

ASTIN COLLOQUIUM 1966 ARNHEM  
SUBJECT ONE  
EXPERIENCE RATING AND CREDIBILITY

# EXPERIENCE RATING AND CREDIBILITY

Introductory report

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I hope that you will allow me first of all to congratulate Dr. Bühlmann on the excellent lecture he has given us and I am sure that you can all agree with me that after his general survey on experience rating and credibility, there is no need at all for me to go into the general aspects of the subject. Therefore I can start immediately to report on the contributions delivered, and I am very happy to state that, though the number of papers is rather small, namely six, their contents are qualitatively as high as has become usual at our ASTIN-Colloquia.

As regards the number of contributions I just mentioned, I like to draw your attention to the fact that because the present subject was on the programme of our last colloquium in Luzern too, in some cases we had to decide whether a contribution should be considered as relevant to subject 1, or as a report on further work done on subjects from previous meetings. If perhaps in some cases our choice is not in accordance with the intention of the contributor I would like to apologize for that in advance. Moreover I wish to remind you that on Friday after the summaries of the discussions, Mr. Jansen will report on those papers submitted, which were not regarded as contributions to subject 1 or 2.

I would like to deal first with the papers of a more theoretical nature, afterwards going on to the contributions on special applications. Among the papers of general interest I ask your attention for the paper of Franckx entitled "Le comportement de l'assureur". I consider this contribution as a very important attempt to compare the method of credibility theory, where as you know credibility is used as a function of the volume, with a method in which decisions are made in advance on the amount of certain levels of significance, in this way fixing what Franckx calls "le comportement a priori".

He states that an insurer should choose a priori the value of

two percentages  $K$  and  $L$ , which constants of, for example 2 or 5 %, have to be fixed not as the answer to a mathematical problem, but as a matter of management. The significance of  $K$  and  $L$ , which are called "les paramètres de comportement" is the requirement that—to say it in a way inspired by the paper—it is "almost-certain" that the total number of claims out of  $n$  assurances is "almost-equal" to its expected value.

To put this in a mathematical form, Franckx uses the central limit theorem, which for sufficiently large values of  $n$  leads to a critical number  $I$  depending only on  $K$  and  $L$ . On this critical number depends a lower-limit for the number  $n$ , this lower-limit being

$$n_0 = \frac{I}{\alpha}, \text{ where } \alpha = \left( \frac{Ex}{\sigma x} \right)^2$$

in the notation of the paper.

The a priori conditions are fulfilled only for values of  $n$  larger than  $n_0$ . As I said before the connection which Franckx lays between these a priori demands and the American practice in credibility theory is very interesting. As is well known credibility has always been defined as a function of the volume of the portfolio or risk-class, where full credibility can be reached only beyond a certain limit in order to reduce the probability of undesirable deviations between the premium paid and the true risk-premium.

In his report to the Luzern-colloquium Bühlmann derived a formula for the credibility from the general solution of experience-rating by considering credibility-theory as its best linear approximation, and found

$$\frac{n}{n+k}, \text{ where } k = \frac{E[\sigma^2(\theta)]}{\text{Var}[\mu(\theta)]}$$

In the case of a Poisson model Franckx shows that the a priori conditions are equivalent to the condition that the expected number of claims should be larger than the critical number  $I$ , whereas in credibility-theory this appears as the a posteriori control that for full credibility the observed number of claims has to be greater than the critical number. As he states at the end of his very clear

contribution the problem of full credibility can in this way be seen as the question whether or not the central limit theorem can be used a posteriori.

The next paper on which I wish to report is that of Welten, who, under the heading "The unearned no claim bonus", studies an effect in experience rating which as far as I know, has so far not been examined in detail.

Welten's starting-point is in fact the same as that of Franckx, namely the danger of deviations from the proper risk-premium, but where Franckx derived conditions for the volume of business to prevent these fluctuations, Welten has tried to estimate their influence in the case of bonus-malus-systems, taking into account not only the individual random factor but also a collective one, having the same effect on all risks of a portfolio.

Of course it is rather obvious that owing to these effects a bonus-system can lead to wrong results. If for instance in a certain year the individual and collective random factors cause a small number of claims, this year itself will show a large difference between the premiums received and the total amount of claims, thus providing a large profit, whereas in the next year the estimation of the risk premium will be too low. To correct this tendency Welten advises the formation of what he calls a bonusreserve and he illustrates the possible size of such a reserve by calculating an example in motor car insurance.

For his model of the total number of claims in a year he uses the same distribution as has mostly been applied by previous authors in the case of motorcar insurances, namely a compound Poisson-model with a gamma-distribution as the structural function of the portfolio. Moreover, as I have already mentioned, he also assumes the presence of a collective random factor. By applying the theorem of Bayes an a posteriori conditional expression is derived for this last distribution function from the observed claim experience in the past. Starting from this model Welten succeeds in deriving an expression for the total amount of unearned bonuses based on the experience up to a certain year. This expression can be further reduced if a large portfolio is assumed and finally a simple formula is obtained for the relative part of the yearly profit that should be reserved to cover future unearned bonuses, namely

$$\frac{D}{W} = b \cdot \sum_{u=1}^{\infty} \frac{r_u}{1 + b \cdot (t + u)}, \text{ where}$$

$D$  = total amount of unearned bonuses,  $W$  = profit

$b$  = one of the parameters of the gamma-distribution

$t$  = age of the assurances in the portfolio, and

$r_u$  = the fraction of the portfolio, still existing after  $u$  years.

Welten has calculated the fraction  $\frac{D}{W}$  for some special cases and found values up to 0.24, which illustrates the importance of the effect.

In the past attention has often been paid to special difficulties of bonus-systems, for example by Gürtler, Derron and others, but their remarks were mainly objections from the point of view of the assured. The concept of the bonus-reserve however seems to be of importance for the assurance company and, though of course the results so far calculated can have only a qualitative significance, they give rise to the idea that a bonus reserve may be one of the necessary reserves in non-life assurance.

Philipson has delivered a paper to our colloquium, the title of which "Comments on Different Deductions of Expressions for Conditional Expectations" already reveals that Philipson makes some remarks and adds certain conclusions of his own to earlier published results.

Firstly he derives, in a way similar to the reasoning introduced by Bühlmann, an expression for the best linear estimate of the conditional mean value of the individual risk parameter. His result corresponds with previous results of Lundberg and Bichsel, submitted in Luzern. As Bichsel did when introducing random fluctuations in time, Philipson again stresses the fact that time-variations should be taken into account for this risk parameter, random as well as non-random.

In connection with this remark of Philipson I am happy to refer to the ideas for handling a linear trend in time, which Bühlmann gave us today in his brilliant opening lecture. His suggestion for a possible way of attack may very well stimulate further investigations on non-random time dependency.

Apart from the comments I have just mentioned Philipson quotes Bühlmann's remark that it might be very useful to search for methods of estimating directly the variables appearing in the credibility formula, derived as a linear approximation to the general conditional expectation, which formula has already been mentioned in connection with Franckx's paper.

Concerning this direct estimation problem Philipson draws attention to some earlier work done by Grenander and Anscombe.

Let us now turn to the paper of Jung.

As is well known to most of us, Bailey and Simon in 1960 published a profound study on the rate-making in motor-car insurance, using for their computations a lot of data on Canadian experiences, and starting from a tariff with a double classification. On this work is based the study which Jung has made on the application of Chi-square-minimum methods which he delivered to our colloquium under the title "On Automobile Insurance Ratemaking". His investigation was prompted by the fact that recently a new tariff has come into force in Swedish motor car insurance. In the preparation of the new rates, of which Andreasson has published full details, not less than eight independent classifications have been considered. Since in a model of this size the original equations of Bailey and Simon mean a considerable amount of work, Jung has tried to find a way to simplify the calculations.

In their study Bailey and Simon considered different models for the function giving the expected value of the relative loss ratios in the tariff classes, but, as was shown by Almer in 1954, the claim frequencies and the loss ratios in Swedish motor-car insurance can be quite well described by a multiplicative model, that is a model in which the expected value of a relative loss ratio is the product of 8 class parameters, each parameter representing the influence of one aspect of the classification. Apart from a set of balance factors, Bailey and Simon estimated the relativities by means of a minimizing Chi-square method. For this method Jung introduces a simplified way of calculating successive estimation, which he illustrates by means of a two-dimensional classification.

For his successive approximations he applies the formulae for the special case of a Poisson-distribution, in each step considering one set of parameters as known, and calculating the other. He

shows that his estimates are unbiased and converge to estimates which render a value of Chi-square only slightly higher than the value corresponding to the solutions of the equations of Bailey and Simon.

To examine the convergence of his iterative process Jung applied it to the Canadian material and shows that his successive estimations converge very rapidly. For this reason I think that his method can be very useful for cases where estimations have to be calculated using a purely multiplicative model.

Though at the end of his paper "Methods of Studying the Risk process in disability insurance" Lundberg states that the results obtained so far neither can, nor should serve as a basis for experience rating, I think that his paper should nevertheless be mentioned in this report. It is a good example of a thorough examination of a closed set of data according to various points of view with the advantage that each individual data record is practically complete. The material consists of the experience gathered from long-term disability insurances taken out by males with a Swedish insurance company and registered as terminated during the period 1955-1960, this termination being due either to attainment of the terminating age, or to death.

The results of the study have been grouped in two categories according to the cause of termination.

To avoid difficulties in determining whether or not subsequent disability periods at short intervals should be considered as one period Lundberg has so far concentrated on the number and the occurrence in time of the first disabilities investigating whether Poisson processes "in the wide sense" or "in the narrow sense", according to the definitions of Philipson, are applicable. Moreover he distinguishes between disabilities lasting at least three months and longterm disabilities of more than 24 months. As Lundberg states himself, the material available has not yet been fully utilized, the work done so far serving as a guide to further examination. However I think that to many of us the knowledge of the present results and even more of the way he attacks a problem of this kind, might be of great interest. I hope that in due course he will inform us further on the results of work still to be done on this data.

As all of you will probably have noticed, in the announcement

of the subjects for our present colloquium the committee added a note to the title of subject 1, stating that though of course theoretical contributions would also be very welcome, it might be useful if some papers were submitted dealing with practical applications.

Though among the papers on which I have already reported, some were closely connected with, or based upon some practical application, I have kept to the end the contribution which fully meets the committee's request. Of course none of us is surprised that this paper has its origin on the other side of the ocean and deals with an application of credibility theory. Of course I mean the paper by Kormes entitled "A practical application of credibility to experience rating plans for hospitalization and medical-surgical insurance".

In this paper detailed information is given on an experience rating plan developed by Kormes in 1949 for the Massachusetts Blue Cross and since adopted by a number of other Blue Cross and Blue Shield organizations. Though there are individual policyholders too, most of the underwriting consists of groups, formed by the whole or a part of the employees of some commercial entity.

As a measure for the credibility of these groups Kormes has chosen the yearly premium income, using the formula

$$z = \frac{P + fK}{P + K}, \text{ where}$$

$P$  = premium income;  $z$  = credibility;  $K$  = a constant and  $f$  = a function of  $P$  which varies from 0 for  $P = Q$  to 1 for  $P = S$ ,  $Q$  being the lower limit for any credibility and  $S$  the upper limit beyond which full credibility is attained.

The limits  $Q$  and  $S$  were chosen in advance.

It is very instructive to see how Kormes deals with some practical difficulties of credibility theory. In what he calls "prospective experience rating" he finally arrives at the formula

$$\text{Rate modification} = 1 + \frac{(R - P)f}{P}, \text{ where}$$

$R$  = adjusted group loss ratio;  $P$  = permissible loss ratio and  $f$  = credibility.

In defining  $R$  and  $P$  effects like the trend in costs and a loading for expenses and contingencies depending on the size of the group, are taken into account.

This rating procedure can be applied to all groups combined to calculate over-all changes in the rates, as well as to each group separately.

Apart from the prospective ratemaking I just mentioned, the rates in Kormes' model are corrected afterwards by a bonus malus system which he calls "retrospective experience rating". By means of simple formulae and taking into account the incurred claims, a claim expense factor and some insurance charge, a refund or a carry-over is calculated.

I think this contribution is indeed a very clear and instructive example of solving a purely practical question of ratemaking.

Now coming to the end of my report I would like to stress that I am fully aware that the limited number of remarks which I was able to make on each paper does not give full credit to their value. Therefore I think it just to express my deep appreciation to all of you who took the effort to deliver such substantial contributions to the development of new ideas concerning experience rating and credibility. I am convinced that your papers will be an invitation to all of us for very valuable discussions.

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