

A Model for the Pulsar Radio Emission

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Abstract. New, self-consistent theory of coherent pulsar radio emission based on the sparking model of the polar gap first suggested by Ruderman & Sutherland (1975) and modified by Gil & Sendyk (1999), is proposed.

1. Introduction

The polar cap with a radius $r_p \simeq 10^4 P^{-0.5}$ cm is populated with about $(r_p/h)^2$ sparks of a characteristic dimension approximately equal to height scale $h \sim 5 \times 10^3 P^{3/7}$ cm of the so-called polar gap. One spark is anchored to the local pole of a sunspot-like surface magnetic field (Gil & Mitra 1999) while others tend to rotate around the polar one due to familiar $\mathbf{E} \times \mathbf{B}$ drift across the planes of field lines converging at the pole (Gil & Sendyk 1999). This prevents sparks from fast motion along the planes of field lines towards the pole, which allows them to reappear in approximately the same places on time scales much longer than $10 \mu\text{sec}$. Each spark, reappearing in approximately the same place on the polar cap, delivers to the open magnetosphere a sequence of e^-e^+ plasma clouds, flowing orderly along dipolar magnetic field lines. Overlapping of particles with different momenta from consecutive clouds leads to effective two-stream instability, which triggers electrostatic Langmuir waves at the altitudes of about 50 stellar radii. However, the Langmuir waves generated due to these instabilities can not produce directly an observed pulsar radio emission. In fact, their frequency about 100 GHz is much higher than the observed pulsar radio frequencies (Asseo & Melikidze 1998; Melrose & Gedalin 1999). Moreover, having an electrostatic nature they can not leave the plasma. The electrostatic oscillations are modulationary unstable and their nonlinear evolution results in formation of ‘bunch-like’ charged solitons. A characteristic soliton length along magnetic field lines is about 30 cm, so they are capable of emitting coherent curvature radiation at radio wavelengths. A perpendicular cross-section of each soliton at radiation altitudes follows from a dipolar spread of a plasma cloud with a characteristic dimension near the star surface of about $h \approx 50$ meters. The net soliton charge is about 10^{21} fundamental charges, contained within a volume of about 10^{14} cm³. For a typical pulsar, there are about 10^5 solitons associated with each of about 25 sparks operating on the polar cap at any instant. One soliton moving relativistically along dipolar field lines with a Lorentz factor of

the order of 100 generates a power of about 10^{21} erg/s by means of curvature radiation. Then the total power of a typical radio pulsar can be estimated as being about 10^{27+28} erg/s. The energy of the soliton curvature radiation is supported by kinetic energy of secondary electron-positron plasma created by the primary beam produced by the accelerating potential drop within the polar gap. A significant fraction of kinetic energy generated by sparks is radiated away in form of the observed coherent radio emission.

2. Theory

The Langmuir waves with frequency $\omega_l \sim 100$ GHz are generated by the following simple mechanism (Usov 1987; Asseo & Melikidze 1998). The repeatable sparking creates a succession of plasma clouds moving along a tube of magnetic field lines, each cloud containing particles with a large spread of momenta. Overlapping of particles with different energies from adjacent clouds ignites strong Langmuir oscillations, which may lead eventually to the generation of coherent pulsar radio emission. This instability is the only one which develops at altitudes of the order of a few percent of the light cylinder radius, where the pulsar radio emission is expected to originate (e.g. Kijak & Gil 1998). The altitude $r_{em} \simeq 50 R R_{50}$ (where $R = 10^6$ cm), at which the two-stream instability can develop depends on the average Lorentz factor of plasma γ_p . This has been calculated by Asseo & Melikidze (1998). Here we can demonstrate a simple kinematic estimate for the altitude of the two-stream instability region. In fact, two adjacent secondary plasma clouds corresponding to the two consecutive sparks are separated by about $\Delta t \sim h/c$ (typically 10^{-7} s), where $h \simeq 5 \times 10^3 \mathcal{R}_6^{2/7} B_{12}^{-4/7} P^{3/7}$ cm is the polar gap height, $\mathcal{R}_6 = \mathcal{R}/R$ and $B_{12} = B_0/10^{12}$ (Ruderman & Sutherland 1975). Let us estimate the time ΔT after which the particles with different Lorentz factors will overcome each other. The corresponding velocity difference is determined by the average Lorentz factor $\Delta v \simeq c/(2\gamma_p^2)$. It is easy to show that $\Delta T \sim h/\Delta v \sim 2\gamma_p^2 h/c$. The distance covered during this time $\Delta r \sim c\Delta T \sim 2\gamma_p^2 h \sim 10^2 (\gamma_p/100)^2 \mathcal{R}_6^{2/7} B_{12}^{-4/7} P^{3/7}$. Since $\Delta r \ll R = 10^6$ cm, one can write the expression $R_{50} \sim (\gamma_p/100)^2 \mathcal{R}_6^{2/7} B_{12}^{-4/7} P^{3/7}$. This can be compared with the empirical relationship for the radio emission altitude $(r/R) = 50 \times R_{50} \sim (55 \pm 5) \nu_{\text{GHz}}^{-0.21 \pm 0.07} \tau_6^{-0.07 \pm 0.03} P^{0.33 \pm 0.05}$ given by Kijak & Gil (1998), where ν_{GHz} is the frequency in GHz and τ_6 is pulsar timing age in million years.

The linear growth rate Γ_l , which should satisfy the condition $\Gamma_l \gg c/\Delta r$, where Δr is characteristic dimension of the instability region, can be written as $\Gamma_l \sim 10^6 (\gamma_p/100)^{-1.5} R_{50}^{-1.5} (\dot{P}_{-15}/P)^{0.25}$, and the condition for the instability development in the resulting plasma cloud is $(\gamma_p/100)^{-1.5} R_{50}^{-1.5} (\dot{P}_{-15}/P)^{0.25} \gg 0.1$ (Asseo & Melikidze 1998). It is obvious that for typical values of magnetospheric plasma parameters $(\gamma_p/100 \sim R_{50} = r_{em}/(50R) \sim \dot{P}_{-15} = \dot{P}/10^{-15} \sim P \sim 1)$ the growth rate of instability is high enough to provide a strong Langmuir turbulence.

A packet of plasma waves propagating through the relativistic electron-positron plasma with phase velocities close to the velocity of light is unstable from the modulation standpoint, and its nonlinear evolution results in the for-

mation of a soliton (Melikidze & Pataraya 1980). Let us emphasize that in the case of electron-positron plasma the role of low-frequency perturbations is played by the nonlinear beatings of plasma waves, and the resonant interaction of beatings with particles determine the nonlinear damping. The soliton charge separation due to relative motion of electrons and positrons is supported by the ponderomotive force. Even small difference in the unperturbed distribution function of electrons and positrons results in the net soliton charge, which can be estimated as being about $Q \sim 10^{21}e$. These charged soliton radiate by means of curvature radiation a power

$$L \sim 10^{22} (\gamma_p/100)^4 \kappa_4 R_{50}^2 P^{-0.93} \dot{P}_{-15}^{-0.64} x \quad [\text{erg s}^{-1}], \tag{1}$$

where $\kappa_4 = \kappa/10^4$ is the normalized Sturrock multiplication factor, and x is the numerical factor of the order of unity. The frequency of radiated waves is much smaller than the characteristic frequency of ambient plasma. The spectra of these waves for small angles between the wave vector and the external magnetic field, which is the case of the curvature radiation, is $\omega = kc(1 - \delta)$, where $\delta \simeq \omega_p/\omega_B$ is negligibly small in the inner magnetosphere. Therefore the radio waves generated by relativistic solitons can propagate like in the vacuum. The total power radiated by a pulsar at radio wavelengths $L_t = LN_t$, where N_t is a total number of solitons contributing to pulsar emission at any instant (see Melikidze et al. (1999) for details) can be written approximately as

$$L_t \sim 10^{28} (\gamma_p/100)^{3.5} \kappa_4^{1.5} R_{50}^{1.5} P^{-2.47} \dot{P}^{0.18} y \quad [\text{erg s}^{-1}], \tag{2}$$

where y is another numerical factor of the order of unity.

Table 1. Observed and calculated pulsar luminosities.

PSR	P [s]	\dot{P}_{-15}	L_{sp} [erg/s]	L_R [erg/s]	L_t [erg/s]	L_m [erg/s]
B0531+21	0.033	421	4.6×10^{38}	1.3×10^{29}	1.7×10^{29}	3.0×10^{32}
B0833-45	0.0893	125	6.9×10^{36}	4.3×10^{28}	8.0×10^{28}	3.0×10^{31}
B1610-50	0.232	493	1.6×10^{36}	2.8×10^{28}	7.7×10^{28}	6.0×10^{27}
B0950+08	0.253	0.23	5.6×10^{32}	2.3×10^{26}	4.6×10^{27}	4.0×10^{27}
B1133+16	1.188	3.73	8.8×10^{31}	7.4×10^{26}	9.6×10^{26}	1.5×10^{28}
B1746-30	0.61	7.9	1.4×10^{33}	3.0×10^{28}	9.4×10^{28}	1.3×10^{28}
B0525+21	3.75	40	3.0×10^{31}	1.1×10^{28}	1.5×10^{28}	9.3×10^{26}
J2144-39	8.51	0.48	3.0×10^{28}	5.0×10^{24}	7.0×10^{24}	3.0×10^{26}

In Table 1 we present results of the luminosity calculations from equation (2) for a number of pulsars with different values of period P and period derivative $\dot{P} = \dot{P}_{-15} \times 10^{-15}$. As one can see, it is easy to obtain the total luminosity L_t close to the observed luminosity L_R for a narrow range of free parameters $\gamma_p \sim 100$, $x \sim 1$ and $y \sim 1$. This means that the pulsar luminosity $L_R \sim L_t$ is determined mainly by the values of P and \dot{P} , similarly to the morphological properties of single pulses and average profiles (Gil & Sendyk 1999). The fraction $f = L_R/L_{sp}$, where $L_{sp} = 3.8 \times 10^{31} \times P_{-15} P^{-3}$ erg/s is the pulsar spindown luminosity, is a small number between $10^{-9} \div 10^{-3}$, increasing towards longer

periods, as observed. This is easy to understand bearing in mind that the soliton pulsar radiation is supported by the kinetic energy generated by the accelerating potential drop within the polar gap. In fact, if a significant fraction (say 30%) of spark maximum luminosity $L_m = N_{sp} \dot{N}_s e \Delta V \simeq 5 \times 10^{27} (\dot{P}_{-15}/P)^{15/14}$ erg/s is radiated away in the form of radiowaves, that is $L_R \sim L_t \sim 0.3 L_m$, then $L_R/L_{sp} \sim 0.3 \times 10^{-4} P^2$. This ratio is about 3×10^{-8} for the Crab pulsar, 2×10^{-7} for Vela pulsar, 3×10^{-5} for one second pulsar, 4×10^{-4} for long period (3.75 s) pulsar and 2×10^{-3} for longest period (8.5 s) pulsar J2144-3933 (Young et al. 1999).

3. Conclusions

The radio emission mechanism proposed in this paper is based on the non-stationary sparking discharge of the inner potential drop with a high scale determined by the Ruderman & Sutherland polar gap model. Sparks, whose existence have been recently demonstrated by Deshpande & Rankin (1999), (see also Gil and Sendyk 1999), should occur in approximately the same places on the polar cap (modulo the $\mathbf{E} \times \mathbf{B}$ drift). This is warranted by the polar spark anchored to the local pole of the sunspot-like surface magnetic field (Gil & Mitra 1999), which forces other sparks to move circumferentially around the pole. Thus, plasma clouds associated with each spark can flow orderly along the same tube of dipolar field lines. This is necessary for high frequency Langmuir oscillations to occur (Usov 1987; Asseo & Melikidze 1999). These oscillations lead to charge relativistic solitons due to nonlinear evolution of electrostatic turbulence. The total power emitted at radio wavelengths by all spark-associated solitons depends strongly on both the potential drop high scale and the number of sparks, parameters which depend mainly on P and \dot{P} values. In this sense our model is completely self-consistent.

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