in the solution of linear algebraic equations for instance the pessimistic predictions of von Neumann and Goldstine are confirmed. The province of interval analysis is that of re-writing known algorithms and developing new ones so that the use of interval arithmetic will lead to satisfactory error estimates. In the applications given here this means that a close investigation of the given mathematical problem has to be made before actually performing the calculations, and then taking it into account when organizing the program of operations.

The book is in two parts. The first is on algebraic problems and the chapter titles are as follows:

1. Introduction to algebraic problems by R. E. Moore.
2. Triplex-Algol and its applications by K. Nickel.
3. Zeros of polynomials and other topics by E. Hansen.
4. On linear algebraic equations with interval coefficients by E. Hansen.
5. On the estimation of significance by J. Meinguet.

The second part is devoted to continuous problems:

6. Introduction to continuous problems by R. E. Moore.
7. On solving two-point boundary value problems using interval arithmetic by E. Hansen.
8. Ordinary differential equations by F. Krückerberg.
9. Partial differential equations by F. Krückerberg.
10. On the centred form by E. Hansen.
11. Distributions in intervals and linear programming by M. Dempster.

The contributions are of varying quality and length. For example the chapter on partial differential equations is three pages long and is a sketch of possible ways of dealing with two types of equation. There is clearly a great deal more work to be done in this direction. But chapter 4 contains a fairly detailed account of the problem of its title. The first two chapters are thoroughly practical and can be recommended as a useful exposition of some of the aims and techniques of interval analysis. However the final chapter, although of theoretical interest, has little to say to the practical numerical analysist.

It is clear to the reviewer from the examples discussed that the marriage of interval analysis with mathematical analysis produces worthwhile results. It remains to be seen if the union can survive all the problems, including those of cost effectiveness, which it will be called upon to solve. In the meantime the book can be recommended as a useful collection of essays on this expanding field of research. D. KERSHAW

NEILL, H. AND MOAKES, A. J., *Vectors, Matrices and Linear Equations* (Oliver and Boyd, 1967), 216 pp.

The teaching of these topics at sixth-form level in schools is now well-established, and these two experienced authors cover them very broadly. I find congenial their introduction of general ideas such as algebraic structure, mapping, equivalence relation, and the definition of an algebra as a set with a binary relation defined on it. These topics are well introduced with many examples which will be familiar and illuminating. It is also good to see linear dependence and the fundamental idea of a vector space being thoroughly explained at this stage in simple terms.

Certainly I think the authors have tried to cram too much into the space, and the development of matrix algebra in chapters 9 and 12 would be heavy going for beginners of less than the highest calibre. The banishing of a vital step in § 12.7 to an appendix is surely a sign of overloading, as is "An appeal to a book on mechanics" on p. 21. Layout is pleasant although occasionally cramped. Misprints are few—"$x+y$" for "x and y" on p. 18 and "vector non-zero" for "non-zero vector" on p. 114 are confusing. One answer is undoubtedly wrong (12(*a*)(2)) and some questions appear to have been poorly set [e.g. 6(*b*)(2) where the wording is misleading, 10(*b*)(1) where the officer must surely not know his men and 12(*b*)(5) where a stochastic matrix is given an incomplete definition] but the general standard is good. The book will certainly be stimulating for the brighter sixth-form pupil.

M. PETERSON

KARLIN, S., *Total Positivity*, Vol. I (Stanford University Press; London: Oxford University Press, 1968), xi+576 pp., 166s. 6d.

Let X and Y be sets of real numbers (or any totally ordered sets) and let $x_1, \ldots, x_m$ and $y_1, \ldots, y_m$ be m-tuples, arranged in increasing order, in X and Y respectively. For a real-valued function $K(x, y)$ denote by $K_{[m]}$ the determinant of $[K(x_i, y_j)]$, $1 \leqq i, j \leqq m$. Call $K(x, y)$ sign-consistent of order m (SC_m) if $\varepsilon_m K_{[m]} \geqq 0$ for all x_i, y_j ($\varepsilon_m = \pm 1$), sign-regular of order r (SR_r) if it is SC_m for $1 \leqq m \leqq r$, totally positive of order r (TP_r) if it is SR_r with $\varepsilon_m = 1$ for $1 \leqq m \leqq r$, and a Pólya frequency function of order r (PF_r) if $K(x, y) = K(x-y)$ and is TP_r. These concepts and their ramifications (such as strictly SC if $\geqq$ is replaced by $>$, confluent forms, etc.) are the central theme of this work.

Total positivity (TP) and related properties play (sometimes indirectly) an important role in surprisingly numerous areas of mathematics including convexity, inequalities, moment spaces, eigenvalues of integral operators, oscillation properties of solutions of differential equations, approximation theory, statistical decision processes, inventory and production problems, reliability theory, stochastic processes of diffusion type, and the study of coupled mechanical systems. The present volume I concentrates primarily on developing the analytical properties of TP_r functions, although some applications (among them applications to approximation theory and to differential equations) are included. A projected second volume will contain further applications, notably to integral operators, statistics, and stochastic processes. The author estimates that more than half of the material presented in the volume under review appears here for the first time; and much of the remaining part is presented for the first time in a unified and sometimes novel form.

After a brief summary of some formulae involving matrices and determinants, chapter 1 contains in 35 pages an overall view of TP and its applications and may be regarded as a prospectus of the entire 2-volume work. The subject of chapter 2 is a hierarchy of notions of sign structure and their interrelations, of chapter 3, operations preserving TP, of chapter 4, smoothness properties of TP_r functions, of chapters 5 and 6, the variation-diminishing property of SR_r functions and its applications, of chapters 7 and 8, PF_r functions in the continuous and discrete case respectively, of chapter 9, periodic PF_r functions, and of chapter 10, the role of TP_r functions in connection with differential equations.

This volume contains an enormous amount of material, the organization of which presents a formidable problem, and it would be idle to pretend that the book is easy to read. The author has performed a very valuable service by relating all this material to a central theme and showing how the concept of sign regularity irradiates and illuminates a great variety of subjects.

A. ERDÉLYI