## Forum

# Standardization Within NATO on the Method of Expressing Navigation Accuracies 

Wing Commander D. W. Broughton mbe, raf

At the Royal Institute of Navigation conferences NAV 84 and NAV 85 considerable interest was shown in the achievement of standardization in expressing navigational accuracy. As a result permission was sought and granted to publish the NATO Standardization Agreement on the subject in the journals of all IAIN members. The text of the Agreement is hereby presented.

NATO Standardization Agreements (stanags) are formal documents promulgated by the Military Agency for Standardization, NATO HQ, Brussells. For each stanag nations are requested to make three formal declarations:

Ratification - 'The declaration by which a nation formally accepts the content of a Standardization Agreement'.

Implementation - 'The fulfilment by a nation of its obligations under a Standardization Agreement'.

Reservation - 'The stated qualification by a nation which describes that part of a Standardization Agreement which it cannot implement or can implement only with limitations'.
stanag 4278, 'Method of Expressing Navigation Accuracies', was promulgated as Edition 2 on 29 April 1986. It has been ratified by Belgium, Canada, Denmark, France, Germany, Greece, Italy, Netherlands, Norway, Turkey, the United Kingdom and the United States of America. It has already been implemented by most of these nations and none has expressed a reservation.

The stanag is offered as a contribution to standardization on a wider basis. NATO would appreciate comments on the content ; these should be addressed to the Director, Royal Institute of Navigation.

Stanag 4278
(Edition 2)
NATO STANDARDIZATION AGREEMENT (STANAG)
Method of expressing navigation accuracies
Annexes
A. Method of Expressing Navigation Accuracies
B. Tables Relating Various Measures of Navigation Accuracy

Aim

1. The aims of this agreement are:
(a) To standardize the method of stating navigation accuracy in NATO operational and requirements documents.
(b) To ensure that new NATO technical documents which give statements of navigation performance include standard expressions of navigation accuracy

## Agreement

2. Participating nations agree:
(a) Whenever navigational accuracies are discussed among NATO members in requirements or operational matters, the accuracies will be described in the following statistical terms.
(1) Linear (single dimension) accuracy. The measure will be given in metres as the distance centred on the true value within which is contained 95 per cent of observations.
(2) Two or three-dimensional accuracy. The measure will be given in metres as the radial distance centred on the true position of a large number of trials which includes 95 per cent of observations.
(3) Radio navigation aids. When it is desirable to express the performance of radio navigation systems, independent of geometrical factors, the measure will represent the limit in appropriate dimensions (e.g. Hertz) within which 95 per cent of observations are contained.
(4) Velocity or speed and angles or angular rates. If performance statements include velocity, speed, angles or angular rates, the measure will represent the limit in appropriate dimensions (e.g. kilometres per hour) within which 95 per cent of observations are contained.
(5) Time and distance-dependent navigation performance. For autonomous navigation systems such as inertial and doppler navigation systems, where errors accumulate over time or distance, the measure will be the distance or radius per unit of time or a percentage of distance travelled which contains 95 per cent of observations (e.g. i kilometre per hour or i per cent of distance travelled). For aircraft inertial navigation systems, accuracy will normally be represented in two dimensions as the radius in kilometres after one hour without up-date from external sources which contains 95 per cent of observations.
(6) All statements of navigation accuracy will indicate the number of dimensions applicable to the statement.
(7) If the system performance under assessment has an error distribution whose mean is removed from zero, the statement will include the magnitude and direction of the bias.
(8) If the system performance under assessment has an error distribution which varies significantly from the Standard (or Gaussian) Distribution, the deviation will be reported.
(9) Input relationships. For systems whose accuracy is affected by the quality of input data (such as Transit, where an error in velocity input results in an error in estimated position) the relationships should be described (e.g. $\circ \cdot 1$ n.m. position error for each i m.p.s. velocity input error) as well as the 95 percentile limit of the input source (e.g. $3 \mathrm{~m} . \mathrm{p} . \mathrm{s}$. in the above example) so that the probable performance of the integrated system can be assessed.
(1o) Precise time and time interval. Where precise time or time interval data are provided, the time or time interval accuracy will be expressed as a single dimensional number (microseconds, nanoseconds, etc.) encompassing 95 per cent of the observations from a large number of trials.
(b) New and revised documentation treating navigation system accuracy will include the measures agreed to above.
(c) This agreement does not preclude the use of additional customary methods of expressing navigation performance, but seeks to ensure that standardized measures are available for comparison and that misunderstandings caused by different terms of expression are avoided.

## Implementation of the agreement

3. This agreement is considered implemented when national instructions have been issued to direct use of standard measures of accuracy in NATO documentation.

## ANNEX A

## Method of expressing navigation accuracies

## 1. Summary

When the precision of a navigational device is stated, it is desirable to add the degree of confidence that may be attributed to the indicated figure. While in reality the error distribution is more often elliptical than circular, it is simpler to cite only one parameter. Moreover, a single parameter is usually sufficient for operational, as opposed to technical, decision making. The most common measure is the radius of the circle, centred on the point observed, within which the navigator has $x$ per cent of chances to actually determine his position. The same concept, when useful, may be extended to speed. This document outlines a proposal to enhance the quality of information exchanged concerning navigation systems accuracies. This enhancement may be obtained by implementing consistency and clarity in the statement of statistics associated with navigational performance parameters. Recommendations are made to establish a single level of confidence for the exchange of data within NATO, and to report significant deviations from the idealized distribution of trials.

## 2. Background

2.1. There are various methods of stating navigation system accuracies. The variations in accuracy statements stem from the fact that several statistical measures of performance (or measures of errors) are in common use. The method of analysis and assumptions made by evaluators may introduce inconsistencies between performance measures using the same basic statistical technique. So that information exchanged between NATO members may have maximum utility, it is desirable to identify which statistical method is used in performance statements; this should include the degree of confidence, expressed as the percentage of trials found within the specified radius, associated with that measure.
2.2. This subject area has been discussed as length by $\mathrm{AC} / 302(\mathrm{SG} / 4)$ and there is general agreement that an orderly method of presenting navigational accuracies should be standardized. It was further agreed that no one statistical method should be used for all statements of accuracy.

## 3. Discussion

3.1. This discussion treats a few of the presently popular statistical measures of performance. Strong and weak points are discussed and some cautionary remarks concerning their applications are made. The instrumentation for evaluating the navigation system's performance need not be taken into account.
3.2. Two types of accuracy measures are commonly cited in requirements and system specification documents; the choice of measure depends upon the purpose to be satisfied or the consequences of failure. The measures are:
'Type A', mission achievement - the interval containing the greatest concentration of trials;
'Type B', safety considerations - the interval containing virtually all trials.
3.2.1. Type ' $A$ ' measures, where only one dimension is involved, are typically either the linear error probable, LEP (by definition, half of the interval containing half of the trials, regardless of how they are distributed), or the standard deviation (the square root of the variance or the root mean square (r.m.s.) error). It should be noted that the contents (number of trials within an r.m.s. distance) are dependent on the statistical distribution. The advantage of r.m.s. measures is that they are suitable for root sum square addition. If the distribution is known, the probable error and the r.m.s. measures are related by a simple constant.
3.2.2. For two-dimensional position measures, say, $x$ and $y$, the errors are often aggregated and the equivalent measures are circular error probable, CEP, and root-mean square distance 2 d r.m.s. If it is reasonable to assume that the two marginal standard deviations ( $\sigma_{x}$ and $\sigma_{y}$ ) are equal, that the $x$ and $y$ axes cross at right angles, and that the two are distributed normally (Gaussian), it is possible to formulate some useful relationships.

$$
2 d \text { r.m.s. }=\sqrt{ } 2 \sigma_{x}=\sqrt{ } 2 \sigma_{y}
$$

or approximately $1 \cdot 1774 \sigma_{x}$

$$
\mathrm{CEP}=\sqrt{ }(2 \log 2) \sigma_{x}
$$

3.2.3. It can be shown that the radial errors, under the assumed conditions of $\sigma_{x}=\sigma_{y}$, etc., have a Rayleigh distribution. For this to be useful, it must be assumed that the magnitude, but not the direction, of the error is important. It is interesting to note that the peak of this distribution is at $1 \sigma_{x}$ (or $1 \sigma_{y}$ ). That is, errors of this size will be the most frequent. From the cumulative Rayleigh distribution curve, it should be noted that within the $1 \sigma_{x}$ (or $1 \sigma_{y}$ ) radius, only about 39 per cent of the trials are included, and that for $\sqrt{ } 2 \sigma_{x}$ (that is, 2 d r.m.s.) 63 per cent are encompassed. Thus, the percentage within an r.m.s. distance in a plane is different from the percentage on a line.
3.3. The process of aggregation could be carried to three dimensions ( $x, y$ and $z$ ). Then the measures of performance would be spherical error probable, SEP, and three-dimensional r.m.s. or $3^{d}$ r.m.s. However, this higher level of combining errors is questionable, since different equipment is usually employed for measuring altitude, that is, $z$, than is employed for measuring position $x-y$. Also, the payoff (or consequences) of accuracy (or errors) is often different for the third dimension. Indeed, if $\sigma_{x}$ and $\sigma_{y}$ are not equal (as assumed earlier) a distorted impression may be obtained by aggregation in even two dimensions. Operational opportunities may arise if it is known which is the smaller, $\sigma_{x}$ or $\sigma_{y}$. Along-track and cross-track errors are then appropriate.
3.4. Fifty-percentile measures are often used in weapon effects descriptions and are common practice in other engineering areas where concentration of effect is the issue. For example, in electronics, filter bandwidth and antenna beam width are usually given as measured at the half-power points. If freedom from interference is the issue, some other measure is used corresponding to some very high level of attenuation. This corresponds to the Type B measure.
3.4.1. The percentage of trials to be included within a Type B measure is sometimes arbitrarily set at some value (examples can be found for 90 per cent, 95 per cent and 99.95 per cent), or multiples of CEP or 2 d r.m.s. may be chosen.
3.4.2. Although 50 per cent is often used for the Type A measure and 95 per cent for the Type B measure, a single measure, say, 95 per cent, would probably be acceptable to all users and would minimize confusion. It should be noted that, where there is reason to believe that the error distributions along two axes are basically normal and orthogonal, commonly accepted practice is to approximate the radius of a circle containing 95 per cent of the trials by multiplying $\sigma_{x}$ (or $\sigma_{x}$ ) by a factor of 2.5 .
3.5. Provided certain critical assumptions are met, it is possible to convert from one to another of the common measures of accuracy. The necessary assumptions and approximate conversion factors are presented as Tables I and 2 in Annex B to this stanag. It should be noted that the necessary assumptions are seldom met, and that these conversions should only be made if precise data are not required. Tables 1 and 2 are useful for rough comparisons between systems or trial results of the same system when data are presented in different forms.

## 4. Radio Navigation Signals

To facilitate the exchange of technical information on a radio navigation system, such as Loran or Omega, it is sometimes preferable to use a unit of measurement given by the system itself - for example, centicycles or microseconds - rather than distance. This represents a measure of the radio-signal performance - transmitter and propagation effects. Accuracies expressed in such units should be at some agreed-upon level of confidence, say 95 per cent. This makes possible a useful partial expression of system performance. Complete expression requires taking into account the geometrical relationship of the user to the system chain.

## 5. Speed

Exactness of speed (rate of movement, independent of direction) is linked to mission performance. Speed errors affect the results during critical phases of flight such as delivering weapons, landing aircraft and avoiding collisions. To be consistent with position-measurement accuracy statements, a figure can be used to represent the deviation from the desired speed; for example, $0.05 \mathrm{~m} / \mathrm{s}$ with a confidence of 95 per cent. This figure will represent the magnitude of the error in three-dimensional space without regard to direction.

## 6. Conclusions

The 'correct' choice of a measure depends upon the application. If safety is the issue, some detailed statistical analysis may be required. If effectiveness is the concern, CEP or 2 d r.m.s. (or LEP or $\sigma$ ) is probably adequate; higher percentile levels are more conservative, but within the added radius very little effect is added. A single uniform measure, set at a high enough value to satisfy safety considerations, would be desirable. In addition, multiples of the simpler measures should be used with caution - particularly if they were arrived at from field data. Such data are often ' edited' to eliminate 'outliers', which are tentatively explained as probably due to weapon duds or electronic lock-on failures. Also, since contents are distribution-sensitive, assumptions such as 'normally distributed ' may prove to be inadequate when multiples are used. (Field data are often fitted to a convenient model such as 'normal', with the fit good only in the central region.) For the exchange of data regarding radio-navigation system performance, probabilistic measures may be used without taking into account geometrical factors. The same concepts that are used to deal with position errors can be extended to speed errors.

## 7. Recommendations

j.i. Whenever navigational accuracies are exchanged/discussed among NATO members, a single number will be used to state explicitly the statistical error (or accuracy).
7.1.2. For one-dimensional or linear error, that number will be the interval containing 95 per cent of the observations. This equates to approximately twice the standard deviation or three times the linear error probable (LEP).
7.1.3. For two- or three-dimensional radial error, this number will represent the radial distance derived from a large number of trials which includes 95 per cent of the observations. Under standard assumptions the 95 per cent circle described by this statistic has a radius equal to about $2.45 \sigma_{x}$ (or $\sigma_{y}$ ) or 2.1 CEP.
7.1.4. When it is desirable to express the performance of radio navigation systems, independent of geometrical factors, performance will be stated as 95 per cent probabilistic measure in terms of portions of a cycle or second.
7.1.5. Speed accuracy will be expressed as a dimensioned number (e.g. $\mathrm{km} / \mathrm{h}, \mathrm{m} / \mathrm{s}$, kt ) of such magnitude that 95 per cent of the observations from a large number of trials will fall within an interval plus or minus one-half of that number. For example, if the speed accuracy of a system is stated as 2 kilometres/hour and the indicated speed provided by the system is 100 kilometres/hour, one is 95 per cent confident that actual speed lies between 99 and 101 kilometres/hour.
7.1.6. The navigational accuracy of an autonomous system such as an inertial navigation system cannot be stated as a single number, since errors accumulate over time or distance. The navigational accuracy of an autonomous system such as an inertial or doppler navigation system is to be stated as distance per unit time or percentage of distance travelled, respectively, without up-date.
7.1.7. All statements of navigational accuracy will indicate whether the distribution described by the 95 per cent statistic has one, two or three dimensions.
7.2. There are some circumstances which will require additional data. These circumstances include the following.
7.2.1. When a system-performance distribution has a mean that is significantly removed from the desired location, the bias magnitude and direction should be reported.
7.2.2. When a system-performance distribution has a significant number of results observed to be far removed from the 'centre', and whose failure to conform is related to the system performance, the location and frequencies of these deviations should be reported.
7.3. The procedures outlined above are designed to standardize the expression of operational performance data. The procedures are not adequate for expression in technical reports precisely describing system performance. However, technical reports will state the computed interval or radial containing 95 per cent of observations in order to expediate incorporating into summary or operational documents. These procedures are intended to make the statement of navigation accuracy independent of the underlying statistical distribution and so avoid the need for complex statistical measures of little use to non-technical tacticians and managers.

## ANNEXB

Tables relating various measures of navigation accuracy

Table 1. MUltiplying factors for approximate conversions between measures of ACCURACY

| To | From |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Linear |  | Planar |  |
|  | LEP | $\sigma$ | CEP | $2 \mathrm{dr.m.s}$. |
| Linear |  |  |  |  |
| LEP | 1 | 0.67 | 0.57 | 0.48 |
| $\sigma$ | 1.5 | 1 | 0.85 | 0.71 |
| Planar |  |  |  |  |
| CEP | $1 \cdot 7$ | 1.2 | , | 0.83 |
| 2d r.m.s. | $2 \cdot 0$ | 1.4 | I. 2 |  |

Table 2. Portion of trials contained in multiples of standard measures of accuracy

|  | Factor |  |  |
| :---: | :---: | :---: | :---: |
|  | $\times 1$ | $\times 2$ | $\times 3$ |
| Measure | $\times 1$ |  |  |
| Linear |  |  |  |
| LEP | $0.5^{\mathrm{b}}$ | 0.82 | 0.96 |
| $\sigma$ | 0.68 | 0.95 | $0.997^{\mathrm{c}}$ |
| Planar |  |  |  |
| CEP | $0.5^{\mathrm{b}}$ | 0.94 | $0.998^{\mathrm{c}}$ |
| 2d r.m.s. | 0.63 | 0.98 | $0.9999^{\mathrm{c}}$ |

For example: LEP $=0.67 \sigma$
(a) Assuming that: bias errors do not exist; distributions are normal (Gaussian). And, further, for two-dimensional distributions, that: axes of the distribution are orthogonal; variances of the orthogonal components of the distribution are equal ; errors along the orthogonal axes are not correlated.
(b) By definition.
(c) Carried to extra precision to show that the figure is not I .

