COORDINATE TIME IN THE VICINITY OF THE EARTH

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ABSTRACT. Atomic clock accuracies continue to improve rapidly, requiring the inclusion of general relativity for unambiguous time and frequency clock comparisons. Atomic clocks are now placed on space vehicles and there are many new applications of time and frequency metrology. This paper addresses theoretical and practical limitations in the accuracy of atomic clock comparisons arising from relativity, and demonstrates that accuracies of time and frequency comparison can approach a few picoseconds and a few parts in 10^{16}, respectively.

1. INTRODUCTION

Recent experience has shown that the accuracy of atomic clocks has improved by about an order of magnitude every seven years. It has therefore been necessary to include relativistic effects in the realization of state-of-the-art time and frequency comparisons for at least the last decade. There is a growing need for agreement about procedures for incorporating relativistic effects in all disciplines which use modern time and frequency metrology techniques. The areas of need include sophisticated communication and navigation systems and fundamental areas of research such as geodesy and radio astrometry.

Significant progress has recently been made in arriving at definitions for coordinate time that are practical, and in experimental verification of the self-consistency of these procedures. International Atomic Time (TAI) and Universal Coordinated Time (UTC) have been defined as coordinate time scales to assist in the unambiguous comparison of time and frequency in the vicinity of the Earth. This paper summarizes the procedures for time and frequency comparisons which have been adopted by the Consultative Committee for the Definition of the Second (CCDS) and the International Radio Consultative Committee (CCIR), and addresses future theoretical and practical limitations in the accuracy of coordinate time and frequency comparisons. Time and frequency measurements are also given showing the need for the construction of an unambiguous coordinate time and frequency network near the Earth, the

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consistency of the proposed comparison methods, and the level of accuracy at which such comparisons break down. Consideration is given to relativistic effects arising from the statics and dynamics of the shape of the earth and its spin instabilities as well as from the gravitational influence of other solar system bodies.

Three of the SI units may now be determined from frequency measurements. It is quite common to compare primary frequency standards situated at great distances from each other on the Earth's surface. Using signals from atomic clocks on Earth-orbiting satellites as transfer standards, the comparison measurement uncertainties are found to be less than the accuracies, of a few parts in 10^{14}, of these primary standards. Relativistic corrections needed to accomplish such frequency comparisons are significantly larger than the accuracies of the primary standards; yet in the measurements reported here the standards agreed to well within their accuracies. Further, plans are now under way to generate an improved coordinate clock on the earth to measure the millisecond pulsar. This pulsar has a theoretical $Q = 10^{19}$ and may be more predictable over the long term than the best atomic clock. An important question the International Astronomical Union (IAU) should address is the unambiguous transformation from Earth coordinate time to barycentric dynamical time or to another appropriate time scale for celestial measurements, so that scientists may communicate the results of their studies without confusion. The future holds some significant challenges as clock accuracies increase. The work addressed here will help to provide tractable clock comparisons for some decades.

2. PROCEDURES FOR CLOCK SYNCHRONIZATION NEAR THE EARTH

In an inertial frame clocks can be synchronized by the Einstein procedure, which is based on the constancy of the speed of light. A light pulse emitted from a reference clock, will arrive at a time L/c later at the position of a second clock, if the latter clock is a proper distance L away from the reference clock. Since the Earth spins, a network of clocks distributed on the Earth's surface cannot be self-consistently synchronized by means of this procedure. Also, clocks carried in jet aircraft or satellites are subject to gravitational frequency shifts and time dilation effects (second-order Doppler shifts) which are path dependent.

To obtain a coordinate time system without inconsistencies arising from relativistic effects, one may introduce a "coordinate time" grid in the following way (Ashby and Allan, 1979). Imagine an underlying nonrotating frame, or local inertial frame unattached to the spinning Earth, but with its origin at the center of the earth. In this frame, introduce a fictitious set of standard clocks available anywhere, all synchronized via the Einstein procedure, and let them run at agreed upon rates such that synchronization is maintained. Call the resulting time scale "coordinate time." Now introduce a set of standard clocks distributed around the surface of the rotating Earth. To each one of these standard clocks a set of systematic corrections may be applied, so that at each instant it agrees with the time on a fictitious standard clock, at rest in the local inertial frame, with which it instantaneously coincides. This set of clocks will therefore all be keeping coordinate time. In other words, coordinate time is equivalent to time measured by standard clocks in the local inertial frame.

In the local inertial frame, tidal potential effects due to other solar system bodies, can be shown (Ashby, 1975) to have effects on clocks rates which are negligible at the present; only the Earth's gravitational potential V needs to be explicitly considered. Keeping first-order corrections, and transforming the invariant interval ds^2 to the Earth's frame rotating with angular velocity $\vec{\omega}$, the metric may be written (Ashby and Allan, 1979):

$$ds^2 = [1 + 2(\phi - \phi_0)/c^2](c\,dt')^2 - 2\vec{\omega} \cdot \vec{r}\,' \times d\vec{r}\,'\,dt' - [1 - 2V/c^2]\delta_{ij}\,dx'^i\,dx'^j \quad , \qquad (1)$$

where c is the speed of light, the gravitational potential ϕ includes centrifugal effects, ϕ_0 is the value of ϕ on the geoid, primes denote quantities measured in the rotating frame, and $t' = (1 + \phi_0/c^2)t$ is a time scale which takes advantage of the combination of effects causing standard clocks on the geoid to beat at equal rates.

For portable clocks with velocity $\vec{v}'$ relative to the ground, Eq. (1) may be solved for dt' and integrated along the clock's path giving:

$$\Delta t' = \int ds\,[1 - (\phi-\phi_0)/c^2 + v'^2/2c^2 + \vec{\omega} \cdot \vec{r}\,' \times \vec{v}\,'/c^2]. \qquad (2)$$

For synchronization along a path by means of electromagnetic (em) signals, ds vanishes. Solving Eq. (1) for the elapsed coordinate time in this case gives

$$\Delta t' = \frac{1}{c}\int d\sigma'\,[1 - (\phi-\phi_0)/c^2 + \vec{\omega} \cdot \vec{r}\,' \times \vec{c}\,'/c^2] \,. \qquad (3)$$

where $d\sigma'$ is the increment of proper distance along the signal path, and $\vec{c}\,'$ is the velocity of the em signal pulse observed in the rotating frame. The right-hand sides of Eqs. (2) and (3) are expressed in terms of measurable or calculable quantities. Using these corrections, the coordinate clocks which read t' may be consistently synchronized by either portable clocks or em signals. Second-order terms which have been neglected in Eqs. (1)-(3) would give additional corrections of order $(v'/c)^4$ and $(\phi/c^2)^2$; these contribute less than one part in 10^{16}.

In the following sections we shall discuss why the above equations are useful, and the limits of applicability of these equations near the Earth. Geocentric coordinate time will clearly be seen to be a useful time and frequency metrology tool in the vicinity of the Earth.

2.1. The CCDS and CCIR Resolutions

At the ninth session of the CCDS (CCDS, 1980), a report was prepared on the above topic. This committee, on considering among other things that the 14th General Conference of Weights and Measures established TAI as the International Time Standard Reference, recognized that it is necessary to consider relativistic effects when comparing time standards on the Earth, and that it is necessary to adopt a model that clearly defines how the comparisons are to be made. The committee declared that TAI should be established as the Coordinate Time Standard defined in a geocentric coordinate frame with the SI second as realized on the rotating geoid as the unit of time, and that in comparing clocks in the vicinity of the Earth it is necessary to incorporate general relativistic corrections which include the velocities of portable clocks, the gravitational potentials involved and the effects of the rotating earth in order to establish a self-consistent coordinate time frame in which to measure state-of-the-art clocks.

The equations adopted by the CCDS and quoted below are specializations of Eqs. (2) and (3) appropriate for clocks near the Earth's surface, and are valid for the estimation of relativistic effects on clock rates to better than one part in 10^{14}.

When transferring time from point P to another point Q by means of a portable clock, the coordinate time accumulated during transport as derived from Eq. (2), becomes

$$\Delta t = \int_P^Q ds \left[1 - \frac{\Delta\phi(\vec{r}\,')}{c^2} + \frac{v'^2}{2c^2}\right] + \frac{2\omega}{c^2} A_E , \tag{4}$$

where $\vec{r}\,'$ is a vector whose origin is at the center of the Earth and whose terminus moves with the clock from P to Q; ds is the increment of proper time given by the clock, $\Delta\phi(\vec{r}\,')$ is the gravitational potential difference between the location of the clock and the geoid ($\Delta\phi$ is positive above the geoid); and A_E is the equatorial projection of the area swept out by $\vec{r}$ in an Earth-fixed coordinate system. In computing A_E, its increment is taken positive when the equatorial projection of $\vec{r}\,'$ moves eastward. The correction term arising from A_E in Eq. (4) is the Sagnac effect, which is the effect on the apparent velocity of light due to the rotating, noninertial reference frame (Post, 1967). If the height of clock above the geoid is less than 24 km, one may take $\Delta\phi(r) = gh$ and still retain an accuracy of a part in 10^{14}, where h is the altitude of the clock above the geoid and g the acceleration of gravity (including rotational effects) at the intersection of $\vec{r}$ with the geoid. For an accuracy of a part in 10^{15}, the approximation $\Delta\phi = gh$ can be used if h is less than about 2.4 km.

When transferring time from one point to another by means of an em signal, the coordinate time elapsed between transmission and reception, Eq. (3), is given by:

$$\Delta t = \frac{1}{c}\int_P^Q d\sigma\left[1 - \frac{\Delta\phi(\vec{r}\,')}{c^2}\right] + \frac{2\omega}{c^2} A_E \tag{5}$$

where $d\sigma$ is the increment of proper length along the transmission path and the other notations are the same as for the first case, except that $\vec{r}\,'$ refers to points on the transmission path.

In 1982 the CCIR (CCIR, 1982), adopted conventions consistent with the CCDS report, but extending the range of applicability of the relativistic corrections to include geostationary satellite orbits. A second set of equations adopted by the CCIR allowed for the consideration of comparisons for two cases; first as viewed from an earth-fixed (rotating) frame and second, from a geocentric, non-rotating, local inertial frame as is appropriate for clocks in orbit.

In the first case, when transferring time by means of a portable clock or em signals, the coordinate time accumulated during transport is as given by Eqs. (4) or (5), respectively.

When h is greater than about 24 km, for one part in 10^{14} accuracy, or greater than 2.4 km for one part in 10^{15} accuracy, the potential difference $\Delta\phi$ must be calculated to greater accuracy as follows:

$$\Delta\phi(\vec{r}\,') = -GM_e\left(\frac{1}{r} - \frac{1}{a_1}\right) - \frac{1}{2}\omega^2(r^2 \sin^2\theta - a_1^2) + \frac{J_2 GM_e}{2a_1}\left[1 + \left(\frac{a_1}{r}\right)^3 (3\cos^2\theta - 1)\right] \qquad (6)$$

where a_1 is the equatorial radius of the Earth; r is the magnitude of the vector $\vec{r}\,'$; θ is the colatitude; GM_e is the product of the Earth's mass and the gravitational constant; and $J_2 = +1.082 \times 10^{-3}$ is the quadrupole moment coefficient of the Earth. Accuracy of a part in 10^{16} can be achieved by including additional known terms in the multipole expansion of the Earth's potential.

The second term in Eq. (5) amounts to about a nanosecond for an Earth-to-geostationary satellite-to-Earth trajectory. The third term can contribute hundreds of nanoseconds for practical values of A_E. The increment of proper length, $d\sigma$, can be taken as the length measured using standard rods at rest in the rotating system; this is equivalent to measurement of length by taking c/2 times the proper time (normalized to vacuum) of a two-way em signal sent from P to Q and back along the transmission path. In practice uncertainties in the proper distances $\int d\sigma$ play a significant role in limiting the accuracy with which the elapsed coordinate time can be determined.

In the second case, as viewed instead from a geocentric, non-rotating, local inertial frame, when transferring time with a portable clock the coordinate time elapsed during the motion of the clock is:

$$\Delta t = \int_P^Q ds \left[1 - \frac{V(\vec{r}) - \phi_g}{c^2} + \frac{v^2}{2c^2}\right] \qquad (7)$$

where $V(\vec{r})$ is the Earth's potential at the location of the clock excluding rotational contributions and $\vec{v}$ is the velocity of the clock, both as viewed from a geocentric non-rotating reference frame. The potential ϕ_g at the geoid still includes the effect on the potential of the Earth's rotational motion. Note $\Delta\phi(\vec{r}) \neq V(\vec{r}) - \phi_g$, since $V(\vec{r})$ does not include the effect of the Earth's rotation. Eq. (7) also applies to clocks in geostationary orbits but should not be used beyond a distance of about 50,000 km from the center of the Earth for 10^{-14} ac-

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curacy, because at greater distances from the center of the Earth, Lunar and Solar tidal potentials have a nonnegligible effects.

From the viewpoint of a geocentric, non-rotating, local inertial frame, the coordinate time elapsed between emission and reception of an electromagnetic signal is:

$$\Delta t = \frac{1}{c} \int_P^Q d\sigma \left[1 - \frac{V(\vec{r}) - \phi_g}{c^2}\right] \tag{8}$$

where $V(\vec{r})$ and ϕ_g are defined as in Eq. (7), and $d\sigma$ is the increment of proper length along the transmission path. The quantities $d\sigma$ appearing in equation (5) and (8) differ slightly because the reference frames in which they are measured are rotating with respect to each other; Lorentz contraction could cause the difference in Δt to 15 ps at most for transmission to a satellite in geostationary orbit.

3. SOME LIMITATIONS ON THE DETERMINATION OF COORDINATE TIME

For the Earth considered in isolation from other solar system bodies, then one can think of the Earth's center of mass as at rest in an inertial frame with the Earth itself in uniform rotation relative to this frame. Establishing a network of synchronized clocks on the surface of the Earth is then simplified by several significant cancellations among relativistic effects.

In the rotating frame, there is a pair of effects which cancel to a high degree because the Earth's surface is nearly in hydrostatic equilibrium. Comparing two clocks at rest on the same meridian, one which is farther from the rotation axis will move faster and will therefore beat more slowly, a consequence of time dilation. However, because of the Earth's equatorial bulge the clock farther from the rotation axis is also higher in the Earth's gravitational field and beats more rapidly due to a gravitational frequency shift. If the Earth were a perfect ellipsoid of revolution, then in the rotating frame the Earth's surface -- or some other equipotential -- could be taken as a reference surface on which, to a high degree of accuracy, all identical standard clocks would beat at the same rate. It is conventional to choose the geoid of the Earth in rotation as the reference surface for international clock comparisons.

If the Sun's gravitational potential is expanded in a Taylor series about the Earth's center of mass, the leading term will contribute to a constant frequency shift which is the same for all clocks near the Earth, and so will not affect comparisons between such clocks. Consider next the terms in this expansion which are linear in the distance from the Earth's center. Of two clocks at different distances from the sun, the closer one should beat at a slower rate due to the gravitational red shift. There must exist yet another effect because, according to the principle of equivalence, gravitational fields can be transformed away locally by introducing an appropriate freely falling local inertial frame. This second effect is the relativity of simultaneity, according to which clocks synchronized in one inertial frame by

Einstein's procedure will not appear synchronous when viewed from a second frame moving with respect to the first. In the present case the terms arising from breakdown of simultaneity have an annual period because the velocity of the Earth changes as the Earth revolves around the sun. This effect gives rise to a contribution which causes net cancellation of all linear terms in distance, in the clock comparisons between clocks in the Earth's local inertial frame, due to the sun. This has been proven elsewhere by explicit calculation even for a model of the Earth's motion which includes the orbital eccentricity (Ashby, 1980). The result is that to a high degree of approximation, the residual effect of the sun, on clocks synchronized in the local inertial frame near the Earth, is due to Newtonian tidal potentials. A similar argument applies to the lunar potential and to potentials arising from other solar system bodies.

Studies of the geoid show that the geoid itself may deviate by up to 105 meters from the reference ellipsoid. This is illustrated in Fig. 1, (Lerch et al., 1979). A height difference of 105 meters between two clocks could cause a difference of $gh/c^2 \approx 1.1\times10^{-14}$ in the fractional frequency difference between two clocks, if not accounted for. However the geoid (not the reference ellipsoid) is the surface of reference for comparison of clock rates; the systematic deviation of the geoid from the reference ellipsoid is well-modelled and can be accounted for to within a few percent.

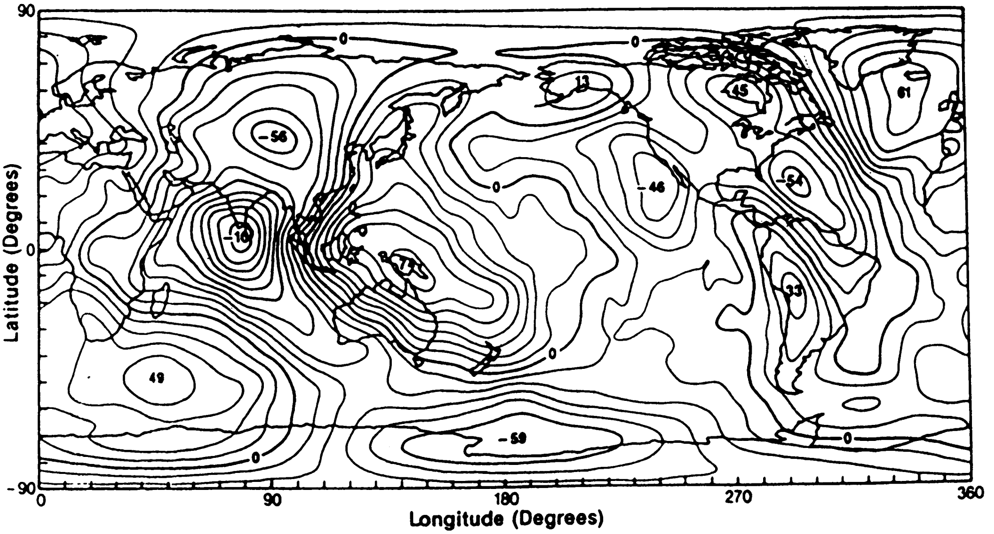

Figure 1. Geoid surface computed from GEM 10 model, with height contours at 10 m intervals above the mean ellipsiod (Lerch et al., 1979)

Mean sea level can be affected by Coriolis forces acting on large scale ocean currents. For example at 45° latitude a persistent northerly current 170 km wide flowing at 2 m/sec would require the eastern edge to be about 3 meters above the western edge. It is thus important to note that mean sea level may differ by some meters from the geoid.

More serious limitations arise from uncertainties in knowledge of higher harmonics of the gravitational potential due to the Earth itself. Goddard Earth Models 9 and 10 (Lerch et al. 1979), result from extensive analysis of ranging from stations on the earth to satellites such as GEOS 3. The resulting gravitational potential can be expressed as an expansion in a series of spherical harmonics with coefficients determined by fitting the data. The determination of the geoidal surface is affected by uncertainties in the knowledge of GM_e, a_1, and in the potential coefficients. It has been estimated (Lerch et al., 1979) that uncertainties in these potential coefficients result in uncertainties in the height of the geoid of about 1.5 meters. This is a global rms value, and leads to an uncertainty in fractional frequency comparisons less than 2 parts in 10^{16}. As knowledge of the Earth's gravitational field continues to improve, one may expect this uncertainty in the determination of the geoid to be reduced to half or a third of its current value.

Other potential effects may cause small time-varying fluctuations in the gravitational equipotential surface. For example, relative to the spinning Earth, the Newtonian tidal potential has a period of one lunar day which is long compared to the period of the normal modes of oscillation of the Earth. The Earth's response to such tidal forces is approximately static. The resulting deviation of the equipotential surface of the earth including lunar tidal potentials may then be estimated from modern theories of Earth tides (Baker 1984). If W represents the tidal potential, then the vertical displacement of the equipotential surface relative to the center of the Earth is $(1+k_2)W/g$, while relative to the deformed Earth surface the displacement is $(1+k_2-h_2)W/g$. The terms W/g in these expressions arise from considering the Earth to be undistorted, and may be obtained using Brun's equation (Heiskanen and Moritz, 1967) while the Love numbers k_2 and h_2 arise from the distortion of the Earth's mass on the total potential. For a variety of layered Earth models, $0.604<h_2<0.630$,, and $0.299<k_2<0.310$.

One finds that the peak to peak range of the displacement of the geoid <u>relative to the deformed surface</u> is 0.37 meters, with an uncertainty of about 0.009 meters. This could produce an error in the comparison of fractional frequency differences of clocks of at most 4 parts in 10^{17}, with an uncertainty of about a part in 10^{18}. A similar computation for the effect of solar tides yields effects of approximately half this size. These are for the most part systematic fairly well-understood effects which can be corrected for to within about 2.5%. Tidal effects due to other bodies--such as Jupiter and Saturn--are much smaller and can be neglected.

3.1. Errors in computation of the Sagnac effect

The Sagnac correction is of the form $\Delta t = (2\omega/c^2)A_E$ where A_E is the area of the path of the portable clock or light ray as projected on the equatorial plane. Consider a synchronization process in which two stations are involved with the area to be projected being that of

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the triangle consisting of the Earth's center and the ground stations as vertices. If due to polar wander the direction of the Earth's axis should change by an angle $\delta\theta$, the change in the Sagnac correction in the worst possible case would be less than approximately $2\omega A\delta\theta/c^2$. For a typical experiment $2\omega A/c^2 \approx 1\mu s$ while the polar wander is no more than about 30 meters, so $\delta\theta < 5 \times 10^{-6}$ giving an error $\Delta t \approx 6$ picoseconds. Such effects are negligible for the time being, as are uncertainties in the value of ω. It is more likely that poorly known positions will lead to significant uncertainties in the area.

4. EXPERIMENTAL REVIEW

In this section we treat some of the experimental measurements of relativistic corrections as well as some of the operational systems which depend on the use of relativistic effects in the measurement of coordinate time and frequency.

A classic experiment which demonstrated the need for all three relativistic correction terms in Eq. (4) was conducted during October 1971 (Hafele, 1971; Hafele and Keating 1972). In this experiment four cesium clocks were carried eastward and then westward, circumnavigating the globe generally at a northern latitude and returning after each trip to the U. S. Naval Observatory for comparison with UTC(USNO). The difference in the average of the portable clocks' readings, due to all three relativistic terms in Eq. (4), upon return of the clock from the westward trip, minus that upon return from the eastward trip was predicted to be 315 ns. The measured value was 332 ns, a difference of 17 ns. The uncertainties involved in the experiment made this a better than 5% validation of the theory for the composite of all three terms. For purposes of comparison, the Sagnac effect for a global circumnavigation of the Earth on the geoid at the equator is 207.4 ns. The operation of the Global Positioning System (GPS) critically depends upon all three terms in the coordinate time Eq. (4). Historically, the GPS is of interest because at the time of launch (23 June 1977) of the NTS-2 Satellite, which contained the first cesium atomic clock to be placed in orbit, there were some who doubted that relativistic effects were truths that would need to be incorporated! A frequency synthesizer was built in the satellite clock system so that after launch, if in fact the rate of the clock in its final orbit was that predicted by general relativity, then the synthesizer could be turned on bringing the clock to the coordinate rate necessary for operation. After the cesium atomic clock was turned on in NTS-2, it was operated for about 20 days to measure its clock rate before turning on the synthesizer (Buisson, et al., 1977). The frequency measured during that interval was + 442.5 parts in 10^{12} compared to clocks on the ground while general relativity predicted 446.47 parts in 10^{12}. The theoretical value minus the measured value was only 3.97 parts in 10^{12}, well within the accuracy capabilities of the orbiting clock. This then gave about a 1% validation of the combined second order doppler and gravitational red shift effects for a clock at 4.2 earth radii.

In using the GPS for navigation, an observer's position and time are calculated from simultaneous observations of signals from four satellites whose coordinate positions and times are known. The verification of the Sagnac correction term has become apparent in these calculations by virtue of the selfconsistency of such navigation solutions. The size of this effect for the GPS depends upon the relative positions of the satellites and of the navigation receiver. In the worst case these corrections can be equivalent to several tens of meters. On the Yuma Test Range the navigation solution was verified at or below the 6 meter level of accuracy, including relativistic corrections. Another experiment was recently conducted (Allan, Weiss and Ashby, 1985; Allan, Davis, et al. 1985) using photons from the GPS to compare primary standards around the globe and to do an around-the-world check on the Sagnac correction term. The prediction was verified at the 5 nanosecond level of accuracy. The size of the Sagnac term for this latter experiment was in the vicinity of 300 nanoseconds making this about a 2% validation of the theory.

In the fall of 1984 Buisson and Oaks of the Naval Research Laboratory carried a GPS receiver to Europe and compared it to several other receivers there. This experiment raised some questions regarding the above-mentioned Sagnac experiment in which GPS photons were used to circumnavigate the globe.

The experiment was repeated with the following results. A weighted average of the common-view signals from SV #8, 9, 11, 12 and 13 between Boulder, Colorado and Braunschweig, Federal Republic of Germany were used to compute the time difference UTC(PTB) - UTC(NBS). Similarly, the signals from SV #6, 8, 11 and 13 were used to compute the common-view time difference UTC(TAO) - UTC(PTB) between the primary standards at the Tokyo Astronomical Observatory (TAO) and PTB. Lastly, the signals from SV #6, 8 and 9 were used to compute the common-view time difference UTC(NBS) - UTC(TAO). Since the Sagnac effect is incorporated in the software of each of the GPS receivers involved, the above three coordinate time differences should add to zero. The experiment was carried out from 15 February through 30 April, 1985 (74 days). The linear least squares to the time difference residuals yielded a slope of -5×10^{-15}, a mean time residual of +6 ns and a standard deviation to the fit of 10 ns. This is comparable to the previous experiment, and gives about a 2% validation of the Sagnac effect.

On 18 June 1976 a Scott-D rocket carried an atomic hydrogen maser oscillator to an altitude of 10,000 km as a test of the equivalence principle (Vessot et al., 1976). Explicit within the experiment was also a test of the second order doppler effect, and a test of the Sagnac effect, as the reference frame used in the experiment was a non-rotating geocentric reference frame which is essentially the coordinate time reference frame discussed in this paper. Confirmation of the equivalence principle in this experiment was at the 2×10^{-4} level of accuracy. The high accuracy achieved in this experiment was because of a three-frequency Doppler cancellation, an ionospheric delay cancellation technique, and because of the excellent clocks involved.

During August 25-29, 1977 a careful portable clock trip was carried to measure the time difference between UTC(USNO) and UTC(NBS)

(Ashby and Allan 1979). The uncertainty in this measurement was about 2 ns and the size of the three relativistic coordinate time correction terms were: - 12.4 ns for gravitational frequency shift, + 4.4 ns for second-order Doppler shift, and - 9.6 ns for the Sagnac effect, accumulating to - 17.6 ns. Hence the experiment was a validation of the theory at about the 10% level of accuracy. Portable clock technology has not improved significantly since the time of this experiment and the coordinate time correction terms still only marginally impact the coordinate time resulting from a portable clock trip.

Additional experiments have been conducted in which time differences as measured by portable clock versus the time differences measured by em signals were compared showing the consistency of the two techniques (Buisson et al., 1977; Allan, Davis, et.al., 1985). Measured time differences have agreed to well within the measurement uncertainties which were limited primarily by the portable clock uncertainties. It seems apparent that in the future there will be opportunities to supplant portable clock trips by carrying GPS receivers to various sites. The receivers can be carried "cold", can be taken to remote locations, and can perform higher accuracy absolute time and frequency transfers than with state-of-the-art portable clocks.

In November 1975, Alley and coworkers flew an ensemble of clocks over the Washington, D.C. area (Alley, 1983), to test the equivalence principle as well as to measure the second order Doppler effect. The combination of these two terms was measured and agreed with theory at the 1.5% level of accuracy. This experiment was not sensitive to the Sagnac effect.

In August 1975 the Radio Research Laboratories (RRL) in Japan in close cooperation with NASA Goddard Space Flight Center and the U. S. Naval Observatory (USNO) in Washington, D.C. conducted time comparisons between the RRL clock and the USNO clock using the ATS-1 Geostationary Satellite (Saburi, 1976). As part of the experiment a portable clock was carried between USNO and RRL and coordinate time corrections were applied to both the em signals and to the portable clock transport. The size of the Sagnac effect for the geostationary satellite was 333 ns and the sum of the relativistic effects for the portable clock were 87 ns gained for the westward trip and 4 ns lost for the eastward trip. The main uncertainty in the experiment was estimated at 200 ns and was due to the portable clock. The measurement of the time difference between the USNO clock and the RRL clock agreed by the two techniques to 1 ns, well within the 200 ns portable clock uncertainty.

The four primary frequency standards (at NBS, NRC, PTB, and RRL) used in determining the SI second for TAI have been compared employing Eq. (5) via GPS satellites in common-view. The largest relativistic effect which arises in this comparison is due to the height above the geoid (approximately 1.6 km) of NBS-6, the NBS primary frequency standard. Theoretically it should be high in frequency by 18 parts in 10^{14} with respect to an ideal earth coordinate clock on the geoid. Comparison with the other three standards gave frequency differences in agreement with the theoretical value and well within the uncertainties of the standards themselves of a few parts in 10^{14} (Allan, Davis, et al., (1985), Yoshimura (1985); Douglas (1985)). The uncertainties contributed by the GPS common view frequency comparison method were less than 1 part in 10^{14}.

5. FUTURE EXPERIMENTS

With atomic clocks improving about an order of magnitude every seven years, (and currently there appear to be no reasons for this trend not to continue for the next few decades), the need for incorporation of relativity effects in operational time and frequency comparisons will only increase. It seems prudent to agree on a coordinate time system so that any state-of-the-art experiments, terrestrial or celestial, can be consistently described and compared.

Future techniques are anticipated in the GPS which, given access to the signals, should provide a nanosecond timing system on a worldwide basis. In addition the geodesy community is working on techniques for using GPS that will allow differential position determination of the order of 1 cm accuracy (Bilham, 1985). Explicit within these experiments are the determination of the satellite ephemerides to about 25 cm. The first German spacelab mission experiment is planned this year (Starker et al., 1982). Cesium and rubidium clocks will be used on the Space Shuttle in which it will be essential to include coordinate time relativistic effects in order to reach the goals of 10 ns synchronization of ground clocks and 30 meter position determination.

Another Space Shuttle experiment has been proposed in which a hydrogen maser would be flown on board (Allen, Alley, et al., 1981). Using a 3-frequency Doppler cancellation technique as in Vessot's rocket experiment, one can even hope for the removal of the cycle ambiguity at L-band to carry the time information. This would imply 1 part in 10^{16} syntonization capability over 1 day as the phase resolution in this system would be of the order of 10 picoseconds. One of the basic limitations in this experiment are the uncertainties in the relativistic effects in the hydrogen maser clock resulting from the uncertainties in its position and velocity as it orbits the earth.

It is anticipated that starting this year, time comparisons between some of the principal Earth timing centers will be set up using two-way communication with geostationary satellites with time stabilities in the vicinity of 100 picoseconds. The accuracies of this international time comparison technique are expected to reach a nanosecond.

There are both theoretical and experimental indications that atomic clocks with absolute accuracy of a part in 10^{15} are realizable in the not too distant future (Wineland, 1984). Dehmelt has shown that a one part in 10^{18} single ion storage standard may theoretically be possible (Dehmelt, 1981). If that were realized the gravitational red shift alone would allow such a clock to be sensitive to elevation changes of one centimeter! The implications this has for studies of the dynamics of the earth's crust, of geodesy, of planetary effects, of movement of the geoid, and of the use of coordinate time are incredibly interesting and complex. There are several gravitational wave experiments which could use ultra accurate clocks and which would be highly

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(1985), Yoshimura (1985); Douglas (1985)). The uncertainties dependent on the relativistic effects we have discussed. Some of these will need coordinate time and/or transformations to barycentric dynamical time in order to be useful.

A development that is in process is the construction of the "best" atomic clock on the earth to look at the millisecond pulsar signal as received at the Arecibo Observatory (Backer, 1982). This clock would be constructed by using coordinate time and frequency comparisons between the clock ensembles and the primary frequency standards at the principal timing centers, then combining these coordinate time readings in an optimum algorithm to obtain the "best" clock stability and rate accuracy. This experiment is pushing the state-of-the-art about as hard as any other at the present.

6. CONCLUSIONS

Tractable equations for relativistic corrections have been developed which allow the consistent generation of coordinate time and frequency at levels of less than 10 ps and a few parts in 10^{16}, respectively, if lunar and solar tidal potentials are also accounted for. These equations provide a basis for international time and frequency comparisons adequate for state-of-the-art clocks and they probably will be adequate for some decades to come. These equations also fulfill a time and frequency metrology need because they provide a basis for self-consistent time and frequency comparisons between sites which are in the vicinity of the earth. This fact has been recognized, and the equations have been adopted, by the CCDS in the definition of and in the generation of TAI and UTC. The CCIR has also adapted them for their needs. SI units, which are based on the unit of time interval, may also be communicated through coordinate frequencies to yield consistent time and frequency comparisons within the limitations of their definitions.

The IAU typically needs barycentric coordinates. Further, the IAU has agreed that: "the time-scales for equations of motion referred to the barycentre of the solar system be such that there be only periodic variations between these time-scales and that of the apparent geocentric ephermerides." Since the coordinate time equations for the generation of TAI satisfy the conditions for these latter time-scales, it seems important to develop a barycentric dynamical time scale related at some level of accuracy to Earth based coordinate time through a constant frequency offset and some additional periodic terms. This needs to be investigated further to determine the level of accuracy with which the transformation between TAI and barycentric dynamical time can be determined.

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DISCUSSION

Grishchuk : what is the significance of observing a pulsar with a stability of 10^{-20} by means of less stable clocks ?

Allan : at present the period of this pulsar is stable, but we know that starquakes may occur. Then a change of period may be observed.

Cannon : you said that synchronizing clock will improve their accuracy. But the accuracy is an internal property of clocks. How can it increase if we simply synchronize them ?

Allan : when we synchronize clocks, we can use standard statistal procedures for improving the global accuracy. For instance, using arithmetic mean values, we will improve the accuracy by a factor $\sqrt{N}$ if we have N clocks.

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