ORBITAL CHANGES IN PLANET-COMET ENCOUNTERS

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Abstract. We concentrate on the traditional model of capture from the Oort Cloud. We improve the method of calculation of captured comet populations by combining large numbers of accurate orbit calculations with Monte Carlo methods. As a first step in the development of a Monte Carlo code we calculate accurate probabilities for changes of orbit in a single planet-comet encounter. These will be later used to study the orbital evolution of the Oort Cloud comets when multiple encounters take place.

1. Introduction

New comets come from the Oort Cloud (Oort, 1950) to the inner Solar system at a steady rate. Usually they arrive in nearly parabolic orbits, and suffer changes in their orbit, then return to a large distance. Sometimes, the new orbit is much more strongly bound to the Sun than the old one and the comet may even become a short period comet. Everhart (1972) estimates that all known short period comets could originate from the Oort Cloud via several strong perturbations of this kind while Joss (1973), Duncan *et al.* (1988) and Wetherill (1991) see problems with the idea. In particular, the inclinations of the Jupiter family comets are only around 10 degrees, in contrast to models where the captured Oort Cloud comets have considerably higher inclinations. Even though the capture process is inclination dependent (Valtonen and Innanen, 1982; Valtonen *et al.*, 1992), the dependence

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does not appear strong enough to explain the properties of the Jupiter family (Fernández and Gallardo, 1993). Therefore, it has been suggested that there may be another source region for the short period comets, the so-called Kuiper Belt (Kuiper, 1951), which consists entirely of low inclination comets just outside our planetary system. As yet, all observed Kuiper Belt objects are far larger than typical cometary nuclei, but the population is expected to include small members as well. The problem is rather if the present flux of objects entering into Neptune-crossing orbits suffices to guarantee a significant influx into the Jupiter family via multi-planet captures (Morbidelli *et al.*, 1996).

2. Numerical Method

The integration of orbits of millions of comets over the lifetime of the Solar system is still a problem which is beyond the capacity of the existing computers. Here we use the method introduced by Rickman and Vaghi (1976), and later used by Everhart (1977), Froeschlé and Rickman (1980), Stagg and Bailey (1989) and Fernández and Gallardo (1994), where accurate data from comet–planet encounters are used to provide the basic cross sections for comet–planet scattering events (Zheng, 1994). Even though the method used here is not very different from earlier work, the very extensive data base of orbital changes makes the current calculation comparable to direct integration of orbits, except for the great saving of time when orbits where 'nothing happens' are not calculated. Considering the chaotic nature of orbits with multiple scatterings, and the unavoidable accumulation of errors in very long orbit integrations, the current method may even be considered optimal for the problem, not merely the fastest 'short cut' in use.

The calculation and storage of information from single comet–planet encounters is discussed by Valtonen *et al.*, (1992) and Zheng (1994). Here, we explain mainly the calculating method of single comet-planet encounters (for detail see Zheng, 1994).

We suppose a single model planet with Jupiter's mass around the Sun (mass $M_{\odot} = 1$) with a circular orbit. The orbital elements are: semi-major axis $a_J = 1$, inclination $i_J = 0$ and eccentricity $e_J = 0$. A comet comes with parabolic orbit and with inclination $i = i_0$, perihelion distance $q = q_0$. Because the planet has a circular orbit, the encounter can be described by only two angular variables: ω – perihelion of the comet and ϕ – phase angle of the planet. Here, ω represents the relative position of cometary orbit to Jupiter's orbit, ϕ gives Jupiter's position when the comet passes the orbit of the planet. After an encounter with certain values of ω and ϕ, the comet would have a new orbit – semi-major axis a decided by the energy-change ΔE, new inclination i_{new} and new perihelion distance q_{new}.

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For a certain orbit with initial inclination i_0 and initial perihelion distance q_0, there is a largest negative energy-change encounter (also a largest positive one). This appears on the $\omega - \phi$ plane as a point $P_O(\omega_0, \phi_0)$ and at any other point of the plane the comet would not lose such big amount of energy. Because of the continuation of energy-change around P_O, we can find several closed curves on which the energy-changes are equal. Because any point $P(\omega, \phi)$ inside this curve produces an energy-change stronger than that value, the size of the area surrounded by the closed curve represents the probability of these encounters which produce energy-change bigger than that corresponding value ΔE_1:

$$\sigma(|\Delta E| \geq |\Delta E_1|) = area/(2\pi \cdot \pi)$$

For certain i_0 and q_0, we have calculated several iso-energy curves by approximate method. On the same iso-energy curve, different points produce the same semi-major axis but give different inclination i and perihelion distance q. We have recorded all values of new inclination and new perihelion distance for all points located on the iso-energy curves and calculated the distribution functions (probabilities) of the new inclinations and the new perihelion distances. Then for a certain set of i_0 and q_0 and certain value of energy-change, we have a probability function of the new inclination. We also have the probability function of the new perihelion distance, but we can get it by using Tisserand's condition which is satisfied very well (about 8 decimals) in our integrations.

We have studied the combinations of following values of i_0 and q_0: $i_0 =$ 2°, 5°, 30°, 60°, 90°, 120°, 150° and 175°, and $q_0 = 0.1, 0.3, 0.5, 0.7, 0.9$ and 0.95 Jupiter's radii. We have obtained the coefficients of the probability functions to fit the distributions of energy-changes and the distributions of new inclinations. We put these coefficients as tables and use them to get the probability functions for Monte Carlo simulation. The results have been published in Zheng (1994).

Here, we would like to point out that the orbital energy change of a comet, due to an encounter with a planet, can be either positive or negative. There are upper limits both for positive and negative energy changes from the original orbit with elements a_0, i_0 and q_0. Generally, for the same original orbit, the cross-sections of positive and negative energy changes are not the same, and neither are the largest energy changes the same. They are compared with each other in Figure 1 which shows the cross-section of negative energy changes divided by the sum of the cross sections of positive and negative energy changes larger than $(40\,\mathrm{AU})^{-1}$. We find Everhart's (1972) "capture region" as JF-region on the corner $i_0 = 0$ and $q_0 = 1$. Another region of "Halley's type" is also marked as HT-region

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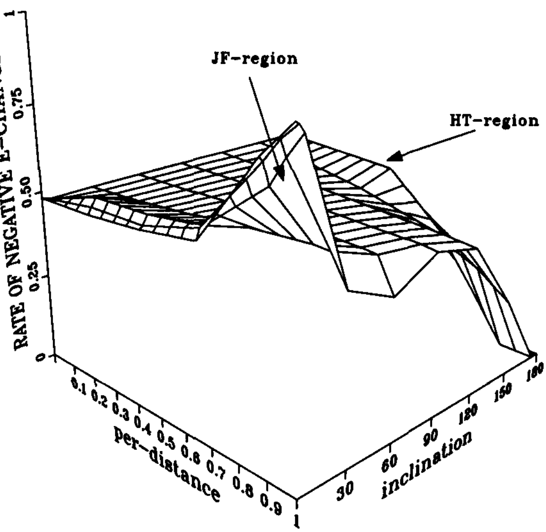

Figure 1. The ratio of the probabilities of negative energy changes over the sum of the cooresponding positive and negative energy changes when Δ E over $(40\,\mathrm{AU})^{-1}$.

in Figure 1. This figure gives us a general idea of the escape and capture processes of comets.

3. Probability of Orbital Energy Change

In order to get cross-sections of weaker encounters, we have simulated single encounter cases by direct a Monte Carlo method. For a certain i_0 and q_0, we choose 100 000 model comets with a random distribution of initial orbits and record their energy changes after one encounter with Jupiter. These results give us the same values as the previous method for close encounters. For weaker encounters, the cross-sections of negative and positive energy changes turn out to be almost the same. We would suppose that the closer encounters with major planets play a great role in forming short period comets, but we should keep our mind open and study weaker encounters because most encounters are not close.

By using the Monte Carlo method we have also simulated the cases of $i_0 = 0°, 0.°5, 1°$ and $177°, 178°, 179°$ to get more information about coplanar systems.

For the range of q_0, we have used $q_0 = 0.99, 1.01, 1.05, 1.10, 1.50$ and 1.80 Jupiter's radii. The probabilities of positive energy changes decrease very fast with q_0. Figure 2 gives us the probabilities of energy changes when $q_0 = 0.99\,a_J$ with $i_0 = 0°$ and $180°$. We can see that the escape probability

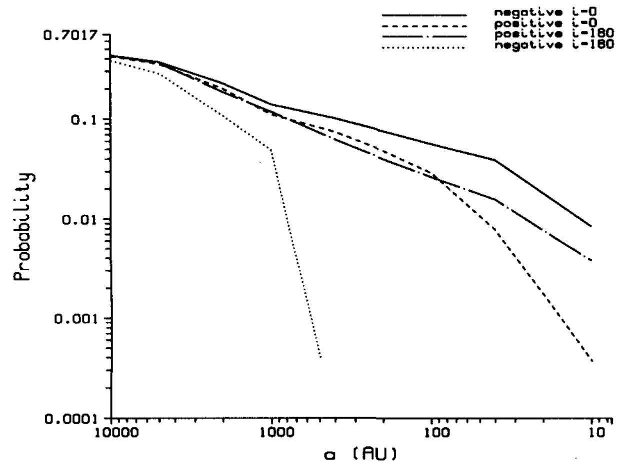

Figure 2. Probability of change of semi-major axis of comet from infinity to a. Single encounter with Jupiter and $q_0 = 0.99\, a_J$. Solid line: $i_0 = 0°$, negative energy changes; dashed line: $i_0 = 0°$, positive energy changes; chain-dotted line: $i_0 = 180°$, positive energy changes; and dotted line: $i_0 = 180°$, negative energy changes.

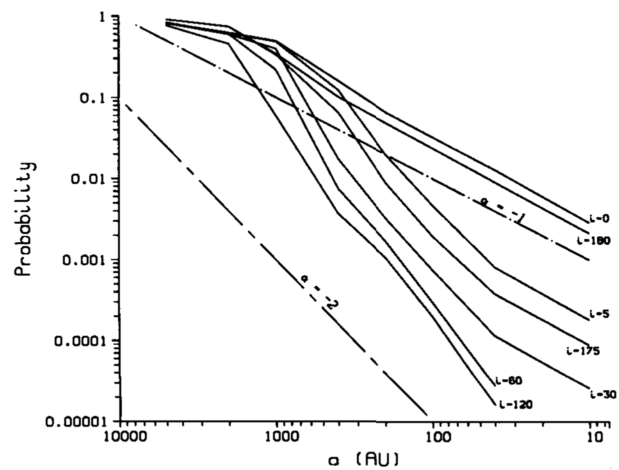

Figure 3. Probability of energy change from 0 to $1/a$. Single encounter with Jupiter and $q_0 = 0.1\, a_J$. The lines are labelled by the value of the initial inclination i_0. The power-law slopes of -1 and -2 are also indicated.

is much bigger than capture probability for retrograde orbits when the energy change is large. For direct orbits with low inclination, it is just the opposite case.

In Figure 3, we give the total probabilities P (the positive energy changes plus the negative energy changes) when $q_0 = 0.1$ with different i_0. We have also drawn the $\alpha = -1$ and $\alpha = -2$ power-law lines for $P \propto (\Delta E)^\alpha$. We

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can see that α is near -2 for most of the cases ($60° < i_0 < 175°$), but when i_0 is near 0° or 180°, it is about -1. For i_0 between 5° and 30°, α is -1 for encounters with $\Delta E < (40\,\text{AU})^{-1}$ and is -2 with $\Delta E > (40\,\text{AU})^{-1}$.
In previous work (Zheng *et al.*, 1996) we have used the power-law index $\alpha = -2$ throughout but we are currently incorporating the proper variation of P as a function of a in the code. The results of this work will be reported elsewhere.

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