are given.

Next, it is shown that if a dual space E^* is weakly locally uniformly rotund, or if E^{**} is smooth, then E^* can be equivalently renormed to be locally uniformly rotund. Conditions which make this a dual norm are investigated.

Finally, spaces which admit Markushevich bases are studied. The main result shows that if a space admits a Markushevich basis with a norming coefficient space, then that space can be equivalently renormed to be locally uniformly rotund. Several applications are given, and some open questions are raised. These results appear in Yorke [2].

References

- [1] A.C. Yorke, "Weak rotundity in Banach spaces", submitted.
- [2] A.C. Yorke, "Locally uniformly rotund norms and Markushevich bases", submitted.