RADAR DETERMINATION OF THE ASTRONOMICAL UNIT

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RÉSUMÉ. — L'auteur donne une description complète des mesures de la distance Terre-Vénus faites par le radar Millstone en 1959-1961. Le temps de parcours des ondes et le décalage Doppler donnent, pour l'unité astronomique 499,0052 ± 0,001 secondes de lumière, soit 149 598 000 ± 300 km avec $c = 299\,792{,}5$ km/s. Avec un rayon terrestre de 6 378,15 km, la parallaxe solaire devient 8″,79416 ± 0″,00002. Ces résultats s'accordent avec ceux d'autres laboratoires à 10^{-5} près.

ABSTRACT. — A comprehensive review is given of the Earth-Venus measurements made with the Lincoln Laboratory Millstone radar in 1959 and 1961. The time-delay and Doppler shift data yield a value for the Astronomical Unit of 499.0052 ± 0.001 light-sec. Using 299 792.5 km/s for the speed of light leads to an AU of 149 598 000 ± 300 km. With the radius of Earth taken as 6 378.15 km, the solar parallax then becomes 8″.79416 ± 0″.00002. This value is consistent with measurements made at various other laboratories to about one part in 10^5.

ZUSAMMENFASSUNG. — Es wird ein vollständiger Überblick über die Bestimmungen der Entfernung Erde-Venus gegeben, die mit dem Millstone-Radargerät des Lincoln Laboratory in den Jahren 1959 und 1961 durchgeführt wurden. Die Messungen der Laufzeit und der Doppler-Verschiebung ergeben einen Wert von 499,0052 ± 0,001 Lichtsekunden für die Astronomische Einheit; das entspricht 149 598 000 ± 300 km bei einer Lichtgeschwindigkeit von 299 792,5 km. Bei Annahme des Erdradius zu 6 378,15 km wird die Sonnenparallaxe dann 8″,79416 ± 0″,00002. Dieser Wert stimmt auf etwa 1 : 10^5 mit den Messungen an verschiedenen anderen Laboratorien überein.

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Резюме. — Автор подробно описывает измерения расстояния между Землей и Венерой при помощи радара Мильстон в 1959 и 1961гг. Метод времени распространения волн также как и метод смещения частот по Допплеру, дали следущие значения астрономической единицы: 499,0052±0,001 секунд света, то-есть, пологая с 299 792,5 км/с, 149 598 000 ± 300 км. Принимая 6 378,15 км для радиуса Земли, получаем для солнечного параллакса, 8″79416±0″00001. Эти результаты согласуются с точностью до 10^{-5} с результатами других лабораторий.

1. **Introduction.** — A determination of the Astronomical Unit (AU) directly in terms of the speed of light has been made from observations of echo delays and Doppler shifts of radar pulses reflected from Venus. Below, we review this determination using the results of the investigations made with the Lincoln Laboratory Millstone radar during the close approaches of Earth and Venus in 1959 and 1961. Extensive use is made of the descriptions given by Pettengill *et al.* [1], and by Smith [2]. The result obtained by Price *et al.* [3] from a 1958 Venus experiment appears to be incorrect; no explanation for the error has been sought because of the extremely lengthy processes involved in a reexamination of those data.

A value for the AU of 499.0052 ± 0.001 light-sec has been deduced from the 1959 and 1961 Millstone measurements. With the speed of light taken to be 299 792.5 km/s, we obtain 149 598 000 ± 300 as the value for the AU in kilometers. The solar parallax, 8″.79416 ± 0″.00002, following from this result (on the further assumption that the radius of Earth is 6 378.15 km) is by now well known to differ significantly from the value determined by Rabe [4], from comparing the observed motion of Eros during the period 1926-1945 whith a very accurate dynamical theory. We therefore expose in considerable detail all facets of the radar-based determination, and also give as complete a discussion as possible of the accompanying errors. Included is a comparison of our results with those obtained by other radar observatories.

2. **Experimental procedure.** — Since many astronomers are relatively unfamiliar with the techniques of radar astronomy, we discuss the experimental procedure in some detail without, however, probing the interiors of the associated electronic and mechanical devices.

A. *Radar characteristics.* — The Millstone radar, whose relevant parameters are listed in table I, was used for the transmission and subsequent detection of sets of very short, uniformly spaced radar pulses (i. e., bursts of energy) that were directed at Venus. A simplified block diagram of the equipment used is shown in figure 1.

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TABLE I.

Millstone radar parameters.

Operating frequency	4.4×10^8 c/s (exactly)
Antenna gain over an isotropic radiator	5 600
Effective aperture of antenna	207 m^2
Peak transmitted power	2.5×10^6 W
Average transmitted power	1.5×10^5 W
Pulse lengths, t_l	0.5×10^{-3}, 2×10^{-3}, 4×10^{-3} s
Over-all system noise temperature (typical)	240°K
Over-all system loss factor	0.75
Site latitude (geodetic)	42°37′03″N
Site longitude	71°29′29″W
Site altitude above M.S.L.	156 m.

All time and frequency operations of the system were governed by a very stable oscillator ([2]). Timing generator A (*fig.* 1) was connected directly to this oscillator and was preset to emit one signal that turned on the transmitter and another that turned it off after an interval equal to the predetermined pulse length, t_l. Generator A was also adjusted to repeat this sequence after the lapse of a fixed, predetermined time interval — the interpulse interval, t_p. Thus, during each such interval t_p, the radar transmitted for a time t_l. These transmissions were continued until just before the expected time of return of the echo from the first emitted pulse. The antenna was then connected to the receiver until slightly after the echo from the last pulse was expected to be received. At that time, the transmitter was reactivated and the cycle repeated. Such cycles (hereafter termed " runs ") were generally continued for several hours during each day that data were obtained.

The signal-to-noise ratio $\left(\frac{S}{N}\right)$ for a single pulse was far too low to allow identification of the signal. Therefore, it was necessary to superpose returns extending over a long period of time (i. e., to add the corresponding contributions from the echo of each pulse) to determine the presence of a Venus echo. This integration was accomplished either incoherently or partially coherently; both methods are described below.

B. *Incoherent processing.* — Processing by incoherent integration was carried out on separate sets of observations made between 6 March and 18 May 1961. At other times the increased range reduced the echo signal strength sufficiently to make reliable detection impossible by this technique.

([2]) There are actually four quartz-crystal oscillator clocks at Millstone; at least two are in operation at any given time and are compared routinely to radio signals that are co-ordinated with the U. S. Naval Observatory atomic clock.

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In incoherent integration, the returns from successive pulses were added without regard to their relative phases. The received voltage was first passed through a matched filter ([3]); the output was squared (i. e., converted to power) and then put in digital form for processing by a computer. Timing generator B, installed on 23 March 1961, determined the rate at which the received power was sampled; i. e., generator B was preset to emit a series of pulses separated from each other by a time interval termed the intersampling interval, t_s. At the time of each such signal, the instantaneous value of the received power in digital form, was placed in storage and subsequently read into the computer. The intersampling interval was always short compared to the pulse length and was chosen so that there was exactly an integral number in each interpulse interval. The computer numbered the samples consecutively and placed each in a separate register corresponding to its number. After each $\frac{t_p}{t_s}$ samples, the numbering was restarted so that corresponding samples from every interpulse interval were added together and the cumulative sum stored in one of the $\frac{t_p}{t_s}$ separate registers (i. e., the number of registers was simply the number of intersampling intervals in an interpulse interval). The sampling continued for as long as the antenna was connected to the receiver. Because of the superposing of samples, the total in the register that contained the signal was enhanced relative to the others whose totals reflected only cumulative noise. Since the standard deviation of the noise decreased as the number of samples increased, the position of the signal could be localized to within a small segment of an interpulse interval.

For pulse lengths usually employed (*see* section 4), and with the criterion that the integrated signal-plus-noise exceed the mean noise by at least five standard deviations of the integrated noise distribution, about 5 mn of incoherent detection were required to render the signal just visible near conjunction. To obtain a better result, the integration was usually continued throughout the several-hour interval during which data were obtained on a given day (*see*, for example, *fig.* 2).

Were generator B to count on the same time scale as generator A, the above-described method of processing would be fruitful only if the Earth-Venus distance were to remain constant during the time of integration. When the range changes, the received interpulse interval is also modified, in particular, by a factor $\left(1 + \frac{2\dot{R}}{c}\right)$, where $\dot{R}$ is the rate

([3]) A matched filter maximizes the output signal-to-noise ratio at a preselected moment for a signal that is a known function of time. In the Millstone equipment, the chosen moment is the time of reception of the trailing edge of the radar pulse.

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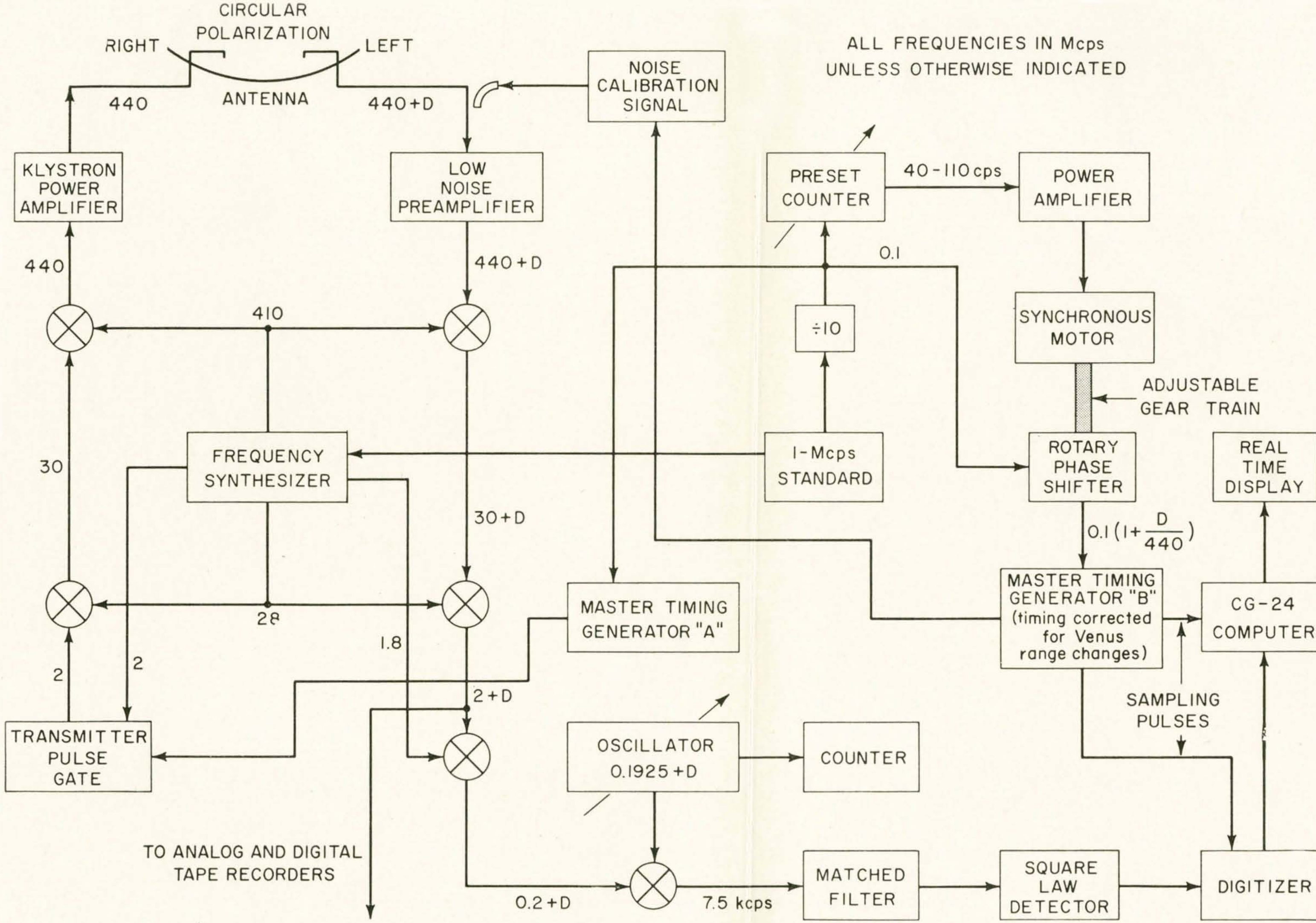

Fig. 1. — Block diagram of the Millstone radar system.

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of change of range and $\frac{2\dot{R}}{c}$ is $\left(\text{to first order in } \frac{v}{c}\right)$ the fractional change, $-\frac{\Delta f}{f}$, undergone by the signal frequency between transmission and reception. Generator B was therefore adjusted so that its counting rate differed from that of generator A by the factor $\left(1+\frac{\Delta f}{f}\right)$. (The Doppler shift introduced was that appropriate for reception at the given time as predetermined from the ephemeris.) This change in the rate

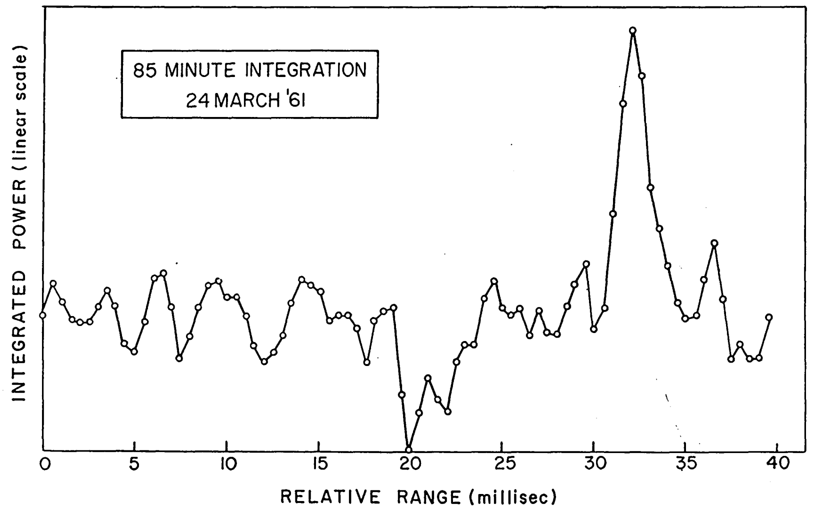

Fig. 2. — Intensity of the incoherently integrated received signal for each intersampling interval in the interpulse interval.

of emission of signals from B ensured that the superposition of returns from successive interpulse intervals was carried out correctly ([4]). Since the Doppler shift, Δf, was itself changing, periodic corrections (based again on the ephemeris) were made to the counting rate of generator B.

To relate the recorded signals to UT and to the time of transmission, signals from both generators A and B were emitted every t_p sec (as measured on their respective time scales) and displayed on an oscilloscope. The images moved relative to each other since the time interval between

([4]) Actually, of course, the accuracy of the precomputed ephemeris was limited, and therefore the totals in each register at the end of each run were preserved for possible later reprocessing. When a more accurate ephemeris becomes available, the totals in the corresponding registers from each of the successive runs will not simply be added but, rather, a new correspondence will be used (based on the more accurate ephemeris) and the totals combined accordingly. In the future, totals for each 5 s interval will be permanently recorded.

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successive signals generated by A was slightly different from that between successive signals from B. In particular, the images periodically (at least partially) superposed. At each such near-coincidence, the UT, as determined by the reference oscillator clock, was recorded by an automatic pen ([5]). Each of these signals from B was, during the receiving mode, coincident with a sampling signal, and served to reset a counter in the computer so that the following samples were renumbered for adding to the respective totals from the previous interpulse intervals. On the other hand, each of these signals from A was, during the transmitting mode, coincident with the signal that turned off the transmitter. But the peak intensity of the processed signal is reached at a time corresponding to that of reception of the trailing edge of the radar echo pulse from Venus (*see* footnote on p. 180). It follows, then, that at times of coincidence of these signals from generators A and B, the first register containing sums of power samples corresponded to an echo time delay equal to an integral number of interpulse intervals as measured by A. The actual Venus echo delay, determined from the register containing the highest total power, was thereby also localized in time to within an integral number of interpulse intervals. (The resolution of the ambiguity is discussed in section 2.D.)

Before the installation of generator B (i. e., before 23 March 1961), the timing was not so accurate. The intersampling interval was governed by generator A and not by the Doppler-shifted intervals. But the echo delay time changes by an intersampling interval after a time t satisfying

$$\left|\frac{\overline{\Delta f}}{f}\right| t = t_s, \tag{2.1}$$

where $\overline{\Delta f}$ is the average value of Δf during the time interval t. The number of interpulse intervals corresponding to this time was therefore calculated in advance, and the computer instructed to shift the addition of new samples by one register after this number of interpulse intervals. For a positive Δf (i. e., for a decreasing delay time), a given sample was then added to the register which previously had contained only samples corresponding to the preceding sample number. Since the Doppler shift changes with time, the number of interpulse intervals elapsing between such unit shifts was also changed in accordance with the precomputed ephemeris. (This procedure is obviously less precise than that involving generator B with which the shifts are, in effect, made

([5]) The times between these near-coincidences were calculated accurately from the known Doppler shifts introduced in generator B, and were checked against the corresponding recorded times to ensure that a significant number of pulses was not "dropped" by the generator and that nothing was grossly wrong with the operation of the equipment.

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continuously.) In place of recording the time of near-coincidences of certain signals from generators A and B, generator A was adjusted to periodically emit a signal that had the effect of placing a large number in the register to which a power sample would next be added. At each such instant (which, by design, was always separated by integral multiples of t_p from all pulses that turned the transmitter off), the set of totals in each register was recorded. Simultaneously the station clock was read visually and the time noted. From these data, the recorded Venus echo was related to UT and the time delay determined to within an integral number of interpulse intervals.

Accurate Doppler shift data were not obtainable from the incoherent processing, but only from the coherent processing technique described below.

C. *Coherent processing.* — Data obtained during September 1959 and from 3 April to 8 June 1961 were processed by coherent integration. After 8 June 1961 the echo signal strength was too weak to detect. No data were obtained for coherent processing during the 1961 conjunction prior to 3 April since the radar was then being used only for real-time, incoherent integration.

Coherent processing is based on a comparison of the phases of successive pulses. For targets whose reflections possess a sufficiently long coherence time, a higher $\frac{S}{N}$ can then be obtained for a given amount of integration. Increased time-delay accuracy is obtained because a considerably higher sampling rate can be employed. Correlating the phases of neighbouring pulses also leads to a very accurate determination of the Doppler shift; such accuracy, of course, is unattainable in incoherent processing. (Because of the uniform spacing of the transmitted pulses, the Doppler shift was actually determined only to within an integral multiple of the pulse repetition frequency. However, the p. r. f. used was large enough for the ambiguity to be resolved easily by using the precomputed ephemeris.)

The general operation of the radar was the same as for incoherent processing but here the entire receiver output, after being translated in frequency for convenience, was simultaneously recorded on magnetic tape with a reference sinusoid from generator A. This latter signal preserved the accurate relative phase inherent in the system. Demodulated radio signals from WWV, which for the present purpose served to define UT, were also recorded on the tape. For processing in the computer all the information was first converted to digital form by sampling the analog recording at a rate of 15 000 per second. To determine a single echo delay time and Doppler shift appropriate for the set of runs made on a given day of observations, the digital data were treated as follows : a copy of the transmitted set of pulses, but delayed by a known amount

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and translated by a known frequency [6], was multiplied by the recorded signal and the product passed through a filter. The purpose of this filter is to take proper account of the phase coherence existing between successively reflected pulses. The coherence lasts for a time given approximately by the reciprocal of the Doppler bandwidth which in this case amounted to about 2 s. Thus, the filter coherently adds appropriate contributions from pulses in the neighbourhood of a given pulse, the contribution decreasing with increasing separation from the given pulse in a manner determined by the coherence time. The output of the filter was squared and then integrated over time.

This computer experiment was performed for each point of a large set located on a grid in the time-delay, Doppler shift space. That combination of time delay and Doppler shift which yielded the largest integrated output provided the estimate of the interplanetary range and range rate. Figure 3 shows the result of such an experiment. The peak output is clearly in evidence, and exceeds the standard deviation (σ_N) of the integrated noise by a significant amount.

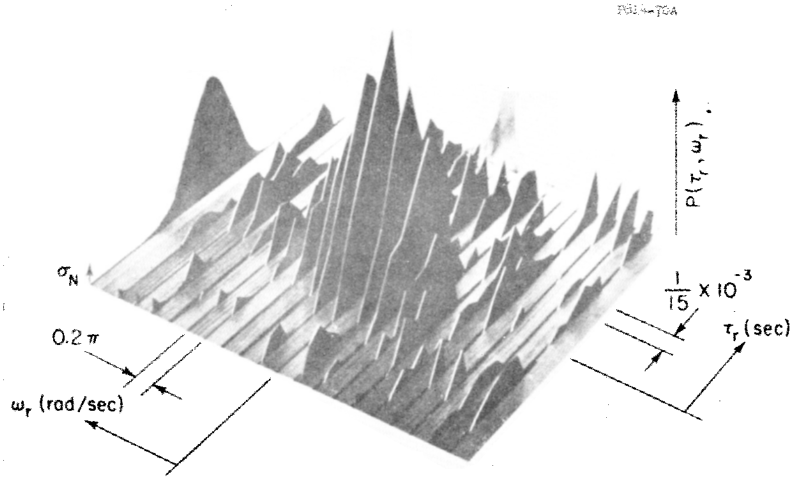

Fig. 3. — Intensity of the coherently integrated received signal processed for various time delays and Doppler shifts.

In principle, the appropriate time delay and Doppler shift can be determined by using this method of processing even if only a very inaccurate precomputed ephemeris is available. In practice, however,

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(6) This copy was, of course, adjusted so that the prechosen time delay and Doppler shift were modified in accordance with the changes expected during the total time interval over which the runs were obtained.

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the ephemeris must be sufficiently accurate to reduce to a manageable size the region of the time-delay, Doppler shift domain to be searched. Hence, coherent processing did not become practical until an improved value for the AU had been obtained from the incoherent detections [7].

D. *Elimination of time-delay ambiguity.* — The incoherent processing method described above leads to an ambiguity in the interpretation of time-delay measurements. As was shown, the time delay is determined only to within an integral number of interpulse intervals. The two-way echo delay, τ, can therefore be expressed as

$$\tau = nt_p + mt_s, \tag{2.2}$$

where m is determined and denotes the number of the intersampling interval containing the peak signal intensity $\left(1 \leq m < \frac{t_p}{t_s}\right)$ [8].

In principle, this ambiguity in echo delay could be resolved by choosing an interpulse interval that exceeded the *a priori* uncertainty in delay caused by the imprecision in the previously determined values of the AU, the radius of Venus, etc. However, since the transmitting equipment was limited to a peak power of 2.5 MW and a maximum pulse length of 4×10^{-3} s, the interpulse interval necessary to resolve the ambiguity (approximately 0.3 s near conjunction) would result in a post-integration signal-to-noise ratio too low for detection. [Aside from geometric limiting factors introduced by the Earth's rotation, integration could not be continued for an arbitrarily long time because the inaccuracies in the *a priori* knowledge of the Doppler shift would lead to a significant drift in generator B, and hence would prevent the achievement of an adequate superposition of the returned signals (*see* section 3.C).]

There are two other obvious ways of eliminating the ambiguity. In one, two trains of pulses are transmitted in parallel (with each at a different frequency to allow separation of the received energies) but with different interpulse intervals. Thus, in addition to (2.2), there will be a similar equation with m replaced by m', t_p by t'_p, and n by n', where t'_p is the second value of the interpulse interval. (The intersampling interval can be kept constant, as here assumed, or changed,

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(7) For coherent processing to be used, it is not really necessary to eliminate the ambiguity present in the incoherently processed time-delay measurements : knowing that the AU has a value close to some one of a number of possible values only increases the time required for the computer experiment by a factor equal to that number.

(8) It is frequently possible to interpolate accurately between adjacent sampling times when estimating the delay associated with the peak intensity. In such cases, m would not be an integer.

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without affecting the analysis.) These two equations yield the diophantine equation

$$n - n'\left(\frac{t'_p}{t_p}\right) = (m' - m)\left(\frac{t_s}{t_p}\right). \tag{2.3}$$

With $\left(\frac{t'_p}{t_p}\right)$ chosen to be irrational, equation (2.3) can have at most one solution for which both n and n' are integers ([9]).

In the second method, the interpulse interval is maintained, but the experiment is repeated at a time when the echo delay is different. Suppose the new echo delay to be $(1 - \varepsilon)\tau$; then the ambiguity equation for this experiment will be

$$(1 - \varepsilon)\tau = n'' t_p + m'' t_s, \tag{2.4}$$

which, when combined with equation (2.2), yields

$$n - \frac{n''}{1 - \varepsilon} = \left(\frac{m''}{1 - \varepsilon} - m\right)\left(\frac{t_s}{t_p}\right). \tag{2.5}$$

Provided that ε is irrational, this equation also has at most one solution with n and n'' both integers.

In practice, of course, the " knowns " in these equations are not exact. Therefore, experimentally, a variety of interpulse intervals (ranging from 0.02 to 0.08 s with corresponding ambiguity intervals for the AU ranging from about 10^4 to 4×10^4 km) were used on different days to ensure a correct resolution. Figure 4 illustrates pictorially the elimination of the spurious values by using data obtained on ten different days.

3. **Errors in the measurement and interpretation of time delays and Doppler shifts.** — The time delays and Doppler shifts obtained from the above-described experiment are to be compared (*see* section 5) with predictions computed from an ephemeris on the assumption that the radar waves propagate in a vacuum. Therefore, it is necessary to deduce from the actual measurements the values that would have been observed were there a vacuum between Earth and Venus. The resultant errors will be determined in part by the measuring process and in part by the interpretation, and will contain contributions from several sources, among which are the radar system itself, the medium

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([9]) Thus, the simultaneous equations

$$na + n'b = c, \qquad n''a + n'''b = c$$

yield the relation $\left(\frac{n'' - n}{n' - n'''}\right) = \frac{b}{a}$. Since the right side, by construction, is irrational and the left side, by assumption, is rational, the equality cannot be satisfied and hence two such solutions of the first equation cannot exist.

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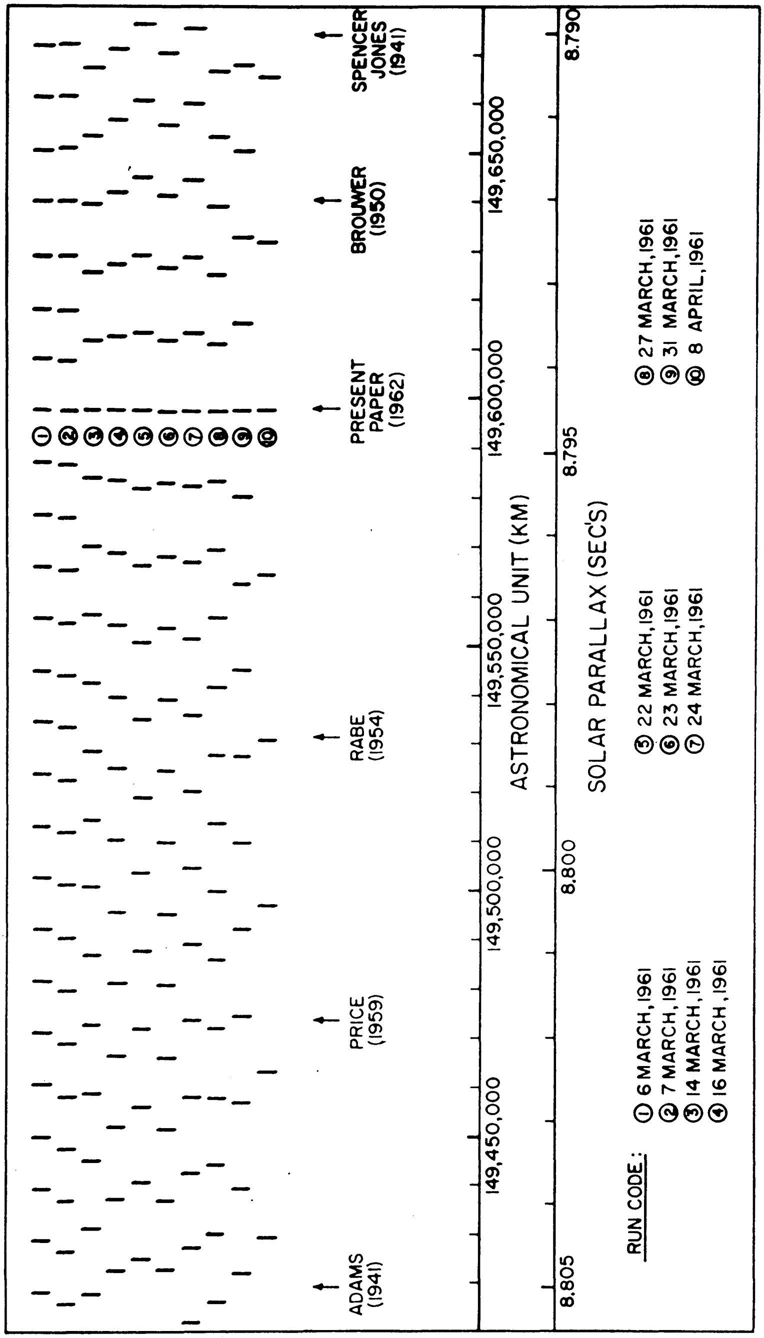

Fig. 4. — Elimination of the time-delay ambiguity.

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of propagation, the precomputed ephemeris, and the surface characteristics of Venus. We discuss in turn the errors associated with each of these sources.

A. *Radar system.* — In the operation of the radar system, the relevant errors are those associated with time and frequency measurements. For convenience, we separate time measurements into two types : (1) the time at which the signal is transmitted (or received); and (2) the time that elapses from transmission to reception (echo delay time). The accuracy required for the first type is, of course, much lower than that required for the second. Since the maximum ratio of range rate to range that occurred during the course of these experiments was less than 3×10^{-7} per second, the maximum error introduced into the calculation of the AU by a time error, Δt, of the first type was never greater than $1.5 \times 10^{-4}\, \Delta t$ light-sec $\approx 45\, \Delta t$ km with Δt measured in seconds. (Within a week of conjunction, near where the range rate goes through a null, this error fell to less than $0.7 \times 10^{-5}\, \Delta t$ light-sec.)

The time given by the reference clock at Millstone is judged to coincide with WWV time (and, hence, for our purposes with UT) to within about 2 ms [10]. However, in the 1961 measurements, before 23 March, the recording of UT at instants related to pulse transmission (*see* section 2.B) had an associated probable error $\Delta t \approx 5$ s [11]. After the installation of generator B, the recording of the coincidences of pulses from it and from generator A were accomplished such that $\Delta t \approx 1$ s. In the coherent processing, as stated previously, demodulated radio signals from WWV were recorded simultaneously with the received voltage and on the same tape, the error in associating the radar data with UT being about 10 ms. Such accuracies were more than sufficient for this experiment.

Since the measurements were to be compared with an ephemeris based on ET, an additional error of about ± 1 s was introduced in associating UT with ET. This error can be substantially reduced after the definitive relation between ET and UT becomes available.

Time errors of the second type have several sources in both incoherent and coherent processing. A possible source involving the incoherent processing performed after 23 March, is the failure of the above-discussed pulses from generators A and B to coincide precisely. The time between such pulses from generator A is t_p and from generator B, $t_p\left(1 - \frac{\Delta f}{f}\right)$.

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[10] This value includes an estimate of the error in calculating the propagation path of the radio time signals.

[11] A systematic error of 25 s in the reference clock used for determining UT was noted on 17 March and the previous data corrected accordingly.

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The time separation at near-coincidences is clearly at most $\left(\frac{\Delta f}{f}\right)\left(\frac{t_p}{2}\right)$ (12). But (*see* section 4) $t_p < 64 \times 10^{-3}$ s and $\frac{\Delta f}{f} < 10^{-4}$; therefore the time error incurred by assuming an exact concidence was always under $3\,\mu$ s, and is negligible for this experiment.

In both processing methods, the intersampling interval placed a limit on the accuracy with which the position of the peak of the integrated samples could be determined. However, the peak could often be located to within one-quarter of the intersampling interval which ranged from 60 to 500 μs (*see* section 4). Its position was of course also influenced by the noise to an extent that depended on the signal-to-noise ratio; the actual contribution to the time-delay error was never greater, and usually far less, than 250 μs.

Systematic time delays between the reception at the antenna and the sampling of the received power varied from 100 to 400 μs, depending on the particular auxiliary equipment used. The necessary corrections were easily calculated to within 50 μs.

All frequencies were rigorously controlled by a standard crystal oscillator that was stable to about two parts in 10^{10} over the echo delay time. This limit on stability implies a probable error in Doppler shift measurements of about 0.1 c/s. The sampling of the received power was probably consistent to about 2 μs over a 3-hour interval (i. e., over the maximum interval during which successive runs were superposed).

B. *Propagation medium.* — The medium through which the radar wave propagates influences both the echo time delay and Doppler shift. In particular, the echo delay will be affected by changes in the speed of propagation and by changes in the length of the propagation path. Each can occur in either of the planetary atmospheres or in the interplanetary medium. Consider first the influences of Earth's atmosphere. The neutral component causes a variation in speed of no greater than 3 parts in 10^4. Since the thickness of the dense portion represents less than 1 part in 10^6 of the entire Earth-Venus distance, this variation (if ignored) clearly introduces an entirely negligible error in the determination of the corresponding vacuum time delay. The ionized component of the atmosphere introduces a more important change in time delay. Thus, the fractional effect on the speed of a radar pulse travelling through a plasma can be approximated in the absence of a magnetic field by

$$\frac{\Delta v}{c} \approx \frac{4.1 \times 10^7 \mathrm{N}}{f^2} \qquad (3.1)$$

(12) The time between these near-coincidences is approximately $t_p\left(\frac{f}{\Delta f}\right)$, i. e. about 10 mn, except very near conjunction when $\Delta f \to 0$.

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where N is the number of charged particles per cubic centimeter and f is the transmitter frequency in cycles per second; the corrections required for the weak fields near Earth are negligible. The increase in two-way time delay caused by the ionosphere is therefore given by

$$\Delta\tau \approx 2\,\frac{4.1\times 10^{7}}{f^{2}c}\int_{0}^{L} N(l)\,dl\,, \tag{3.2}$$

where L is the length of the (geometric) path of the radar wave that extends from the antenna to the top of the atmosphere, and where $N(l)$ is the electron density along the path. In the zenith direction, the integrated electron density has been estimated at the latitude of Millstone (Evans [5]) to be less than 6×10^{13} per square centimeter. The integrated density also appears to increase in accordance with the secant law, i. e., in a direction α from the zenith this density is enhanced by a factor sec α. For an angle from the zenith as great as 80° (the highest attained during the Venus measurements), the change in time delay indicated by equation (3.2) is still less than 4.5 μs and is therefore negligible for this experiment. Path-length changes caused by atmospheric refraction changing the direction of the path, are also easily shown to be minute.

Changes in time delay attributable to a change in speed in the interplanetary medium can be estimated using the recent results from the Venus probe, *Mariner II*. These indicated an average charged particle density of less than 5 per cubic centimeter which would have caused a change from the vacuum delay time of less than 0.3 μs. Even allowing for *Mariner II*'s measurements being taken during a quieter part of the Sun-spot cycle, and for the possible presence of regions of relatively high density, we deem it unlikely that the plasma could have caused a change in echo delay of as much as 10 μs which would still be negligible.

The radar path length would be varied most effectively for a given angular change in path direction, were the shift to occur midway between the planets. It can be shown that a plasma wedge in this position will produce a fractional change in path length given by

$$\frac{\Delta R}{R} \approx 2\times 10^{14}\left[\frac{\sin\beta}{\cos\varphi\,\cos(\varphi-\beta)}\right]^{2}\frac{N_1^2}{f^4}\,, \tag{3.3}$$

where φ is the angle of incidence of the radar wave on the wedge, β is the angle between the edges of the wedge, and N_1 is the plasma density interior to the edges (an outside plasma density of zero is assumed). Even a density of 10^3 electron/cm^3 inside the wedge would produce a path-length change of only 1 part in 10^{11}, unless φ is very close to 90°.

Effects on time delay of the atmosphere of Venus are, of course, more difficult to estimate. First of all, it is not clear *a priori* whether the radar signals would be reflected from the surface of Venus or from its iono-

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sphere. Were the latter to be the reflecting agency, time delays would be less than expected, and would lead to a lower value for the AU. The percentage error would be largest at inferior conjunction when reflection from a Venus altitude of 300 km would result in an underestimate of the AU of about 3.5×10^{-3} light-sec ≈ 1000 km. Aside from an argument of " minimum astonishment " (i. e., that the 2.5×10^{9} electron/cm^3 plasma density required to reflect waves transmitted at the 440 Mc/s Millstone frequency is " unreasonably " high), little can be said about the electron density in Venus' ionosphere. However, the rather similar but low values for the radar cross-section (approximately one-tenth of the geometric) obtained at 440 Mc/s [1] and at 2 388 Mc/s [6], support the assumption that reflection took place at Venus' surface, as do the relatively narrow Doppler bandwidths observed in the received signals. The good agreement (*see* section 5.C) found between the AU determined from time delays and that determined from Doppler shifts also indicates that the 440 Mc/s waves were doubtless not reflected at a level much above the surface of Venus ([13]). Further, radar echos at a frequency of 50 Mc/s (*see* section 6) indicated coherence times as long as 10 s and a radar cross-section commensurate with the values obtained at the higher frequencies [7]. While inconclusive, both of these results are certainly in accord with the assumption that, even for the 50 Mc/s waves, reflection occurred at Venus' surface ([14]).

If in fact the 50 Mc/s waves penetrated to the surface, then the maximum electron density in Venus' ionosphere did not exceed about 4×10^{7} electrons/cm^3. Even were such a density maintained over an altitude range of 500 km, the consequent change in time delay of the Millstone radar pulses would have been less than 25 μs, which is still of no significance. In any event, the effect of the neutral component of the Venus atmosphere can certainly be ignored with impunity (*see* de Vaucouleurs and Menzel [8]).

The medium between the radar antenna and the near-point of Venus can also cause the Doppler shift measurements to be altered from the values that would be obtained in vacuum. In particular, systematic changes in the intervening medium may cause a significant displacement of the Doppler shift. (Random changes only increase the bandwidth of the observed shift.)

The neutral component of Earth's atmosphere is very slowly varying; therefore, it can introduce a possibly observable displacement in the

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([13]) If the waves were reflected from the ionosphere, the Doppler shifts would not in general be affected in the same manner as the time delays.

([14]) It perhaps should be emphasized that these possible difficulties in determining the point of reflection of radar waves will not be present in a similar investigation of Mercury whose ionosphere could not be very dense.

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Doppler shift only in virtue of a change in the effective path length of the radar wave caused by the antenna's following of Venus. For radio wave lengths sufficiently far removed from the atmospheric absorption lines, this path-length change is independent of frequency. The corresponding change in the Doppler shift is given by

$$\delta(\Delta f) = \frac{f}{c}\frac{d}{dt}\int_0^{L} [n(l) - 1]\, dl, \tag{3.4}$$

where $n(l)$ is the index of refraction along the path of the radar wave. The time rate of change of the integral is always less than 0.1 cm/s for zenith angles less than 80° [9]. Hence, during the Venus experiment the neutral component of Earth's atmosphere never caused a displacement in the two-way Doppler shift of more than 0.003 c/s, which is considerably below the level of detectability.

The change in Doppler shift caused by a systematic change in the plasma encountered by a radar wave is given by

$$\delta(\Delta f) \approx -\frac{4.1\times 10^7}{cf}\frac{d}{dt}\int_{\text{path}} N(l)\, dl. \tag{3.5}$$

To obtain an upper bound on the contribution of Earth's ionosphere to such a two-way Doppler shift, we consider the conditions for which the average angle to the zenith was greatest. These occurred on 17 April when the antenna scanned from a 47° to an 80° zenith angle during the course of a 3-hour experiment. The mean change in the resulting Doppler shift is comparable to the measurement accuracy, and can be approximated by

$$\begin{aligned}\overline{\delta(\Delta f)} &\approx -2\,\frac{4.1\times 10^7}{cf}\,\frac{\overline{N}L_0}{t_2-t_1}[\sec\alpha(t_2) - \sec\alpha(t_1)] \\ &\approx -0.1\ c/s.\end{aligned} \tag{3.6}$$

where $\overline{N}L_0$ denotes the integrated electron density in the zenith direction and is taken to be 6×10^{13} per square centimeter, the upper limit found experimentally by Evans and Taylor for temperate latitudes [10]. In general, when the inclination of the antenna to the zenith increases with time, the systematic change in Doppler shift caused by Earth's ionosphere is negative, while for a decreasing angle to the zenith the change is positive. Exceptions may occur, for example, when measurements are taken near dawn since the integrated electron density in a given direction has been observed to increase by as much as 50 % per hour during this period [5]. However, the absolute effect is lessened because the electron content is at its low ebb just before dawn.

To estimate the effects on the Doppler shift of systematic changes in the interplanetary medium, consider a plasma wedge moving perpen-

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dicular to the path of a radar wave. Such a movement produces the maximum displacement of the Doppler shift; its magnitude is given by

$$(3.7) \qquad |\delta(\Delta f)| \approx \frac{v_p}{c}\left[\frac{8.2\times 10^7 N_1}{f}\right]\tan\left(\frac{\beta}{2}\right),$$

where v_p is the plasma velocity, N_1 the plasma density inside the wedge, and β the angle between its edges. For $v_p \approx 300$ km/s, $N_1 \approx 10^3$ electron/cm^3, and $\beta \approx 30°$, $|\delta(\Delta f)|$ would be about 0.05 c/s, which is somewhat smaller than the experimental error.

The effect on the Doppler shift of the ionosphere of Venus is perhaps the most serious and certainly the most difficult to estimate. However, only radar waves incident normally on Venus' atmosphere contribute substantially to the echos detected by the antenna on Earth. Therefore, systematic changes in the observed Doppler shift can be caused only by changes in the integrated electron density along the path of the normally incident radar pulses as successive ones pass over adjacent portions of the surface of Venus. For changes at the rate of 10 % per hour, the two-way effect would be of magnitude 0.35 c/s when we assume an integrated electron density of 2×10^{15} electron/cm^2, as before. This estimate of the change in the Doppler shift is most likely far larger than the true value if reflection of 50 Mc/s waves does in fact take place at the surface of Venus.

We conclude from this discussion that the medium between the antenna and Venus caused only insignificant effects on the echo delays of the Millstone radar pulses unless reflection took place in Venus' ionosphere. (Although the evidence is not really conclusive, this latter possibility is generally felt to be very unlikely.) If reflection actually did take place in the ionosphere, the time delays were probably decreased at inferior conjunction by no more than 1 part in 10^5, and proportionately less away from conjunction. In either event, the plasma effects may possibly be important for the proper interpretation of the Doppler shift measurements, although for the last six observation dates (when the Doppler shifts were large) any changes from the vacuum values probably did not exceed 1 part in 10^5.

C. *Ephemeris.* — An ephemeris was needed in these experiments both for the proper recording of the data and for the prediction of time delays and Doppler shifts to be compared with those deduced from the observations. We discuss here only the important errors associated with the first function and defer a comprehensive treatment of prediction errors to section 5.

In determining time delays by the incoherent integration method, an ephemeris was used to adjust the frequency shifts in generator B and its predecessor. The error in this predetermined Doppler shift had

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only to be small enough to insure that the error in superposing returns over the duration of a day's measurements was small compared to the intersampling interval; i. e., the error in the frequency shift, $\delta(\Delta f)$, had only to be small compared to $\left(\frac{t_s}{T}\right) f$, where T was the total time interval over which samples were superposed. With $T \approx 2.5$ h and $t_s \approx 5 \times 10^{-4}$ s (*see* section 4), the condition becomes $\delta(\Delta f) \ll 24$ c/s. Since Δf itself did not exceed 4×10^4 c/s, the requirement on $\delta(\Delta f)$ was equivalent to the requirement that the error in the ephemeris calculation be much less than 6 parts in 10^4. Before 23 March, the value for the AU obtained by Price *et al.* [3] was used in preparing the ephemeris. As it later turned out, this value was only accurate to 1 part in 10^3 so that, in superposing returns, the received signal was actually distributed among several intersampling intervals. This relatively slight spreading was sufficient to prevent a clear-cut detection of the echo signal. However, on 23 March (which accidentally happened to coincide with the installation of generator B), the integrated signal was sufficiently strong to detect, partly because of the reduced Doppler (and the consequent reduced spreading of the integrated signal) and mostly because of the decreased range. Since the totals for each of the earlier 5 mn integration periods had been preserved, these were then recombined using the newly-found value for the AU. The result was a detectable (albeit weak) integrated signal. With the new value for the AU, the requirements on accuracy for presetting generator B were, of course, easily met.

In the coherent integration method, an ephemeris was used to determine trial values of, and expected changes in, time delays and Doppler shifts (section 2.C). The original processing of the 1959 data also failed to detect an echo because the incorrect, 1958 radar value for the AU was incorporated into the ephemeris (i. e., the wrong region of the time-delay, Doppler shift domain was searched). For the processing of the 1961 data (and for the reprocessing of some of the 1959 data) the improved value of the AU was used, the corresponding neighbourhood in the time-delay, Doppler shift space was explored, and *bona fide* echos were detected. More records containing some-what weaker signals from the 1959 inferior conjunction are available for analysis at a time when the relative orbits of Earth and Venus are sufficiently well known to make feasible a search for these signals.

D. *Surface characteristics of Venus.* — Even with the reflection of the radar waves occurring at the surface of Venus (*see* above), there remains the problem of identifying the actual point of reflection that corresponds to the peak of the integrated received signal (or, equivalently, of determining the time delay that corresponds to reflection of the radar pulse from the " near-point " on Venus' surface). As illustrated in figure 5,

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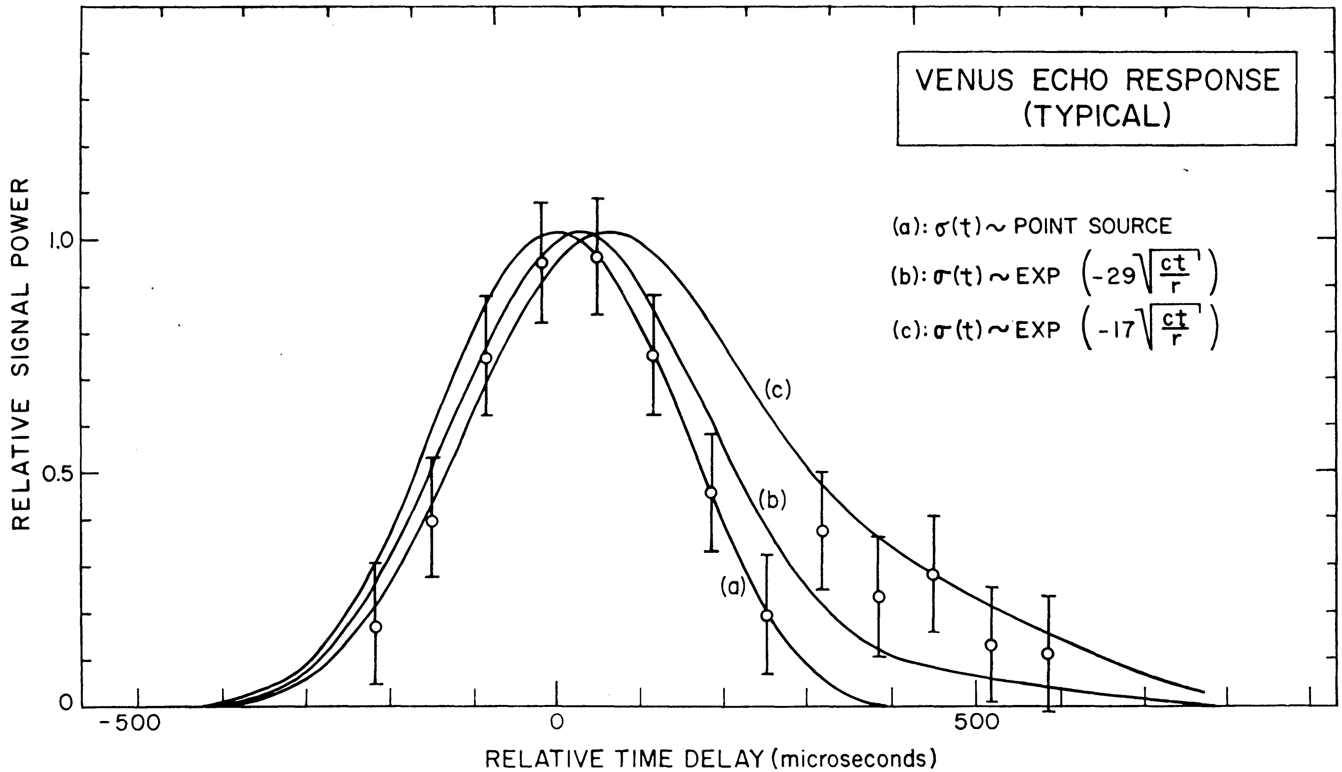

Fig. 5. — Intensity of the coherently integrated received signal compared with theoretical results to be expected from different models for the target's scattering properties.

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TABLE II.

Summary of time-delay and Doppler shift observations.

Year.	Month.	Time of transmission (UT) day.	h.	mn.	s.	Observed time delay (s).	Observed Doppler shift (c/s).	Total receiving time (mn).	Pulse length (ms).	Interpulse interval (ms).	Inter-sampling interval (ms).
1959.	Sept...	14	17	13	50 ± 1	308.63178 ± 0.0003	− 18 046.0 ± 0.2	52	–	–	–
1961.	Mar....	6	21	30	55 ± 5	425.2230 ± 0.0008	–	42	2	30	0.500
		7	18	44	01	419.4121	–	70	2	35	0.500
		14	16	22	44	376.3695	–	88	2	35	0.500
		16	17	37	56	364.4973	–	48	2	35	0.500
		22	18	09	28	332.9159 ± 0.0005	–	60	2	35	0.500
		23	20	57	32 ± 1	327.6940		72	2	32	0.500
		24	00	33	33	327.0435	–	22	2	34	0.500
		27	19	21	49	311.1564	–	42	2	29	0.500
		31	19	35	02	297.7762	–	20	2	35	0.500
	Apr...	3	19	41	10	290.3877	–	34	2	35	0.500
		3	21	21	10	290.25589 ± 0.00005	9 525.80 ± 0.1	29	–	–	–
		3	21	28	10	290.2464 ± 0.0005	–	19	2	35	0.500
		5	23	37	14	286.6264		29	2	35	0.500
		8	18	14	02	283.7176 ± 0.0002	–	122	4	64	0.250
		8	20	27	35	283.67188 ± 0.00007	2 247.76 ± 0.1	48	–		
		10	21	22	27	283.0845 ± 0.0002		33	0.5	10	0.250
		12	17	27	31	283.5996		29	2	30	0.250

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TABLE II (*continued*).

Summary of time-delay and Doppler shift observations.

Year.	Month.	Time of transmission (UT) day.	h.	mn.	s.	Observed time delay (s).	Observed Doppler shift (c/s).	Total receving time (mn).	Pulse length (ms).	Interpulse interval (ms).	Inter-sampling interval (ms).
1961	Apr...	12	21	55	17	283.73579 ± 0.0001	− 4 143.99 ± 0.1	25	–	–	–
		12	22	01	22	283.7396 ± 0.0002	–	24	2	50	0.250
		17	19	35	11	290.32687 ± 0.00003	− 11 284.75 ± 0.1	53	–	–	–
		18	20	15	00	292.5937 ± 0.0002	–	92	0.18	35	0.180
		20	18	00	34	297.5433		30	2	35	0.250
		21	17	05	00	300.4085		75	0.18	35	0.180
		24	19	06	13	311.1873		36	2	40	0.250
		26	17	30	01	319.0662		133	0.36	63	0.180
		28	17	30	00	328.0724		137	0.36	63	0.180
	May...	1	18	55	00	343.36472 ± 0.00005	− 27 723.30 ± 0.1	51	–	–	–
		3	14	08	14	353.11229 ± 0.0002	− 28 205.76 ± 0.1	41	–		–
		3	14	14	21	353.1356 ± 0.0003	–	70	4	64	0.250
		16	14	17	50	436.1257		100	4	64	0.250
		16	18	15	25	437.30151 ± 0.001	− 36 709.63 ± 0.2	36	–	–	–
		18	09	42	09	448.9659 ± 0.0003	–	67	4	64	0.250
		18	12	30	06	449.78497 ± 0.0007	− 36 078.40 ± 0.2	112	–	–	–
		31	15	41	27	548.51981 ± 0.0005	− 40 149.08 ± 0.2	64	–	–	–
	June...	8	15	43	14	611.58523	− 41 005.60 ± 0.2	71	–	–	–

Note 1 : **Entries including Doppler shift** observations were processed **coherently**; all others were processed **incoherently.**

Note 2 : **The error in a quantity is identical** to that of the entry directly above except where otherwise noted.

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the time delay associated with the peak of the received energy corresponds to a depth that depends on the target's reflection properties. These are characterized by the power impulse response function, $\sigma(t)$, with t being the delay measured from the near-point of the target. Curve (*a*) in the figure corresponds to the return to be expected from a point source, and curve (*c*) to thatfrom the moon; the shift in delay time of the peak intensity is clearly in evidence. (For further details, *see* Pettengill *et al.* [1].) For the present experiment, Smith [2] estimates an upper bound of 70 μs on the error in two-way time delay incurred by assuming that the peak intensity corresponds to the near-point of Venus' surface.

Since returns are detected from regions other than that in the immediate neighbourhood of the near-point, the integrated echo exhibits an appreciable Doppler bandwidth (approximately 0.5 c/s) which can be used to infer the rotation rate of Venus. (As discussed above, random changes with time of the electron content that is integrated over the path of the radar wave, also add to and complicate the interpretation of this bandwidth.)

4. **Summary of time-delay and Doppler shift data.** — In table II, we present a summary of all time delays and Doppler shifts deduced to date from the laboratory's radar detections of Venus during the 1959 and 1961 inferior conjunctions. Each result was obtained by integrating returns over the indicated time interval, and is referred to a time that corresponds approximately to the mid-point of the integration interval. (This time is only given to the nearest second, but is known in some instances to within 10 ms; *see* section 3.A.) Also included in the table are data describing the transmitted wave form and the processing technique employed. The measurement errors shown were obtained by combining the various contributions discussed above ([15]); they are intended to represent probable errors although, of course, the actual error distributions are unknown. Although always possible, it is nevertheless considered very unlikely that significant systematic errors have been overlooked.

Some astronomers have expressed the opinion that the " actual data " from these interplanetary radar experiments should be published rather than, for example, simply one two-way echo delay for each extended period of observation. We therefore wish to emphasize that many minutes of transmitting and receiving data usually resulted in only one weak measurement : the detection of an echo from a single pulse

([15]) These errors do not, however, include estimates of the effects of the medium or of the target each of which more properly relates to the interpretation of the measurements.

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was impossible; only by summing the contributions from a great number of pulses was detection enabled. Presenting the results for each separate set for which detection was barely achieved would indeed yield a host of independent results ([16]). But these would then certainly have to be averaged before being used for any interpretative purpose. The computer performs this averaging (without introducing any significant systematic error), and yields a single, strong measurement (i. e., a " normal place ") from the totality of individual runs made during a given period of observation. While the choosing of a single reference time from an extended set of measurements is admittedly somewhat arbitrary each selection actually made corresponded closely to the mid-point of the observing interval.

5. **Determination of the astronomical unit.** — The primary goal of this early series of time-delay and Doppler shift measurements was to determine the AU accurately in terms of " terrestrial " units, such as light-seconds or kilometers. The determination can be made from either a single interplanetary time-delay or Doppler shift measurement and the corresponding prediction based on the appropriate ephemeris. For example, a time delay, τ_0, measured in seconds, and a delay, τ_c, computed in AU, leads to

$$AU = \frac{\tau_0}{\tau_c} \text{ light-sec.} \tag{5.1}$$

For our calculations two sets of predictions were prepared, one based on the original Newcomb theory and the other on the ephemeris as corrected by Duncombe [11]. For both sets it was assumed that the radar waves propagated in vacuum.

A. *Prediction of time delays and Doppler shifts.* — The assumed relevant vacuum trajectory of the radar pulse extends from the antenna at the time of transmission to the near-point on the surface of Venus at the time of reflection and back to the position of the antenna at the time of reception. Given the geometric ephemeris and the speed of light, the two-way echo time delay can be calculated accurately enough by a simple iterative process : first the transit time between the antenna and Venus' surface is calculated for the time of transmission; then the position of Venus at the later time (transmission plus transit) is used for determining a new flight time, etc.. The first iteration yields an accuracy of better than 1 part in 10^7 and hence a second is not necessary. The return time can be calculated in the same manner. In the limit of coplanar, circular orbital motion, and for a pulse transmitted at time t_1

([16]) Actually, these would be independent only in so far as the correlations introduced by systematic errors were ignored.

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this prescription leads to an echo time delay between the centers of the planets given by

$$(5.2)\qquad \tau(t_1) = \frac{2\,R}{c}\left\{1 + \frac{R_E R_V}{c\,R}(\dot{u}_V - \dot{u}_E)\sin(u_V - u_E) + O\left(\frac{v_d^2}{c^2}\right)\right\}\ (^{17}),$$

where lengths are measured in AU, where c is the speed of light in AU per second, and where R_E, R_V are the heliocentric radii and u_E, u_V are the orbital longitudes of Earth and Venus, respectively. The Earth-Venus range is denoted by R and the difference in the orbital speeds of Earth and Venus by v_d. All quantities on the right side of equation (5.2) are geometric and are to be evaluated at the time t_1. The error term is of order 10^{-8}. An equivalent formula, which is very similar to that actually used in the Lincoln Laboratory calculations, is

$$(5.3)\qquad \tau(t_1) \approx \frac{2\,R(t_1)}{c - \dot{R}(t')} \approx \frac{2\,R(t_1)}{c}\left[1 + \frac{\dot{R}(t')}{c}\right]; \qquad t' = t_1 + \frac{R(t_1)}{c}.$$

Surface corrections to $\tau(t_1)$ are easily made to the required accuracy: For Venus, it is necessary only to subtract a delay equal to the diameter of the planet, expressed in light-seconds. For Earth, the vector from its center to the radar site is projected along the appropriate Earth-Venus direction at the time of transmission and at the time of reception; the sum of these projections, in light-seconds, is also subtracted from the echo delay.

The effect on the echo time delay of special relativistic corrections is small. If we assume the ephemeris to be valid in the solar reference frame, observers in this frame would then measure a time delay, τ, as described above; however, an observer in the terrestrial frame would measure a delay $\tau^* = \tau\left[1 - \left(\frac{v}{c}\right)^2\right]^{\frac{1}{2}}$, where v is the orbital speed of Earth. Since $\left(\frac{v}{c}\right)^2 \approx 10^{-8}$, the resultant difference in time interval measurements can be ignored in this experiment. Similarly, the effect of general relativistic corrections on the speed and path of the radar waves (caused by the changes in the gravitational potential along the path) are about 1 part in 10^8 and can also be neglected, as can the effect on interplanetary distances [12].

To first order in v_d, the two-way Doppler shift is simply given by the rate of change of the two-way time delay, multiplied by the negative of the transmitted frequency:

$$(5.4)\qquad \Delta f = -f\left[\dot{\tau} + O\left(\frac{v_d^2}{c^2}\right)\right] = -f\dot{\tau}\left[1 + O\left(\frac{v_d}{c}\right)\right].$$

(^{17}) A dot placed above a symbol signifies differentiation with respect to time.

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Since the time derivative of the echo delay is proportional to $\frac{v_d}{c}$ and since some Doppler shifts were measured with accuracies exceeding 1 part in 10^5, the second-order terms in $\frac{v_d}{c}$ become important. A rigorous derivation in accordance with the theory of special relativity shows that a terrestrial observer of radar echos from Venus should measure a Doppler shift given by

$$\Delta f = f\left(\frac{1-\beta_1^2}{1-\beta_3^2}\right)^{\frac{1}{2}}\left(\frac{1-\vec{\beta}_2.\vec{e}_{12}}{1-\vec{\beta}_1.\vec{e}_{12}}\right)\left(\frac{1-\vec{\beta}_3.\vec{e}_{23}}{1-\vec{\beta}_2.\vec{e}_{23}}\right)-f, \tag{5.5}$$

where all quantities on the right side (other than f) are computed directly from the ephemeris; where $\vec{\beta}_1$, $\vec{\beta}_2$, and $\vec{\beta}_3$ denote the velocities (expressed as fractions of the speed of light) of the antenna at the time of transmission (t_1), of Venus at the time of reflection (t_2), and of the antenna at the time of reception (t_3), respectively; and where $\vec{e}_{12}$ and $\vec{e}_{23}$ are unit vectors pointing in the direction from the antenna at t_1 to the position of Venus at t_2 and from Venus at t_2 to the position of the antenna at t_3, respectively ([18]). If we neglect the motion of the antenna between t_1 and t_3, and hence set $\vec{\beta}_3 = \vec{\beta}_1$ and $\vec{e}_{23} = -\vec{e}_{12}$, we obtain

$$\Delta f \approx -2f(\vec{\beta}_2-\vec{\beta}_1).\vec{e}_{12}\left\{1-(\vec{\beta}_2-\vec{\beta}_1).\vec{e}_{12}\right\} \approx -f\dot{\tau}\left\{1-\frac{\dot{\tau}}{2}\right\}. \tag{5.6}$$

The $\dot{\tau}$ appropriate for the rate of change of the echo delay measured between the planetary centers in the limit of coplanar, circular motion is given by

$$\left\{\begin{aligned}\dot{\tau}(t_1) &= \frac{2\,R_E R_V(\dot{u}_V-\dot{u}_E)}{cR}\\ &\quad\times\left\{\sin(u_V-u_E)+\frac{R(\dot{u}_V-\dot{u}_E)\cos(u_V-u_E)}{c}+O\left(\frac{v_d^2}{c^2}\right)\right\}\\ &= \frac{2\dot{R}(t')}{c}\left\{1+O\left(\frac{v_d^2}{c^2}\right)\right\};\\ &\qquad t' = t_1+\frac{R(t_1)}{c},\end{aligned}\right. \tag{5.7}$$

where, as in equation (5.2), all quantities are geometric and are to be evaluated at t_1, unless otherwise noted. Surface corrections are again

([18]) A "classical" derivation (in which the velocity of light is assumed to be $\vec{c}$ with respect to the ether) leads to the same result, except that the first parenthetical term is absent. Thus, the "classical" and the special relativistic formulas would be identical were the velocity of the antenna to be the same at t_1 and at t_3.

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easily made; it is necessary only to add to $\dot{R}(t')$ the sum of the rates of change of the projections along $\vec{e}_{12}$ and along $-\vec{e}_{23}$ of the vector extending from the center of Earth to the radar site at times t_1 and t_3, respectively.

B. *Preparation of ephemeris.* — The geometric ephemeris used for comparison with the observations was based on the assumption that ET-UT equalled 34 s at the 1961 conjunction and 33 s at the 1959 conjunction. For the Newcomb theory, the geocentric polar co-ordinates of the Sun and the heliocentric polar co-ordinates of Venus, referred to the equinox and ecliptic of date, were obtained at four-day intervals from volumes 14 and 15, Part III, of the *Astronomical Papers* of the *American Ephemeris* ([13], [14]). The center-to-center distances from Earth to Venus were then calculated at one-day intervals using Lagrangian interpolation; two further Lagrangian interpolations provided both the center-to-center geometric distance and its first time derivative at 1 mn intervals of UT.

The projection along the apparent Earth-Venus direction of the vector defined by the center of Earth and the radar site was obtained for times and directions appropriate for transmission and reception. Lagrangian interpolation was then used to determine the values, at 1 mn intervals of UT, of these projections and of their first time derivatives ([19]).

The radius of Venus was taken to be

$$0.02030 \pm 0.00002 \text{ light-sec} = 6\,089 \pm 6 \text{ km},$$

as determined by de Vaucouleurs and Menzel [8].

The procedures described above were then used (in conjunction with a trial value for the AU, expressed in light-seconds) to obtain predictions for time delays and Doppler shifts.

The Duncombe ephemeris was calculated with the same program as was Newcomb's except that corrections to the tabular values, as provided personally by Duncombe, were used. These values were based on his solution for the elements of Earth and Venus and were intended specifically for use with the appropriate entries of volumes 14 and 15, Part III of the *A. P. A. E.* (The corrections he inserted for the centennial variations, however, were not those determined from the observations but were the ones theoretically derived using the set of planetary masses given by Clemence and Brouwer [15].)

Numerical experiments were performed with our recently-completed double-precision computer program to study the errors introduced into

([19]) In these calculations, aberration was treated only approximately; the resulting error in the Doppler shift was, however, always less, and usually far less, than 0.1 c/s.

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our predictions both by the interpolation procedures and by the round-off errors in the tabulated values of the basic ephemerides. These errors in the time-delay computations are judged to be no greater than 0.2 ms, which amounts to at most 7 parts in 10^7. However, possibly because of the limited accuracy of the tabular values used, the Doppler shift calculations appear sometimes to be in error by as much as 0.4 c/s which, unfortunately, is larger than the uncertainty in the measurements.

An independent check of the time delays computed from our Newcomb ephemeris was made at JPL. From orbits determined by numerical integration [16], JPL calculated echo delays for many of the transmission times listed in table II. These delays were all smaller than ours, but the differences rarely exceeded 0.1 ms and were nowhere greater than 0.3 ms. Another similar but less exhaustive check was made with an ephemeris prepared at RCA [17] and also yielded good agreement.

No direct independent check of the time delays computed from our Duncombe ephemeris has yet been made. However, the time delays predicted with the JPL version of this ephemeris were deduced from a recent determination of the AU made by JPL [18] using some of the Millstone data. A comparison with the corresponding time delays calculated with our program shows the JPL predictions to be systematically larger by about 0.2 ms somewhat before conjunction, but smaller by about 0.7 ms after conjunction. The Duncombe corrections to the time, delays deduced from figures 5 and 6 of the article by Muhleman *et al.* [16] also seem to be smaller than the corresponding ones computed with our program by about 0.7 ms. The cause of this discrepancy has not yet been isolated.

No detailed independent check has been made of the Lincoln Laboratory program for Doppler shift determinations. It is expected that JPL and the U. S. Naval Observatory will soon provide us with values appropriate for direct comparison. Meanwhile, from figure 4 of Muhleman *et al.* [16], we can deduce approximately the Duncombe corrections that were applied to the Doppler shift calculations. These agree with our results to within about 0.2 c/s, when the comparison is made with respect to the Millstone frequency.

C. *Comparison of measurements and predictions.* — For each of the reduced time-delay and Doppler shift measurements, our Newcomb and Duncombe ephemerides were used to generate the corresponding predictions, based on a trial value for the AU of 499.005 light-sec. These results, subtracted from the corresponding measurements, are shown in figures 6 and 7. The residuals for the measurements reduced from the 1959 Venus observations (not shown in figures 6 and 7) are — 1.1 ms and 0.3 c/s for the calculations based on the Duncombe orbits, and — 0.4 ms and 0.5 c/s for those based on the Newcomb orbits.

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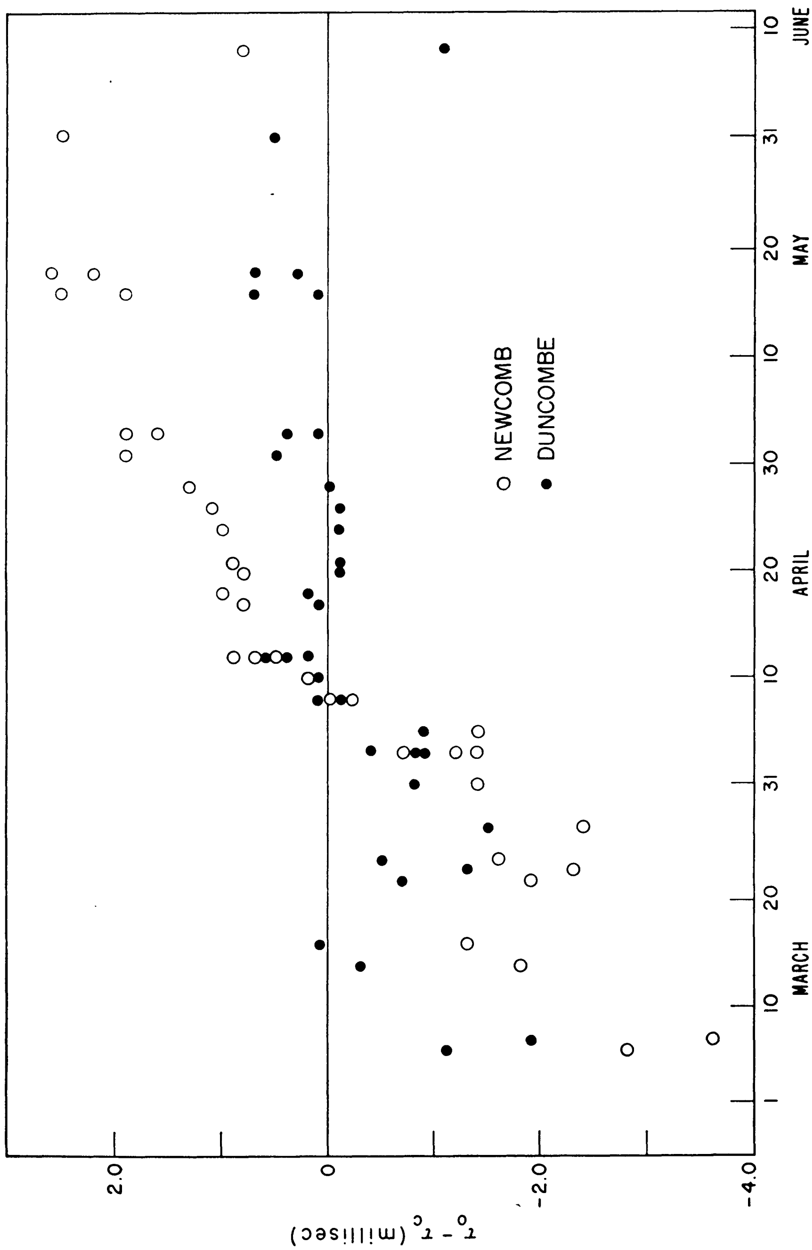

Fig. 6. — Residuals from time-delay observations.

Figure 6 shows clearly that the time-delay data are more consistent with the Duncombe than with the Newcomb predictions. However, even the residuals from the Duncombe predictions are in some cases considerably larger than would be expected from the error estimates. Since the latter are felt to be conservative, the explanation for these large values is related to errors either in the interpretation of the measurements, in the basic ephemerides or in some combination. Previous

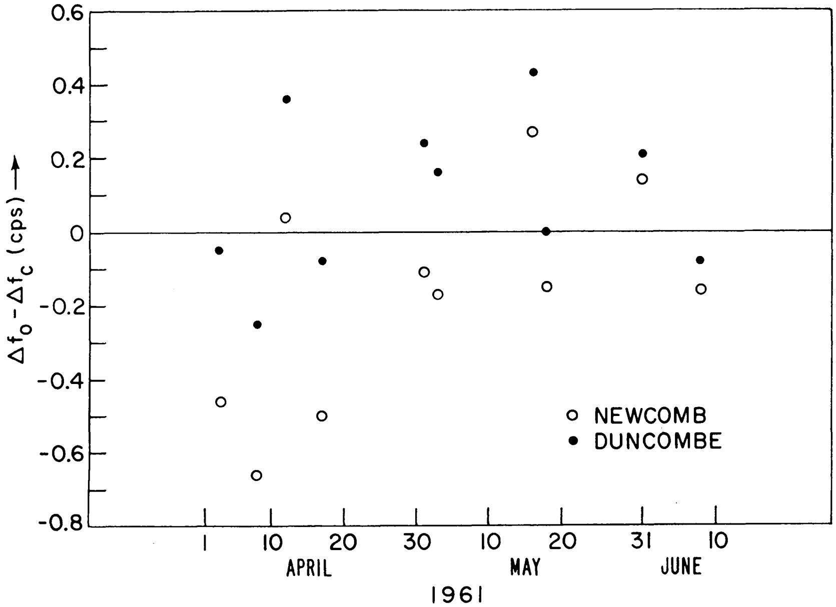

Fig. 7. — Residuals from Doppler shift observations.

attempts to understand the residuals involved postulating either a 2 % change in the accepted value for the Moon's mass, or a plasma effect that has the period of the Sun's rotation. Neither seems cogent, especially the former which is clearly incompatible with all accurate determinations of the Moon's mass, as well as with the post-conjunction residuals in figure 6. The latter, proposed by Priester *et al.* [19], presupposes a very complex, *ad hoc*, dynamic structure for Venus' ionosphere that would reflect the Millstone radar waves but not the higher frequency ones of JPL, and yet would lead to the same radar cross-section at both frequencies. Their model also implies that, even on the dark side of Venus, the ionosphere is highly disturbed and this implication appears to be in conflict with the relatively long coherence times obtained for radar echos at 50 Mc/s (*see* section 3.B). In addition, when the time-delay residuals of all the Millstone data are compared with the decimeter

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radiation from the Sun, the existence of an inverse correlation is somewhat less evident than it is in the article by Priester *et al.* [19]. A different explanation for the observed residuals is discussed in the following section and is based on the possibility of an error of about 1 part in 10^6 in the orbital eccentricities of both Earth and Venus.

From the previous section, it would appear that the Doppler shift residuals are perhaps mainly representative of random errors present in the predicted values. The actual measurement errors are expected to be somewhat smaller, and so the inherent measurement accuracy cannot be utilized fully until the Doppler shift calculations are improved. (In particular, it is probably fruitless to seek any indication of plasma effects.) Although the residuals are not even convincingly more consistent with the Duncombe than with the Newcomb ephemeris [20], they do provide an experimental confirmation of the second-order term in $\frac{v}{c}$ in the Doppler shift formula. For example, this correction reached almost 2 c/s at the time of the last 1961 measurement, and is significantly larger than the sum of the residuals and the combined uncertainties in the measurements and computations.

It might be possible to obtain improved values for the orbital elements of Earth and Venus, as well as for the AU, by performing an appropriate least-squares analysis. However, following such a procedure would seem to be premature; in the next few years many more, higher-precision, interplanetary radar experiments will be conducted and the accumulated data (radar and optical) can then be used to refine the orbital elements, and perhaps even the masses and radii of the inner planets. For the present, then, we ignore the possibility of errors in the basic ephemerides and determine a value for the AU from a weighted, least-squares analysis of the data listed in table II (p. 196). We note that both the time delay and the Doppler shift are proportional to the unit of distance and that an error in either introduces a linearly related error in the AU :

$$\frac{\delta\tau}{\tau} = \frac{\delta(\mathrm{AU})}{\mathrm{AU}}; \qquad \frac{\delta(\Delta f)}{\Delta f} = \frac{\delta(\mathrm{AU})}{\mathrm{AU}}. \tag{5.8}$$

The error in the AU determined from a single measurement is therefore inversely proportional to the value of the measured quantity. Since $\Delta f = 0$ at conjunction, Doppler shift measurements in that immediate vicinity are essentially useless for determining an accurate value for the AU.

[20] Note that the Duncombe corrections to the Doppler shifts reached a maximum of about 0.4 c/s near inferior conjunction and decreased gradually to less than 0.1 c/s on 8 June 1961.

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This insensitivity is, of course, automatically reflected in the least-squares procedure. In particular, we obtain as the exact least-squares solution :

$$(5.9) \qquad \mathrm{AU} = \frac{\displaystyle\sum_{i=1}^{M} \frac{x_o^i x_c^i}{\sigma_i^2}}{\displaystyle\sum_{i=1}^{M} \frac{x_c^i x_c^i}{\sigma_i^2}} \mathrm{AU}_{t0},$$

where AU_{t0} is the trial value used to compute the time delays and Doppler shifts. The subscripts *o* and *c* denote observed and computed values while x^i denotes the *i*th of a total of M measurements which include both time delays and Doppler shifts (e. g., x^j may be a τ and x^{j+1} a Δf measurement). The error, σ_i, associated with each measurement, is arbitrarily taken to be the combined measurement and computational error (*see* section 5.B). Under the (probably inadequate) assumption that such errors are independent, unbiased, and Gaussianly distributed, the resultant standard deviation for the AU is given by

$$(5.10) \qquad \sigma_{\mathrm{AU}} = \left[\sum_{i=1}^{M} \left(\frac{x_c^i}{\sigma_i} \right)^2 \right]^{-\frac{1}{2}} \mathrm{AU}_{t0}.$$

The AU determined in this manner from the Newcomb ephemeris is 499.00608 ± 0.00004 light-sec, whereas including the Duncombe corrections yields an AU of 499.00516 ± 0.00004 light-sec. Only the latter value will be used in the sequel. But in view of the probability that significant errors still remain in the basic planetary ephemerides, we prefer to adopt a considerably larger value for the probable error in this determination of the AU, i. e., ± 0.001 light-sec. Assuming a value of 299 792.5 km/s for the speed of light ([21]) yields an AU of 149 598 000 ± 300 km. Further, assuming 6 378.15 km to be the exact value for the mean equatorial radius of Earth leads to a solar parallax of 8".79416 ± 0".00002.

We emphasize the value of the AU expressed in light-seconds since that is the basic unit being determined by these experiments. In addition, the accuracy in light-seconds is higher or at least no lower than in kilometers; the percentage error in the adopted value of the vacuum speed of light might even be larger than that in some radar time-delay measurements. The result for the solar parallax is also degraded by the inaccuracy in the value used for the mean equatorial radius of Earth, recent determinations having ranged at least from 6 378.084 km (by Spencer Jones [20]) to 6 378.165 km (by Kaula [21]).

([21]) *See*, for example, Froome [22].

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D. *Sensitivity of measurements to ephemeris errors.* — Since the measurements may have disclosed systematic errors in the planetary ephemerides, we investigate the sensitivity of time delays and Doppler shifts to small changes in the (elliptical) elements of the orbits of Earth and Venus [22]. (We ignore all effects associated with the moon, the other planets, and the mutual perturbations of Earth and Venus, although when more precise radar measurements are made some of these effects will definitely be of interest.) To first order

$$\delta\tau_c = \sum_{i=1}^{6} \left(\frac{\partial \tau_c}{\partial a_E^i} \delta a_E^i + \frac{\partial \tau_c}{\partial a_V^i} \delta a_V^i \right), \tag{5.11}$$

$$\delta\Delta f_c = \sum_{i=1}^{6} \left(\frac{\partial \Delta f_c}{\partial a_E^i} \delta a_E^i + \frac{\partial \Delta f_c}{\partial a_V^i} \delta a_V^i \right), \tag{5.12}$$

where a_E^i and $a_V^i (i = 1 \to 6)$ denote the orbital elements of Earth and Venus, respectively. The partial derivatives, to lowest order in eccentricity [23], have been evaluated for the set of elements a, e, ω, T, Ω, and i, where a is the major semi-axis, e the eccentricity, ω the argument of perihelion, T the time of passage through perihelion, Ω the right ascension of the ascending node, and i the orbital inclination. Since a is known with considerably higher precision than the others, and since, in our approximation, errors in T and ω are indistinguishable, we omit the sensitivities to errors both in a and in T. The values of the remaining partial derivatives, in the vicinity of the 1961 Venus conjunction, are plotted in figures 8 and 9. As is to be expected for nearly-circular, nearly coplanar orbits, the sensitivity of τ and Δf to errors in the argument of perihelion is almost equal to the sensitivity to errors in the longitude of the node. Similarly, the sensitivities to errors in ω_E and Ω_E are opposite in sign and almost equal in magnitude to those in ω_V and Ω_V. Both time delays and Doppler shifts are least sensitive to inclination angle errors.

It is also apparent, again in agreement with simplified quantitative considerations, that time delays are insensitive to errors in relative longitude $[(\delta\omega_E + \delta\Omega_E) - (\delta\omega_V + \delta\Omega_V) \neq 0]$ at conjunction, whereas Doppler shifts are most sensitive at this point but are insensitive about 70 days from it.

The short-period variation in the time-delay residuals that seems to be present before conjunction is similar to that which would be intro-

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(22) At present, the uncertainty in c is about 1 part in 10^6 whereas (*see* table II) four of the time-delay measurements have estimated probable errors as small as 2 parts in 10^7.

(23) i. e., to the lowest nonvanishing order.

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duced by a small error in the eccentricity of Earth. For the amplitude and phase to correspond, δe_E would have to be approximately $10^{-6} \approx 0''.2$ (i. e., if $0''.2$ were subtracted from e_i, the preconjunction observed residuals would be reduced). A somewhat smaller error (but of the same

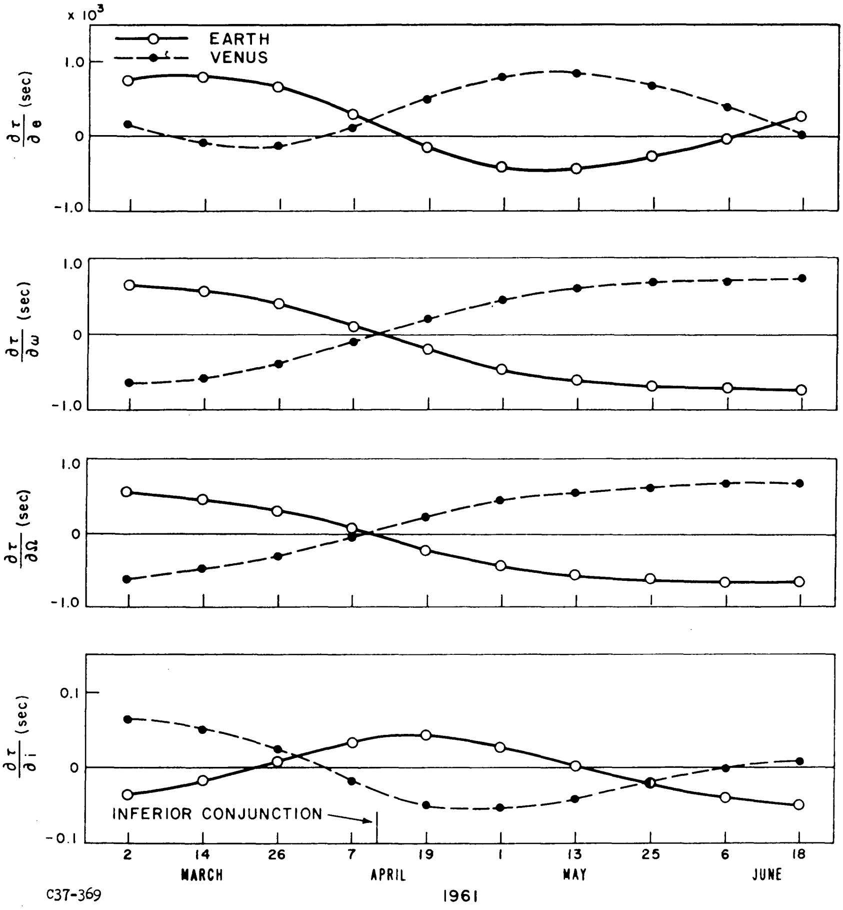

Fig. 8. — Partial derivatives of the time delay with respect to the elements of the orbits of the Earth and of Venus.

sign) in the eccentricity of Venus would combine with δe_E to maintain approximately the same observed small residuals after conjunction. (The effects of such changes in eccentricity on the Doppler shift residuals would be quite small compared to the corresponding measurement and computational errors.) While the presence of small errors in the planetary orbital elements thus appears to be a possible source of the observed residuals, more experiments must be performed before a definitive conclusion can be reached.

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6. **Comparison with radar results of other observatories.** — Other radar determinations of the AU from the 1961 inferior conjunction of Venus have been reported by JPL and RCA in the United States,

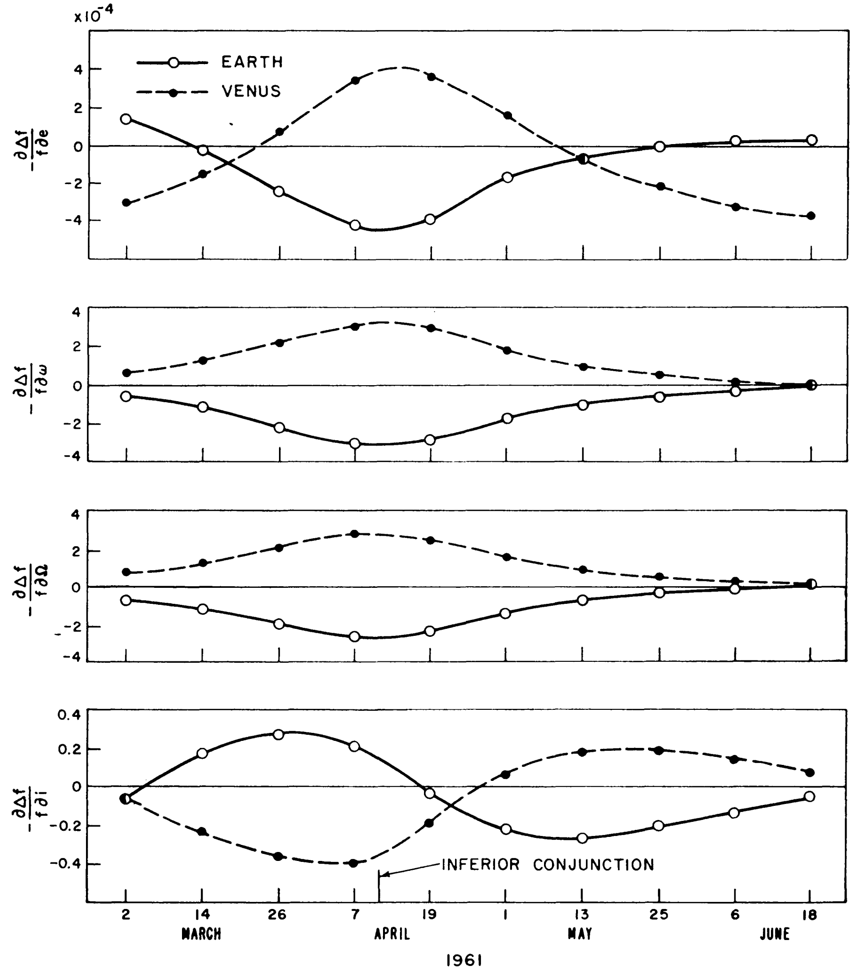

Fig. 9. — Partial derivatives of the Doppler shift with respect to the elements of the orbits of the Earth and of Venus.

by the University of Manchester (Jodrell Bank) in the United Kingdom, and by the Institute of Radio Engineering and Electronics of the Academy of Sciences of the U. S. S. R.

The AU given by JPL [16] was 149 598 850 ± 250 km. This result seems to differ significantly from the 149 598 000 ± 300 km obtained

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here using the Millstone data. However, closer examination discloses that :

1. JPL used a value for the speed of light higher by 0.5 km/s, which resulted in an AU higher by approximately 250 km ([24]).

2. The Duncombe corrections applied by Lincoln Laboratory and by JPL resulted in different predictions (*see* section 5.B); and

3. JPL did not obtain a final value for the AU by a least-squares analysis but, rather, by an extrapolation procedure based on the assumption that the measurement errors were less significant than the ephemeris errors and that the latter were mainly in the relative longitudes of the two planets. The sensitivity of time-delay measurements to such errors vanishes at conjunction. Therefore, JPL fitted a straight line to the AU's calculated from these measurements and took as the resultant AU the ordinate at conjunction of this straight line. The AU's obtained from the Doppler shift measurements were processed in a similar manner: The two branches of the curve (representing approximately the fractional sensitivity of Doppler measurements to errors in relative longitude before and after conjunction) were fitted to the set of AU's calculated from the observed Doppler shifts and were extrapolated to the elongation points. The co-ordinates at these two points were selected as estimates for the AU ([25]), and were combined in a straightforward manner with the estimate from the time-delay measurements to yield a final value.

The actual time-delay and Doppler shift measurements were not tabulated in the JPL article; only graphs of the resulting values for the AU were given. It is therefore not easy to reprocess their data on the basis

([24]) JPL now uses the lower value $c = 299\,792.5$ km/s (*see* the paper presented to the Symposium by Muhleman).

([25]) Actually, the sensitivity of Doppler shift measurements to errors in relative longitude does not vanish at elongation. For the simple example of coplanar, circular orbits, the corresponding interplanetary range rate $\left(\text{which, to first-order accuracy in } \frac{v}{c},\ \text{is proportional to the Doppler shift}\right)$ is given to first order in $\frac{v}{c}$ by

6.1 $$\dot{R} = \frac{R_E R_V}{R}(\dot{u}_E - \dot{u}_V)\sin(u_E - u_V),$$

and the sensitivity of $\dot{R}$ to an error in relative longitude is therefore

6.2 $$\frac{\partial \dot{R}}{\partial(u_E - u_V)} = \frac{R_E R_V}{R}\left\{\cos(u_E - u_V) - \frac{R_E R_V}{R^2}\sin^2(u_E - u_V)\right\}(\dot{u}_E - \dot{u}_V).$$

The right side vanishes at approximately 70 days before and 70 days after inferior conjunction (see *fig.* 9). That this sensitivity does not vanish at the elongations has only a small effect on the values obtained for the AU by extrapolating to these points.

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of our ephemeris. An attempt was made, however, to deduce measurements from the values of the AU that JPL obtained using the Newcomb ephemeris (which was previously shown to be closely related to ours, at least for range predictions). A least-squares analysis was then performed on the basis of our Duncombe ephemeris and assumed standard errors of 0.1 ms for all time-delay measurements and 0.2 c/s for all Doppler shift measurements. The resulting value for the AU is 499.0058 ± 0.001 light-sec $\approx 149\,598\,200 \pm 300$ km ([26]), where the error represents mainly our inability to interpolate accurately from JPL's graphs.

It appears probable, therefore, that the discrepancy between the values for the AU determined by JPL and by Lincoln Laboratory is more apparent than real. The differences may lie more in the interpretation than in any inherent incompatibility of the two sets of data.

The experiment at the Moorestown facility of RCA [17] was intended primarily to show that *bona fide* radar echos were received. Considering the over-all transmission and detection system, Maron [23] estimates the probable error in the three Moorestown time-delay measurements to be about 2×10^{-3} s. Processing these measurements using our Duncombe-corrected ephemeris leads to an AU of

$$498.998 \pm 0.003 \text{ light-sec} \approx 149\,596\,000 \pm 900 \text{ km}.$$

This result differs from ours by slightly more than twice its estimated probable error. The accuracy of the measurements made at Jodrell Bank [24] was limited by long (30×10^{-3} s) pulse lengths; the result obtained there on the basis of the Newcomb ephemeris was given as

$$499.011 \pm 0.017 \text{ light-sec} \approx 149\,600\,000 \pm 5\,000 \text{ km}.$$

(This rather large error implies that the Duncombe corrections would not change the result significantly.)

The experiment performed in the U. S. S. R. [25] yielded a value of 499.0095 ± 0.002 light-sec which disagrees with our determination by about twice the quoted root-mean-square error. Since the Soviet report included neither the measured nor the predicted values of the echo delays, it is not possible for us to ascertain reliably the cause of this disagreement. However, the formula given there for time delay is similar to our equation (5.3) except for a factor of $\frac{3}{2}$ which appears as the coefficient of $\dot{R}$ ([27]). On the assumption (confirmed by Kotel-

([26]) Of course, in view of the possibly large errors in our calculations of Doppler shifts (*see* section 5.B), this agreement with the Millstone result might not be particularly significant.

([27]) Their formula led to a larger computed time delay (during the period that their measurements were made), and hence to a smaller value for the AU.

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nikov [26]) that this formula was in fact used in the reduction of the Soviet data, we recalculated the AU using our equation for time delay, and found the value 499.0141 ± 0.002 light-sec $\approx 149\,600\,700 \pm 600$ km, which makes the disagreement even more serious.

No reports have yet been published on determinations of the AU from radar observations made during the inferior conjunction of Venus in 1962 ([28]). However, Klemperer [7] revealed that the Jicamarca facility of the U. S. National Bureau of Standards obtained an echo time delay of 316.8490 ± 0.001 s, appropriate for a radar pulse transmitted on 2 December 1962 at UT 15 : 02 : 43. Using our Duncombe ephemeris in conjunction with this measurement leads to an AU of

$$499.0017 \pm 0.001 \text{ light-sec} \approx 149\,597\,000 \pm 300 \text{ km}.$$

In view of the uncertainty in determining the reflecting point of the radar signals, it is possible that the 3.5 ms discrepancy between the AU's determined from the 50 Mc/s and from the 440 Mc/s experiments might arise from a 330 km difference in the altitudes from which the respective waves were reflected. However, no meaningful conclusions can be drawn from such a limited comparison.

The U. S. S. R. also established radar contact with Mercury in 1962 [27]. This experiment yielded for the AU the approximate value

$$499.01 \pm 0.03 \text{ light-sec},$$

and supports the other radar-based determinations. (*See*, also, the paper presented by Muhleman where preliminary results are reported concerning a 1963 radar contact with Mercury.)

The above-discussed values for the AU, and the corresponding values for the solar parallax, are summarized in table III ([29]). The accompanying uncertainties are intended to approximate probable errors (except where otherwise noted).

Considering all these radar results, we can conclude with perhaps 90 % confidence that the true value for the AU lies within the interval 499.005 ± 0.007 light-sec $\approx 149\,598\,000 \pm 2100$ km. More radar echos obtained at different frequencies from the various inner planets will probably reduce this uncertainty by one or two orders of magnitude within the next few years, provided that sufficiently accurate ephemerides are used in the data reduction. Since, for a proper interpretation of

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([28]) Note, however, the discussion in the paper presented to this Symposium by Muhleman.

([29]) Although the radar determination of the solar parallax differs significantly from that found by Rabe (in fact by ten times the probable error quoted by him), the former, being more direct and more extensively checked, is undoubtedly more reliable.

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TABLE III.

Comparison of radar determinations of the astronomical unit.

Facility.	Reference.	Radar frequency (Mc/s).	Astronomical unit: light-sec.	Astronomical unit: km (c = 299 792.5 km/s).	Solar parallax (R_o = 6 378.15 km).
Millstone (MIT).......	Present paper	440	499.0052 ± 0.001	149 598 000 ± 300	8.79416″ ± 0.00002
Goldstone (JPL)......	Muhleman *et al.* (1962)	2 388	499.0071 ± 0.001	149 598 600 ± 250	8.79412 ± 0.00002
			(499.0058 ± 0.001) (*)	(149 598 200 ± 300) (*)	(8.79415 ± 0.00002) (*)
Moorestown (RCA))...	Maron *et al.* (1961)	438	498.998 ± 0.003	149 596 000 ± 900	8.79428 ± 0.0005
Jodrell Bank (U. of Man).............	Thomson *et al.* (1961)	408	499.01 ± 0.02	149 600 000 ± 5000	8.7940 ± 0.0003
U. S. S. R...........	Kotelnikov *et al.* (1961)	700	499.0095 ± 0.002 (**)	149 599 300 ± 600 (**)	8.79408 ± 0.00003 (**)
			(499.0141 ± 0.002) (*)	(149 600 700 ± 600) (*)	(8.79400 ± 0.00003) (*)
	Kotelnikov *et al.* (1962)		499.01 ± 0.03	149 600 000 ± 9000	8.7940 ± 0.0006
Jicamarca (NBS)......	Klemperer (1962)	49.92	499.0017 ± 0.001	149 597 000 ± 300	8.79422 ± 0.00002

(*) Estimated in present paper (*see* section 6).
(**) Root-mean-square error.
Note. — None of the errors shown includes a contribution from the uncertainty (about 1 part in 10^6) in c or from the uncertainty (at least 2 parts in 10^6) in R_o.

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these experiments, the accuracy of the computations is as important as that of the measurements, the different groups engaged in radar astronomical research should compare calculations as well as processed results.

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