

# THE DISTRIBUTION OF MASSES AND RADII OF WHITE-DWARF STARS

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## 1. INTRODUCTION AND SUMMARY

The status of determinations of white dwarf radii by model atmosphere methods is reviewed in this paper. Details will appear elsewhere (Shipman 1978). In brief, the results are that (i) the mean radius of a sample of 95 hydrogen-rich stars with parallaxes is  $0.0131 R_{\odot}$ ; (ii) the mean radius of a sample of 13 helium-rich stars is  $0.011 R_{\odot}$ , indistinguishably different from the radius of the hydrogen-rich stars; and (iii) that the most serious limitation on our knowledge of the mean radius of white dwarfs is the influence of selection effects. An estimate of the selection effects indicates that the true mean white dwarf radius is near  $0.011 R_{\odot}$ .

## 2. METHOD AND RESULTS

The calculation of white dwarf radii from model atmospheres and photometric observations is best considered in terms of the fundamental relation  $f_{\nu} = 4\pi H_{\nu}(R/D)^2$  between stellar flux  $f_{\nu}$ , monochromatic Eddington flux  $H_{\nu}$ , stellar radius  $R$  and stellar distance  $D$ . Photometry, here from Greenstein (1976) corrected to the Hayes and Latham (1975) flux calibration where available and intermediate- or broad-band colors from McCook and Sion (1977), determines the flux  $f_{\nu}$  and a color index. Model atmospheres (Shipman 1972, 1977) provide  $H_{\nu}$  as a function of colors. The results are shown in Fig. 1.

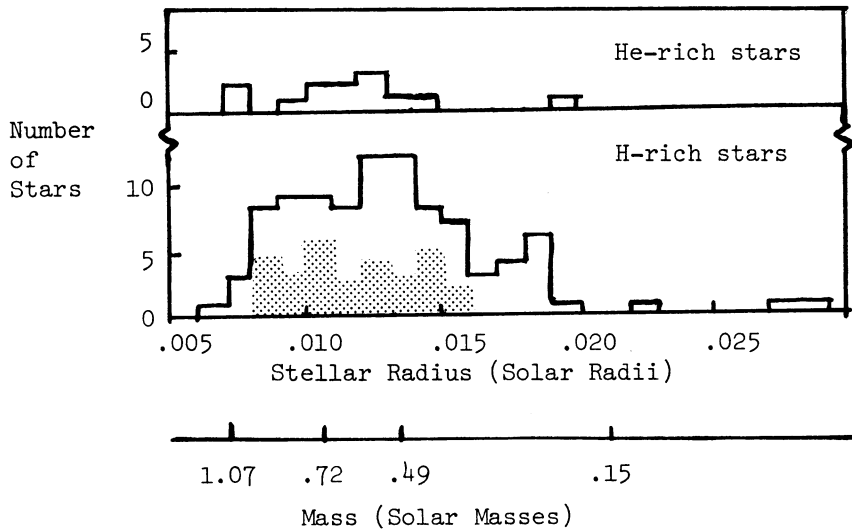


Fig. 1. Distribution of the radii of white dwarf stars. Top panel: Helium-rich stars (types DB, DC, DF, DG,  $\lambda 4670$ , DXP). Bottom panel: Hydrogen-rich stars (type DA, with a few very cool DC and DK stars). Stars in the shaded region have high-quality distances and the radii are known to better than 10%. The Hamada-Salpeter (1961) mass-radius relation for carbon white dwarfs is shown.

Systematic errors, affecting the mean radius of the sample, come from errors in the flux calibration and from the model atmosphere determination of the  $H_v$  vs. color relation. They are, expressed as  $\Delta R/R$ , about 4% for the hydrogen-rich DA stars and about 10% for the helium stars. White dwarf radii (or masses) can also be found by (i) using the five white dwarfs in binary systems, (ii) using hydrogen line profiles to estimate gravities, (iii) using the gravity dependence of the (u-v) colors for  $8000 < T_{\text{eff}} < 14\,000$  K to estimate gravities, or (iv) by measuring gravitational redshifts. When method (iii) is applied with the Hayes-Latham flux calibration and convective, line-blanketed model atmospheres, updating the results of Wehrse (1975), all but method (iv) are consistent with the results of Fig. 1.

The larger white dwarf stars will, in general, be visible over greater distances and any magnitude-limited sample will tend to include more of them. Consequently the distribution of Fig. 1, averaged uncritically, may substantially overestimate the average white dwarf radius and underestimate the average white dwarf mass. Not all white dwarfs have the same radius; the

variation in radius of stars with accurate radii, in the shaded region of Fig. 1, is greater than that which can be explained by observational errors. White dwarfs in binary systems have masses that vary from  $0.43 M_{\odot}$  (40 Eri B) to  $1.01 M_{\odot}$  (Sirius B). A magnitude-limited sample will include stars from a distance  $D = (4\pi H_V/f_V)^{1/2} R$ , so that assuming that selection effects are independent of temperature, the volume sampled is proportional to the white dwarf radius  $R$ . We can then correct the observed distribution of radii  $N(R)$  by dividing  $N(R)$  by  $R^3$  to get a corrected distribution, and then average to obtain a corrected radius of  $0.0105 R_{\odot}$ . This procedure overestimates the selection effect because it assumes that the sample is completely magnitude-limited and that the variation in radius illustrated in Fig. 1 is all intrinsic. One can also try to eliminate the selection effect by considering only those stars hot and near enough so all stars, both large and small, would be included. This second procedure produces a mean radius of  $0.0111 R_{\odot}$  but the sample is small--9 stars. The agreement between the results of the two procedures argues that selection effects do occur and that the mean white dwarf radius is most likely about  $0.011 R_{\odot}$  rather than the  $0.0131 R_{\odot}$  given by the raw data.

The mean white dwarf radius, corresponding to a mass of  $0.7 M_{\odot}$  indicates that mass loss in the red giant stage is substantial (Weidemann 1977). One can also use this number to estimate the density of white dwarfs in the Galaxy; with Sion and Liebert's figure of 0.005 visible ( $M_V < 15^m.5$ ) white dwarfs per cubic parsec, we have only  $0.0035 M_{\odot} \text{ pc}^{-3}$  in the form of visible white dwarfs. Preliminary calculations (Shipman 1978) show that observational constraints on stellar birthrate functions indicate that there are fewer than 8 times as many invisible ( $M_V > 15^m.5$ ) as visible white dwarfs, placing an upper limit of  $0.03 M_{\odot} \text{ pc}^{-3}$  to the density of visible and invisibly cool white dwarfs.

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## DISCUSSION

*BELL*: As I understand it, you have determined the stellar radii from absolute fluxes at a particular wavelength, making use of the stellar parallaxes. You then get the masses from the mass radius relation. This then gives you the gravity. If you then determine gravities from fitting the spectral scans over a range of wavelengths, how well do they agree with the other values?

*SHIPMAN*: They agree quite well; we determine a mean  $\log g = 8.0$  from the separation of white dwarfs in the (u-v), (g-r) diagram.

*HARRINGTON*: We have measured almost 500 parallaxes at the Naval Observatory and can make a statement about the true errors. Several lines of attack indicate that we are publishing true one-sigma errors for our mean errors. Therefore, you can accept our published errors as real errors.

*SHIPMAN*: I'm glad to hear that.

*WALBORN*: What is the lowest white dwarf mass for which there is credible observational evidence?

*SHIPMAN*:  $M/M_{\odot} = 0.43$ .