Lie Algebras, by Nathan Jacobson. Interscience Tracts in Pure and Applied Mathematics, Number 10. John Wiley and Son, Inc. New York, 1962

This is the first systematic account of the algebraic theory of Lie algebras in textbook form. It has arisen out of lectures to graduate students. In the author's own words, 'the subject of Lie algebras has much to recommend it as a subject for study immediately following courses on general abstract algebra and linear algebra, both because of the beauty of its results and its structure, and because of its many contacts with other branches of mathematics.' Add to this the author's gift for precise, but always lucid and exciting presentation --in this book in particular, the author's enthusiasm and his enjoyment of the subject is felt all the way through--and it is clear that the book is a must for students of algebra of all ages.

As indicated by the quotation above, the book builds on a general knowledge of algebra; but, for example, the author's 'Lectures on abstract algebra' are ample preparation. There is no attempt to include the 'contacts with other branches' in any detail, though the relevance of concepts and situations under discussion to other mathematical disciplines is frequently pointed out, or hinted at; and the pointer will often be more explicit when the parent subject of Lie groups is concerned.

The first four chapters contain the structure theory. 'Basic concepts' (Chapter I) include the derivations, the centre and the derived algebra, representations and modules, and the definition and elementary properties of nilpotency and solvability. The explicit determination of all Lie algebras of dimension at most three is particularly welcome at this stage. Chapter II exploits the relation between Lie algebras and associative algebras to deal more closely with nilpotent and solvable algebras, deriving in particular the theorems of Lie and Engel and the decomposition into weight spaces of the vector space of certain nilpotent Lie algebras of linear transformations. Chapter III brings the study of Cartan subalgebras and of Cartan's criteria for solvability and semi-simplicity, leading to the structure theorems for finite dimensional semi-simple Lie algebras in the case of characteristic zero and to conditions for the complete reducibility of finite dimensional representations in this case. Chapter IV achieves the complete classification of simple Lie algebras over an algebraically closed field in terms of the Dynkin diagrams associated with the Cartan matrices. Linear representations for the split simple Lie algebras corresponding to all but two of the connected Dynkin diagrams are constructed explicitly. The next chapter prepares for the representation theory of Lie algebras by introducing the universal enveloping algebra and its properties; in particular we find here the discussion of free Lie algebras, the

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Campbell-Hausdorff formula for exponentials, and an introduction to the cohomology theory of Lie algebras. Finally, the 'restricted' Lie algebras arising in the case of prime characteristic are discussed in some detail. Now follow the proofs of the existence of a faithful finite dimensional representation for every finite dimensional Lie algebra, both for characteristic zero and for prime characteristic (Chapter VI), the classification of the irreducible representations by means of Cartan's dominant integral functions and, as a by-product, independent proofs for the existence of split simple Lie algebras corresponding to every connected Dynkin diagram (Chapter VII), and Weyl's formula for the simple characters derived by means of Freudenthal's purely algebraic approach (Chapter VIII). The two final chapters are devoted to the determination of the automorphism groups of the non-exceptional split simple Lie algebras over an algebraically closed field of characteristic zero and to applications of the results: the split simple Lie algebras obtained before are now shown to be non-isomorphic, and there emerge methods to obtain classification of finite dimensional simple Lie algebras over arbitrary fields of zero characteristic. In both chapters, the possibility of extending the results to the case of prime characteristic is mentioned, but the reader is referred to the literature.

Each chapter ends with a collection of interesting examples, many of them highly non-trivial, designed to supplement the text in various ways.

The bibliography of about 150 items is meant to provide the principal references for the text and further reading on the various applications and related subjects. It is full, but is not--and is not intended to be--exhaustive.

Hanna Neumann

Elementary Differential Equations, by William Ted Martin and Eric Reissner. Second edition. Addison-Wesley, Reading, Massachusetts, 1961. xiii + 331 pages. $6.75.

The first edition of this book was published in 1956 and reviewed by J. Korevaar in the American Mathematical Monthly, Vol. 65, No. 6, June-July 1958, pp. 457-9. The second edition contains many new exercises but is not essentially different from the first. Consequently, the cited review is equally pertinent to the second edition, and this review will be brief.

The book is designed as a text for an introductory course and deserves to be recommended for this purpose. The introductory chapter on the nature and origin of differential equations contains