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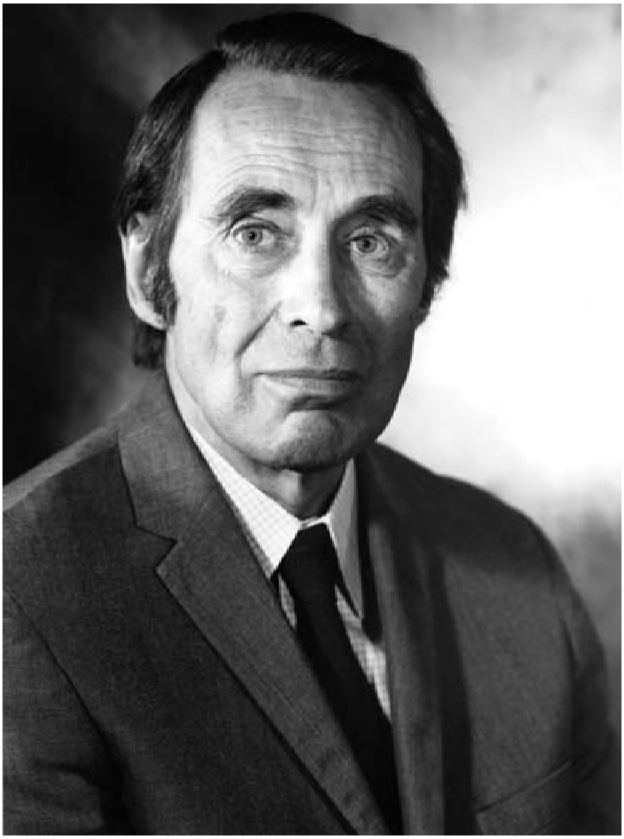
OBITUARY

ALBERT CYRIL OFFORD, FRS, 1906–2000



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Cyril Offord was one of the most distinguished and certainly the most senior of British mathematical analysts. With J. E. Littlewood, he pioneered the subject of probabilistic analysis by studying the typical behaviour of the zeros of a random polynomial with real coefficients, or an entire function.

1. *Life and career*

Cyril's father, Albert Edwin Offord, was a printer for the publishing firm of Eyre and Spottiswoode. His mother, Hester Louise, had a Protestant and Irish background from County Donegal, although she was born in India. She would have liked to go on the stage as an opera singer, but met Albert instead. The parents were convinced Plymouth Brethren, and Cyril had unhappy early recollections of the meetings.

Cyril had two younger brothers, Horace and Frank. They had careers in medicine, Horace as a radiologist and Frank as a GP. All the boys went to Hackney Downs Grammar School. Cyril took his first degree at University College London, and did postgraduate work at St John's College, Cambridge, where he was a Fellow from 1937 to 1940. He started in Cambridge as a pupil of G. H. Hardy.

His early work was on Fourier analysis and he lectured in the subject [**17**] at the International Congress of Mathematicians at Oslo. However the work that shaped the rest of his career was his collaboration with Littlewood [**19**, **20**, **23**, **24**, **26**] on random polynomials and entire functions. This is what turned him from real to complex analysis and in particular to probabilistic analysis.

Offord left Cambridge in 1940 for a temporary assistant lectureship at University College, Bangor. In 1942 he went to King's College, Newcastle (as it was then called), where he became a professor in 1945. Here he stayed until 1948 when he went to Birkbeck College, University of London.

In Newcastle, Cyril met Margaret Yvonne Pickard, known as Rita, and they were married in 1945. Rita had been in the English Department at Newcastle, teaching and doing research on Early English texts. She gave up teaching upon marriage but continued with research, publishing two texts in the Early English Text Society. Their daughter, Margaret, was born in 1949 and has had a career in publishing, distance learning and educational technology.

In January 1947 Cyril appointed me to my first job. He must have come out especially to meet me, but I took him for a porter and asked him for Professor Offord's room. I recognized my mistake only when he sat behind his desk. This illustrates how Cyril, in spite of his great distinction, was very unassuming. Several of his former colleagues have told me what a good and supportive boss he was.

To me Cyril and Rita could not have been kinder. They took me to live with them in their house in Whitley Bay until they had helped me to find suitable lodgings. Cyril gave me a very light teaching load, which left me plenty of time for research. On one day in the week I had just one lecture and asked Cyril whether it was really worth travelling down from the coast for that. He might reasonably have bitten my head off, but just replied that it was necessary. Offord told me about his collaboration with Littlewood (see also ⟨**3**⟩). At that stage they were just writing their big paper [**26**]. Littlewood made him change the draft several times, until finally it came back nearly to what it had been in the first instance. Cyril advised me to do my research with a pencil having a rubber at the end, and I have stuck to this method of working ever since. Such first-rate scientists as G. H. Hardy, Dame Mary Cartwright and Cyril Offord did what was perhaps their best work in number theory, differential equations and complex analysis, respectively, jointly with Littlewood.

Offord had recently appointed W. W. Rogosinski as Reader. So we three complex analysts formed a little seminar to study Nevanlinna theory. Thus, Offord created an extremely stimulating research atmosphere, which I was sorry to leave when financial considerations and the wish to get married induced me to move to Exeter in October 1947. The Offords moved to London in 1948, leaving the department in the capable hands of Rogosinski, who continued to develop analysis, and particularly functional analysis, there. From then until his retirement Cyril was in London, first in Birkbeck College and from 1966 at the London School of Economics (LSE).

In London, Offord again showed his flair for quality in his appointments. At Birkbeck he appointed David (later Sir David) Cox in statistics and Roger (later

Sir Roger) Penrose in applied mathematics. He gave his staff the fullest support and let them get on with things in their own way. Yael Dowker tells me that when her husband Hugh started at Birkbeck and asked Cyril what to teach, he said, 'whatever you think appropriate'. He also supported Hugh during Senator McCarthy's anti-Communist witch hunt and nominated Hugh for an exchange at Moscow University.

In 1966 Cyril left Birkbeck after a disagreement over a full-time mathematics degree and went to the LSE to set up a new mathematics degree there, in which statistics and computing replaced mechanics and conventional applied mathematics. The degree changed the bias in pure and applied mathematics to mathematics as applied to social sciences. Here Offord started a trend in mathematical education. Among Offord's appointments at the LSE was Kenneth Binmore, now Professor of Economics at University College London.

Cyril retired from the LSE in 1973 and came to Imperial College as Senior Research Fellow. There he continued his research and participated in and contributed to our seminar. In 1980 the Offords moved to Oxford, where Rita had done her degree in English literature at Lady Margaret Hall. Cyril attended meetings and continued publishing until the 1990s. Although Cyril was in frail health and poor sight owing to the loss of an eye because of a tumour in the 1970s, he and Rita greatly enjoyed their retirement, receiving guests in their small north Oxford house, while Cyril indulged in his passions for early music (of which there are many concerts in Oxford), gardening and French literature, benefiting from the proximity of the Maison Française. He created a beautiful walled garden, making a particular feature of the high walls with climbing plants such as clematis and roses. In summer the area was a feast of crimson and purple. Although musical, Cyril had never learned to play an instrument. He took up the lute in his retirement, receiving lessons from Diana Poulton. The somewhat doleful strains of the lute seemed to fit in with the world-weariness that beset him in his later years. Rita died in 1998 and at the age of 92, and after 53 years of marriage, this blow was hard to recover from. Despite his diminishing sight and increasing frailty of body Cyril stayed mentally alert until the end, but a few years before he died he wrote to me saying, 'it's not much fun being ninety'.

After Rita's death, Cyril and Margaret drew closer together, and despite his physical frailty Cyril was able to offer great support to Margaret. He advised her on what to buy, and the day before his death he teased her that she did not really need to worry about booking her holiday in Orkney as, given the bad weather, there would hardly be a run on rooms.

Cyril was not religious for most of his life. He held strong left-wing views and was on good terms with Hua Lo Keng and Yang Le, who dominated Chinese mathematics in the 1970s. My wife and I benefited from this in being able, with Cyril's support, to visit China in 1977. Thus, Cyril was influential in building links between Chinese and Western scientists. Before that time there was very little contact between the two groups.

Not long before his death Cyril was received into the Church of England, where he was confirmed by the Bishop of Oxford. Professor Michael Screech was a weekly visitor from the church that he had attended regularly with Rita, who was herself a confirmed member of the Church of England. Professor Screech has testified to Cyril's deep faith but also said that he was so bitter about his narrow Plymouth Brethren upbringing that he did not like to talk about his early years.

Cyril was gentle and gracious and, although frail in his last few years, he continued until he died to be an interesting companion, talking about religion and the state of the world and intellectual topics generally.

John Aldington met Cyril first in 1936, and he and Jack Shapiro told me of the warmth of Cyril's friendship for more than 40 years. John Aldington confirmed that Cyril was a Marxist and looked on Marxism as a science. After the war, many of the staff at Birkbeck College were left-wing. At the time, Cyril believed that in China there was a better understanding of Marxism, but in the end he was critical of certain aspects of Chinese communism.

Yael Dowker said that Cyril made her feel that all was not lost. Cyril was a kind and gentle person and the world is poorer for his passing.

2. *Students*

Cyril Offord had several research students at Birkbeck College and at least one at the LSE. Among his mathematical descendants are the following.

Joseph E. A. Dunnage, 1949–56. Dunnage taught at Chelsea College, which was absorbed into King's College in 1985, where he became Reader in Mathematical Analysis, and is now Emeritus. His research has been on sums of random variables, such as random polynomials, on concentration functions and large deviations. Joseph recalls that Cyril spent a lot of time during his supervisions sitting back in his chair with his eyes shut, but is not sure whether Cyril was deep in thought or whether he was waiting for Joseph to make some intelligent suggestions. Joseph's students are Professor K. Farahmand at the University of Ulster, whose main work is in random polynomials, and Dr A. W. Matz, whose thesis (1974) was on data fitting. Dr Matz was head of mathematics at the North East London Polytechnic (now University of East London).

Roy Perry, 1950–52. Perry lectured at Queen Elizabeth College. Roy reports how he and Cyril used to meet in Russell Square if the weather was fine, because space was restricted at Birkbeck College, where Cyril shared an office.

Ronald Maude, 1953–56. Maude lectured at Aberystwyth, Leeds, Ibadan, Nigeria, and Lima, Peru. Ronald recalls Cyril saying that he had never been on a committee that decided anything.

Gar de Barra, 1958–61, PhD 1962. Gar became a senior lecturer at Royal Holloway College. He worked on random polynomials and the central limit theorem. Gar writes about Cyril:

> he was very particular about his appearance. I was struck by the fact that he did not work with the usual pads of paper to scribble on. Instead, sheets of the best quality paper were stacked at hand and he had a collection of pencils ready at all times with wedge-shaped points with which he wrote with an italic script.

Professor G. Samal, 1959–61. Samal worked on random polynomials and became a professor in Orissa, India. He collaborated with M. N. Mishra and founded a group doing valuable work in this area. He describes how he completed his PhD in two years although Cyril thought this would be impossible. When he asked for a problem two months after his arrival, Cyril's response was, 'There are plenty of problems. If you take up a problem you may get the result and a PhD very soon. Sometimes people get a PhD without knowing much mathematics. So what is needed is that you go on studying and studying.' There was cordial correspondence between Samal and Cyril until the latter's death. Cyril wrote on 6 September 1994:

> the first few years of retirement can be very pleasant and I hope your wife and yourself enjoy them fully. I have now reached 88 and things are not so pleasant when you get to that age. However, I have managed to re-write some of the work I did with Littlewood. It is now a bit more concise and I think easier to read. I do hope we shall keep in touch. I should like to hear of what you are doing.

Edward A. Evans, 1960–63, PhD 1965. Evans worked in turn at the Universities of Exeter, Hull, Sokoto (Nigeria) on secondment from Hull, and finally at St Mary's College, Twickenham. Evans's research was on sums of independent random variables. He writes as follows.

> He was very kind and helpful, but I didn't get to know him. Having seen in *Peace News*, I think, that his address was in the advertisement for the Chinese Friendship Society, I did once ask Professor Offord about the society but I got the feeling that he did not want to talk about the society or his political views.

Cyril Offord moved to the LSE in 1966. The first person to be awarded a PhD in mathematics there was Cyril's last student.

Professor P. L. Davies, 1967–70, Universität Gesamthochschule, Essen, Germany. Davies's thesis was on zeros of random entire functions. Professor Davies writes as follows.

> During the disturbances at LSE in 1968 I found myself sitting in the road outside Bow Street police station. Several of us were arrested and I spent about 4 hours in police custody. I told Cyril about this and he immediately took me out for a meal at an Italian restaurant. It was the only time he talked politics with me.

A student of Davies's was Professor R. Grübel, now at the Institut für Mathematische Stochastik at the University of Hannover. A student of Grübel's is Susan Pitts, now a lecturer at the Statistical Laboratory of Cambridge University. Thus, Cyril's influence continues in both analysis and statistics.

3. *Work*

After taking his first degree at University College, London, Offord went to Cambridge to study, originally under G. H. Hardy. The next decade was extremely productive. Offord did about half of his published research, including some of his best, during this period and he laid the foundation for the research during the remainder of his life.

3.1. *Fourier analysis*

3.1.1. *Trigonometric series* [**1**, **2**, **4**, **5**, **8**, **13**, **21**, **27**]. In the early papers of this section the author establishes conditions for the summability of power series and trigonometric series. For instance, in his first paper [**1**] he obtained substantial results on the summability of power series, which was then a very active topic of research. He obtained necessary and sufficient conditions for a power series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

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to be summable (C, γ) to $S(\theta_0)$ at a point $z = e^{i\theta_0}$ and used these conditions to deduce a number of consequences. We assume that $\gamma > 0$ and that

$$|a_n| = o(n^\gamma).$$

This condition is necessary for f to be summable (C, γ) at any point on $|z| = 1$.

Among the results in this paper are the following.

THEOREM 2. *If $f(z) \to s$ along a path within $|z| = 1$, then $\sum a_n e^{in\theta}$ is summable (C, γ) to s. The result had been proved by Dienes* ⟨**4**⟩ *for $\gamma \geqslant 1$.*

THEOREM 3. *If $f(re^{i\theta})$ is bounded for $\alpha \leqslant \theta \leqslant \beta$ and $0 \leqslant r < 1$ then, for $\alpha < \theta < \beta$, $\sum a_n e^{in\theta}$ is summable (C, γ), whenever it is summable A; that is,*

$$f(re^{i\theta}) \to \ell(\theta) \qquad \text{as } r \to 1.$$

The converse is classical.

In the companion paper [**2**], corresponding results for trigonometric series are obtained.

Perhaps the finest paper in this group is [**21**]. Let $f(x)$ be a function of period 2π and let $U_n(f, x)$ be the unique trigonometric polynomial, of order at most n, which assumes at the $(2n + 1)$ equidistant points

$$x_i^{(n)} = \frac{2\pi i}{2n+1}, \qquad i = 0, 1, \ldots, 2n, \tag{1}$$

the same values as $f(x)$. It was well known that, even if f is continuous, $U_n(f, x)$ need not converge for any x as $n \to \infty$.

Nevertheless, Offord shows that if $V_n(f, x)$ is the trigonometric polynomial that assumes at the points (1) the 'smoothed out values'

$$\tfrac{1}{2} f\left(x_i^{(n)}\right) + \tfrac{1}{4}\left\{f\left(x_{i-1}^{(n)}\right) + f\left(x_{i+1}^{(n)}\right)\right\}$$

and if $f(x)$ is continuous, then $V_n(f, x)$ tends uniformly to $f(x)$.

3.1.2. *Transform theory* [**3**, **6**, **7**, **9**–**12**, **14**–**18**]. In this group of papers the author is concerned with Fourier transforms (and in [**3**, **7**] Hankel transforms). Specifically the author asks: if f belongs to a class of functions H, what can we say about the class B^* of the transforms F of f?

In [**6**, **18**] Offord proves uniqueness theorems. Thus in [**16**] he obtains the trigonometric integral analogue of Cantor's uniqueness theorem for trigonometric series. He proves the following theorem.

THEOREM. *If $\phi(u)$ is L-integrable in every finite interval and*

$$\lim_{\omega \to \infty} \int_{-\omega}^{\omega} \phi(u) e^{ixu}\, du = 0 \tag{2}$$

for all x, then $\phi(u) = 0$.

In [**18**] the author proves that the conclusion still holds if the integral is only $(C, 1)$ summable to zero instead of condition (2). In his book on Fourier integrals ⟨**19**, p. 166⟩, Titchmarsh calls the $(C, 1)$ result 'a remarkable Theorem'. He notes that it is best possible in two ways. One cannot replace $(C, 1)$ by $(C, 1 + \delta)$, nor can the condition (2) be relaxed even at a single point x_0. In fact, e^{ixu} is $(C,1)$ summable to zero for $x \neq 0$ and ue^{ixu} is $(C, 1+\delta)$ summable to zero for every x, when $\delta > 0$.

Titchmarsh also quotes six other papers by Offord. However, fashion changed and the work of Offord on Fourier analysis was largely ignored by later authors.

3.2. *Probabilistic analysis*

3.2.1. *Polynomials* [**19**, **20**, **23**, **34**, **43**]. Offord, with Littlewood [**19**, **20**, **23**] and Erdős [**34**], pioneered the subject of probabilistic analysis. In this area we consider a class $\{P(z,\mu)\}$ of functions $P(z)$ endowed with a probability measure μ and investigate what is 'typical' behaviour of the functions $P(z)$, that is, the behaviour of $\{P(z,\mu)\}$ outside a set whose μ-measure is small or zero. In this group of papers we are concerned with polynomials P of degree n and random coefficients, and ask how many real zeros P typically has. The crude answer is 'rather few'.

I am greatly indebted to Professor K. Farahmand for the following account.

> *Offord's contribution to random polynomials.* The study of random polynomials can be traced back to 1782 when, as reported by Holgate ⟨**7**⟩, Waring ⟨**20**⟩ and later Sylvester ⟨**18**⟩ used random properties and probability theory to solve some classical algebraic problems. However, modern studies in this area began with the work of Littlewood and Offord, who were themselves motivated by a paper by Bloch and Pólya ⟨**1**⟩ on the maximum number of real zeros of certain classes of algebraic polynomials. Let
>
> $$P_n(x) = \sum_{j=0}^{n} a_j x^j$$
>
> be a polynomial of degree n with randomly selected coefficients a_j. For a variety of choices of distributions for these coefficients, which covers both continuous and discrete cases, they found a surprisingly small number of real zeros for $P_n(x)$. For example, in the second paper in their series [**19**, **20**, **23**, **26**] they showed that in the interval $(-\infty, \infty)$ all, apart from a certain exceptional set of polynomials of measure less than $(12 \log n)/n$, have at most $25(\log n)^2$ zeros.
>
> As a result of this series of works, begun in 1938, the subject of random polynomials was born. Since then it has developed in many directions. On the basis of the unexpected results already gained, Offord, together with Erdős, in [**34**] considered one type of distribution for the coefficients a_j, which proved to be one of the most difficult cases studied. Their result was also more significant than just obtaining an upper bound for the number of real zeros. In fact for n sufficiently large and if the coefficients a_j, $j = 0, 1, 2, \ldots, n$, possess values -1 and $+1$ with probability $\frac{1}{2}$, they showed that most of the $P_n(x)$ have
>
> $$(2/\pi)\log n + o\Big\{(\log n)^{1/3} \log(\log n)\Big\}$$
>
> real zeros. The measure of the exceptional set of polynomials is smaller than $o\{\log(\log n)^{-1/3}\}$. These works were fundamental to the discovery of the formula known these days as the Kac–Rice formula ⟨**11**, **17**⟩. The latter gives a simple expression for the expected number of real zeros of any polynomial of the form $P_n(x)$ with mild assumptions on the distributions of coefficients. This formula, important and significant in its own right, formed the basis of discussions on crossing problems for wide classes of stochastic processes. Because of their importance, there is no doubt that these developments on stochastic processes would have been achieved with or without assistance from random polynomials. However, the results obtained for random polynomials, and in particular random algebraic polynomials, have contributed significantly to their development. Random polynomials certainly opened the paths for these developments. These

influences of random polynomials, and therefore of Offord's works, surely deserve greater acknowledgment than they have received.

Offord's works, as discussed above, covered many general distributions for the coefficients, and therefore for the polynomial $P_n(x)$. However, for the obvious reason of simplicity, the formula for the expected number of level crossings of stochastic processes, that is real roots of the equation $\xi(x) = K$, was first developed for stationary ξ. Indeed, the results later developed to a wide class of non-stationary stochastic processes, of which random polynomials are a special case. In a sense, for the polynomial $P_n(x)$ the expected number of real roots of the equation $P_n(x) = K$ in the interval (a, b) is given by

$$\int_a^b dx \int_{-\infty}^{\infty} |y| \, p_x(K, y) \, dy,$$

where $p_x(t, y)$ is the joint probability density function of $P_n(x)$ and its derivative $P_n'(x)$. Therefore our problem of level crossings reduces to evaluating a joint probability density function and a double integral. This opens the way for obtaining results that could not previously have been derived, or could not have been as easily derived. They give a good understanding of the mathematical behaviour of random algebraic polynomials, much of which is surprising, and motivates future works, some of which are still under way. However, most of these works were for Gaussian distributed coefficients, and it was not until the late 1960s and early 1970s that other distributions were seriously considered. These works were initiated by Logan and Shepp ⟨**13**, **14**⟩ and further developed by Ibragimov and Maslova ⟨**8**–**10**⟩. The latter work, indeed, covers the cases that were introduced by Erdős, Littlewood and Offord.

Later, Dunnage ⟨**5**⟩, when Offord's research student, obtained for random trigonometric polynomials

$$T_n(\theta) = \sum_{j=1}^{n} a_j \cos j\theta, \qquad 0 \leqslant \theta \leqslant 2\pi,$$

a result corresponding to the one found by Erdős and Offord for algebraic polynomials. In this work it is shown that for normally distributed coefficients, all except a set of polynomials $T_n(\theta)$ whose measure does not exceed $(\log n)^{-1}$ have a number

$$\tfrac{2}{\sqrt{3}} n + O\Big\{ n^{11/13} (\log n)^{3/13} \Big\}$$

of real zeros.

This work was the basis for the later results, which, compared with the algebraic polynomials, show a different pattern of behaviour for the two cases. For instance, unlike algebraic polynomials, the number of real zeros of trigonometric polynomials is unchanged when the mean of the coefficients assumes a non-zero constant value. The zeros of the trigonometric polynomials are generally more uniformly distributed than those of the algebraic polynomials. However, so far all the results obtained for this type of polynomial are for the Gaussian case. Those methods successfully used for the algebraic case have proved so far to be too complicated to yield any results for trigonometric polynomials. This is also the case for other types of random polynomials such as random hyperbolic polynomials, and for polynomials with orthogonal elements. A comprehensive review of the general properties for random polynomials is given in the book by Bharucha-Reid and Sambandham ⟨**1**⟩. In addition, some detailed comparisons of the mathematical behaviour of different types of random polynomial, categorized by their level crossings' properties, can be found in ⟨**6**⟩. However, it is obvious that there is no analogue for the above results for the expected number of real roots in the interval $(-\infty, \infty)$, if one considers the entire function

$$f(x) = \sum_{j=0}^{\infty} a_j x^j.$$

The problem of entire functions is of a different level of difficulty. Among the contributions of Offord in this direction, [**46**] is remarkable because it was published when he was 89 years old. We return to this subject in the next section.

3.2.2. *Entire functions* [**24**, **26**, **29**, **30**, **35**, **44**–**46**]. This group of papers constitutes some of the most significant as well as the most difficult of Offord's work. The basic results constituted joint work with Littlewood; they were announced in [**24**] and proved in [**26**].

The paper [**24**] begins with the following introduction by the authors.

> The elliptic function $\sigma(z) = \sigma(z;\omega_1,\omega_2)$ is an integral function of z of order 2 with a behaviour of the greatest simplicity. It is exponentially large except in regions of exponentially small area round the zeros, and these regions are distributed uniformly over the z plane. If we map the values of $\log^+|\sigma(z)|$ on the z plane, we obtain a surface like a bowl (the height at distance R from the origin being of order R^2), with uniformly distributed 'pits' each exponentially small in area.
>
> Many who have speculated on the subject must have entertained the conjecture that this behaviour, generalized on obvious principles from $\rho = 2$ to any non-zero order ρ_s is to be expected, approximately, for 'almost all' integral functions of order ρ. We have now obtained results that can fairly be said to establish this conjecture.
>
> The concepts 'most' and 'almost all' integral functions imply a basis of probability. Some such bases are more restricted than others; at the extreme both the moduli and arguments of the coefficients are subject to random variation; at the other one set, say the moduli, are arbitrarily given and only the arguments are random. Since the assertion of the conjecture has a far wider scope in the second case we have made that our principal study.
>
> We suppose a set of coefficients a_n to be given, subject only to $\sum a_n z^n$ being an integral function of finite non-zero order ρ. A 'random' factor ± 1 is then superimposed and we have a family $\mathcal{F}$ of functions $f(z) = \sum \pm a_n z^n$. We find now that there is a subclass $\mathcal{F}^*$ containing 'almost all' the f of $\mathcal{F}$, such that all f of $\mathcal{F}^*$ show the 'pits' behaviour; f is exponentially large except in pits of exponentially small area. If now D is any set of a-values containing $a = 0$, it follows from Rouché's theorem that each pit of an f of $\mathcal{F}$ except for a finite number, contains the same number of a-values as it does zeros; thus all a-values behave essentially alike. As for the distribution of the pits in the z-plane, we are able to assert that for all f of $\mathcal{F}^*$ it is approximately the same. And finally this common distribution has considerable regularity in respect of direction and as much regularity in respect of distance (from the origin) as is compatible with the nature of the case.

There are two ways in which the typical behaviour of $\sum \pm a_n z^n$ may differ from that of $\sigma(z)$. The first is referred to in the last sentence above. We define $\mu(r)$ and $N = N(r)$ by

$$\mu(r) = \sup_n |a_n| r^n = |a_N| r^N. \tag{3}$$

Then it is quite possible for $f(z)$ to be given asymptotically by the largest term for $|z| = r$,

$$f(z) = (1 + o(1)) a_N z^N, \qquad r_N < |z| < r'_N, \tag{4}$$

where $r_{N'}/r_N \to \infty$. This phenomenon occurs for instance when f is highly lacunary, that is most of the coefficients a_n vanish. Clearly, $f(z)$ will have no zero in the ranges (4) and so no pits.

The second possible deviation from the behaviour of $\sigma(z)$ is more subtle. In the case of $\sigma(z)$ all the pits have multiplicity one; that is, each pit contains exactly one zero. In [**26**, Theorem 10] the authors give an example where all functions in $\mathcal{F}^*$

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have an infinity of double pits and state that the example can be extended to yield pits of unlimited multiplicity.

The arguments leading to the proofs of their results in [**26**] are deep and complicated and beyond the scope of this memoir. They depend only on the quantity (3) defined above and so on the moduli $|a_n|$ of the coefficients. The authors also state that changing the probability base, for example by varying the moduli instead of the arguments of the coefficients, does not lead to essentially different conclusions.

One consequence of the authors' results is the proof of a conjecture of Pólya ⟨**16**, p. 622⟩ that all the functions of $\mathcal{F}^*$ have many pits in every sector $\alpha < \arg z < \beta$ so that f assumes every value infinitely often in each such sector.

Both authors return to the general area of [**24**] and [**26**]. For instance, Offord announced in [**29**] and proved in [**30**] that the behaviour described above, although typical in a measure-theoretic sense, is exceptional in the sense of Baire category. The family $\mathcal{F}^*$ has first category; that is, it is the union of a countable number of non-dense sets, when $\mathcal{F}$ is supplied with a suitable metric.

In [**35**] and [**46**] Offord shows that the main conclusions described above still hold if $\sum \pm a_n z^n$ is replaced by $\sum a_n(u) z^n$, where in [**35**] the $a_n(u)$ are independent random variables with some mild additional hypotheses and in [**46**] the $a_n(u)$ are symmetric; that is, $a_n(u)$ and $a_n(-u)$ are equally distributed. In [**44**] and [**45**] the author shows that the conclusions also hold for all functions

$$f(z) = \sum_{n=0}^{\infty} a_n z^{\lambda_n}$$

that are sufficiently lacunary, that is, for which

$$\limsup_{n\to\infty} \frac{\log n}{\log \lambda_n} = \alpha < 1.$$

If $a < \frac{1}{2}$, and f has finite order and regular growth then, if $\varepsilon > 0$, the author proves in [**45**] that

$$\frac{\log |f(z)|}{\log M(|z|)} > 1 - \varepsilon$$

when z is outside the pits and $|z|$ is large.

3.2.3. *Functions in the unit disk* [**36**–**39**, **42**, **43**]. In this group of papers the author (with M. Jacob for the last two) considers in general the typical behaviour of

$$f = f(z, \omega) = \sum_{n=0}^{\infty} a_n(\omega) z^n, \qquad |z| < R,$$

in which the $a_n(\omega)$ are random variables and the series converges in the unit disk ($R < \infty$) or is entire ($R = \infty$). In [**38**] the author proves that if $\sum |a_n| = \infty$ the range of f is the plane almost surely. In [**39**] the author asks whether f is recurrent on radii; that is,

$$\liminf_{r\to R} |f(re^{i\theta}) - a| = 0$$

for every complex a and θ in $[0, 2\pi]$, or transient so that

$$\lim_{r\to R} f|(re^{i\theta})| = \infty.$$

He states that if

$$\liminf_{n\to\infty} -\frac{\log|a_n(\omega)|}{n\log n} > 0,$$

then f has the transient property outside a set G of measure zero, but dense and of second category, but is recurrent in G.

He quotes some conditions of Kahane ⟨**12**⟩ for f in the unit disk to be almost surely recurrent on every radius and other conditions for f to be almost surely transient on every radius.

In [**41**] the authors prove that if λ_n is an increasing unbounded sequence of positive integers and a_n is a sequence of complex numbers such that

$$|a_n|^2 r^{2\lambda_n}$$

converges for $r < 1$, but tends to ∞ as $r \to 1$, then

$$\sum_{n=0}^{\infty} \pm a_n z^{\lambda_n} \tag{5}$$

almost surely has unbounded Nevanlinna characteristic, so that the image of the unit disk is almost surely the whole plane with the exception of a set of capacity zero. The case when a_n is unbounded turns out to be particularly hard.

In [**42**] the authors prove that if

$$\limsup_{n\to\infty} |a_n|^{1/n} = 1$$

and

$$\limsup_{N\to\infty} \frac{\log\sum_1^N |a_n|^2}{\log N} > 0,$$

then almost all the functions (5) take every complex number infinitely often in every sector

$$\alpha < \arg z < \beta,$$

of the unit disk. This beautiful result extends significantly an earlier theorem of Murai ⟨**15**⟩, itself an extension of the above quoted result in [**41**].

Offord's last two papers [**45**, **46**] show that if

$$f(z) = \sum_{n=0}^{\infty} a_n z^{\lambda_n}$$

is an entire function with sufficiently large gaps and satisfying certain order conditions, then $f(z)$ necessarily has the sort of behaviour that was described earlier for almost all functions in a probability space. In particular, all the zeros of f occur in small pits and $f(z)$ is large outside the pits. These two papers, written with the author's deep insight and formidable analytic technique while he was in his late eighties, form a worthy conclusion to Offord's life's work.

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